

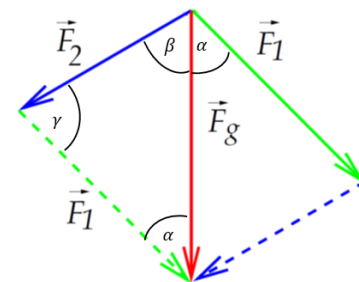
Exercises and Complements for the Introduction to Physics I
 for Students
 of Biology, Pharmacy and Geoscience

Sheet 3 / 5.10.2020

Solutions

Exercise 11.

First, a parallelogram of forces is formed by moving F_1 or F_2 parallel, resulting in a triangle of F_1 , F_2 and the total force F_g .



The angle γ can be calculated by:

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 45^\circ - 60^\circ = 75^\circ$$

According to the sine theorem it follows:

$$\frac{F_1}{\sin \beta} = \frac{F_2}{\sin \alpha} = \frac{F_g}{\sin \gamma}$$

Therefore:

$$F_1 = F_g \frac{\sin(\beta)}{\sin(\gamma)} = 0.5 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \frac{\sin(60^\circ)}{\sin(75^\circ)} = 4.4 \text{ N}$$

$$F_2 = F_g \frac{\sin(\alpha)}{\sin(\gamma)} = 0.5 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \frac{\sin(45^\circ)}{\sin(75^\circ)} = 3.6 \text{ N}$$

Exercise 12.

a) The total system of masses m_1 and m_2 is accelerated by the difference in the downhill forces of each individual car. It follows:

$$F_{D1} = F_G \cdot \sin \alpha = 100 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \sin 20^\circ = 335.52 \text{ N}$$

and

$$F_{D2} = F_G \cdot \sin 2\alpha = 100 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \sin 40^\circ = 630.57 \text{ N}$$

So, the acceleration of the cars is:

$$a = \frac{(F_{D2} - F_{D1})}{m} = \frac{(630.57 \text{ N} - 335.52 \text{ N})}{200 \text{ kg}} = 1.48 \text{ m/s}^2$$

b) For $\Delta t = 10 \text{ s}$:

$$v = a \cdot \Delta t = 14.8 \text{ m/s}$$

Exercise 13.

(a) For the uniformly accelerated movement applies:

$$s(t) = 0.5 \cdot a \cdot t^2$$

For free fall, the fall height h corresponds to the distance and the gravitational constant g to the acceleration. So:

$$h(t) = 0.5 \cdot g \cdot t^2$$

Therefore for t follows:

$$t(h) = \sqrt{\frac{2h}{g}}$$

(b) Inserting results in:

$$t(1 \text{ m}) = \sqrt{\frac{2 \cdot 1 \text{ m}}{1.62 \text{ m/s}^2}} = 1.11 \text{ s}$$

(c) The velocity is:

$$v = at = g\sqrt{\frac{2h}{g}} = \sqrt{2hg} = \sqrt{2 \cdot 1 \text{ m} \cdot 1.62 \text{ m/s}^2} = 1.8 \text{ m/s}$$

Exercise 14.

(a) The geostationary orbit has to rotate with the same angular velocity around the Earth as the Earth itself is rotating.

$$\omega = \frac{2\pi}{86400} = 7.27 \cdot 10^{-5} \frac{1}{\text{s}}$$

The satellite just stays on a circular path if the centrifugal force F_C (script 103-7) is equal the gravitational force F_G (script 103-5):

$$\begin{aligned} F_C &= F_G \\ m\omega^2 r &= \gamma \frac{mM}{r^2} \\ \omega^2 r &= \gamma \frac{M}{r^2} \end{aligned}$$

m mass of the satellite

M mass of the Earth

γ constant of gravitation

r distance to the center of the Earth

from this it follows that the distance to the Earth's center is:

$$r = \sqrt[3]{\frac{\gamma M}{\omega^2}} = 42300 \text{ km}$$

and the distance to the Earth's surface is therefore 36000 km.

(b) The orbital plane of the satellite has to go through the Earth's center. In the cross sectional view with the Earth's surface, it is a great circle. The satellite is really geostationary only if the great circle is at the equator. Otherwise, the satellite would oscillate with a period of one day between northern and southern hemisphere.

Exercise 15.

Regarding to the conservation of rotational momentum it is:

$$L_{\text{before}} = L_{\text{after}}$$

With respect to the moments of inertia and the angular velocity this results in:

$$J_{\text{before}} \cdot \omega_{\text{before}} = J_{\text{after}} \cdot \omega_{\text{after}}$$

After inserting, it follows:

$$\frac{2}{5} m_{\text{earth}} r_{\text{before}}^2 \cdot \frac{2\pi}{T_{\text{before}}} = \frac{2}{5} m_{\text{earth}} r_{\text{after}}^2 \cdot \frac{2\pi}{T_{\text{after}}}$$

Cancelling results in:

$$\frac{r_{\text{before}}^2}{T_{\text{before}}} = \frac{r_{\text{after}}^2}{T_{\text{after}}}$$

Therefore:

$$T_{\text{after}} = T_{\text{before}} \cdot \frac{r_{\text{after}}^2}{r_{\text{before}}^2}$$

We know, that $r_{\text{after}} = 0.6 \cdot r_{\text{before}}$. So, it follows:

$$T_{\text{after}} = T_{\text{before}} \cdot 0.6^2 = 24 \text{ h} \cdot 0.36 = 8.64 \text{ h}$$