

Exercises and Complements for the Introduction to Physics I
for Students
of Biology, Pharmacy and Geoscience

Sheet 5 / 19.10.2020

Solutions

Exercise 21.

At the height of $h = 2000$ m the potential and the kinetic energy are equal:

$$mgh = \frac{mv^2}{2} \quad \Rightarrow \quad v = \sqrt{2gh} = 198 \text{ m/s}$$

The initial velocity is described by:

$$v = \sqrt{-2gh + v_0^2} \quad \Rightarrow \quad v_0 = \sqrt{2}v = 280 \text{ m/s}$$

Exercise 22.

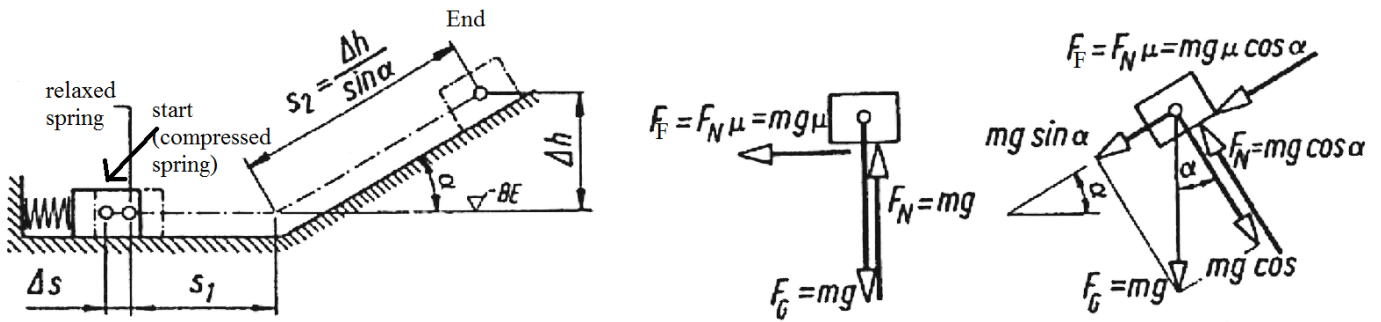
Lifting and friction work have to be performed. The first one is given by mgh , and the second one by $\mu F_N s$, where $F_N = mg \cos \alpha$ is the normal force and $s = h / \sin \alpha$ is the distance of the path on the inclined plane is:

$$\begin{aligned} W &= mgh + \mu mgh \frac{\cos \alpha}{\sin \alpha} = mgh(1 + \mu \cot \alpha) \\ &= mgs(\sin \alpha + \mu \cos \alpha) = 61.85 \text{ kJ} \end{aligned}$$

The lifting work depends only on the difference in height h from the starting and the final position of the movement. The frictional work depends on the actual covered distance s . The gravitational force is conservative and the frictional force is non-conservative.

Exercise 23.

a) Sketch:



b) The energy at the end of the movement E_E is equal to the kinetic energy E_A , at the beginning, minus the loss due to the friction:

$$E_E = E_A \pm W_{+,-}$$

$$mg\Delta h = 0 + \frac{k}{2}\Delta s^2 - mg\mu(s_1 + \Delta s) - mg\mu \cos \alpha \frac{\Delta h}{\sin \alpha}$$

$$\Delta h = \frac{\frac{k}{2}\Delta s^2 - mg\mu(s_1 + \Delta s)}{mg(1 + \mu \cot \alpha)} = 1.65 \text{ m}$$

Exercise 24.

For the initial velocity of the object it follows from the law of conservation of momentum:

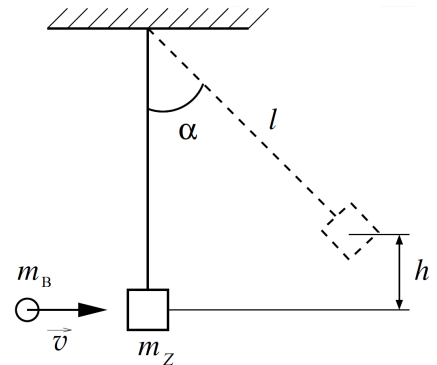
$$v_0 = \frac{m_B v}{m_Z + m_B}$$

The height (see figure) can be calculated from the law of energy conservation:

$$\frac{(m_Z + m_B)v_0^2}{2} = (m_Z + m_B)gh \Rightarrow h = \frac{v_0^2}{2g}$$

In conclusion:

$$\cos \alpha = 1 - \frac{h}{l} = 1 - \frac{m_B^2 v^2}{(m_Z + m_B)^2 2gl} \Rightarrow \alpha = 73^\circ$$



Exercise 25.

a) System is in equilibrium, so nothing happens.

b) We zero the total energy in the resting state = 0 (any other value or constant is also possible since it gets later canceled out anyway). If the mass m_1 moves downwards by the distance x , then:

$$m_1gx - m_2gx + \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{x}^2 = 0$$

$$2g(m_2 - m_1)x = (m_1 + m_2)\dot{x}^2$$

$$2g\frac{m_2 - m_1}{m_1 + m_2}x = \dot{x}^2$$

The derivative with respect to the time is:

$$2g\frac{m_2 - m_1}{m_1 + m_2}\dot{x} = 2\dot{x}\ddot{x}$$

Divide by \dot{x} , $\dot{x} \neq 0$

$$\ddot{x} = a = g\frac{m_2 - m_1}{m_1 + m_2}$$

Alternative solution:

$$Z - m_1g = m_1a$$

$$m_2g - Z = m_2a$$

with Z as tension force. Since the wheel has no friction, the value of the tension force in the rope is the same. Consequently we obtain the same result:

$$a = g\frac{m_2 - m_1}{m_1 + m_2}$$