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Exercises and Complements for the Introduction to Physics I for Students of Biology, Pharmacy and Geoscience

Sheet 6 / 26.10.2020 **Solutions**

Exercise 26.

(a) In this case the equations for an elastic collision are valid, according to them the velocity is given by: $v_1' = -v_2'$. The negative sign indicates that the objects are moving in opposite directions. From the equations (4-5) and (4-6) in Trautwein page 39 it results for v_1' and v_2':

$$\frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2} = \frac{(m_2 - m_1)v_2 + 2m_1v_1}{m_1 + m_2}$$

$$\frac{(m_1 - m_2)v_1 + 0}{m_1 + m_2} = \frac{0 + 2m_1v_1}{m_1 + m_2}$$

$$\frac{(m_1 - m_2)v_1}{m_1 + m_2} = \frac{2m_1v_1}{m_1 + m_2}$$

$$(m_1 + m_2)v_1 = -2m_1v_1$$

$$m_1v_1 - m_2v_1 = -2m_1v_1$$

$$-m_2v_1 = -3m_1v_1$$

$$m_2 = 3m_1$$

$$\Rightarrow m_2 = 6 \text{ kg}$$

(b) According to the previous equation it follows:

$$v_1' = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}$$

$$\Rightarrow v_1' = -3.35 \text{ m/s}$$

Since $v_1' = -v_2'$ the velocity of the second object is $v_2' = 3.35$ m/s. It is also possible to calculate v_2' directly from the equation for v_2' mentioned in (a). The absolute value of the velocity for both objects is 3.35 m/s.

Exercise 27.

The correct answer to the first question is a) and to the second question b).

In both cases, the momentum of the ball changes when it hits the block. Before the impact, both impulses are the same, but after the impact they are different because the rubber ball performs an elastic collision, i.e. rebounds, while the aluminium ball inelastically deforms the block. The momentum of the aluminium ball is completely transferred to the block, which provides the necessary impulse to stop it. In the case of the rubber ball, however, the impulse transmitted from the block is greater, since the block not only has to deliver the impulse to stop the ball, but also an additional impulse to throw the ball back. Therefore, the rubber ball is much more likely to knock the block over.

In the second part of the question, the rubber ball transmits the greatest momentum to the block, but does not deliver the most energy. If the ball rebounds at a fairly high speed, it means that it retains a lot of kinetic energy, while the aluminium ball stops and therefore releases all of the kinetic energy as deformation work.

Therefore, the rubber ball gives off a lot of momentum but only a little bit of energy to the block, while the aluminium ball gives a lot more energy but less momentum to the block.

So, it is essential to distinguish between momentum and energy.

Exercise 28.

The common velocity v' of the vehicles after the crash (inelastic collision) results from the law of conservation of momentum:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v' \quad \text{then} \quad v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

The energy which gets transformed into heat in this process is:

$$\Delta E = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} - \frac{(m_1 + m_2)v'^2}{2}$$

(a) Given in the problem was that: $m_1 = m_2 = m$, $v_1 = v$ and $v_2 = -v$. From this it follows $v' = 0$ and $\Delta E = mv^2$, i.e. the original available energy of the vehicles $E_{kin} = 2 \cdot (mv^2/2)$ is completely used for the deformation of the vehicles.
(b) In this case it was given that $v_1 = 2v$ and $v_2 = 0$. Under these conditions the original available kinetic energy is $(m/2)(2v)^2 = 2mv^2$, twice as much as in (A). From this, it follows that $v' = v$ and $\Delta E = mv^2$. Accordingly the same amount of the kinetic energy is used for the deformation as in (a). Since the initial energy was higher, after the collision each vehicle has a kinetic energy of $mv^2/2$.

Exercise 29.

Due to the conservation of angular momentum it is necessary that the momenta for the outstreched arms $L_0 = J_0\omega_0$ and with the arms closer to the body $L_1 = J_1\omega_1$ have to be equal, $L_0 = L_1$.
By solving this we obtain $\omega_1 = \omega_0 \cdot \frac{J_0}{J_1}$.

For the moments of inertia we calculate:

$$J_0 = J_P + J_C + 2mr_0^2 = 1.95 \text{ kg} \cdot \text{m}^2 + 0.27 \text{ kg} \cdot \text{m}^2 + 2 \cdot 2 \text{ kg} \cdot (0.75 \text{ m})^2$$

and

$$J_1 \;=\; J_P + J_C + 2mr_1^2 \;=\; 1.95\text{ kg}\cdot m^2 + 0.27\text{ kg}\cdot\text{m}^2 + 2\cdot 2\text{ kg}\cdot(0.1\text{ m})^2$$

and with $\omega_0 \;=\; 1\,\frac{\pi}{s}$ we obtain $\omega_1 \approx 2\,\frac{\pi}{s}$.

Exercise 30.

(a) The centripetal force acting on the dust particle can be calculated by:

$$F_Z = mr\omega^2 = 4\pi^2\cdot m\cdot r\cdot f^2 = 4\pi^2\cdot 10^{-5}\,\text{kg}\cdot 0.06\,\text{m}\cdot(100\,\text{Hz})^2 = 0.24\,\text{N}$$

(b) The rotational energy can be estimated by:

$$E_{rot} = \frac{1}{2}\cdot J\cdot\omega^2$$

Since the CD can be considered as a flat square cuboid, the moment of inertia can be calculated by:

$$J_{CD} = \frac{1}{2}\cdot m\cdot r^2$$

Therefore E_{rot} is:

$$E_{rot} = \frac{1}{2}\cdot\frac{1}{2}\cdot m\cdot r^2\cdot 4\pi^2 f^2 = mr^2\pi^2 f^2 = 5.33\,\text{J}$$

(c) The angular momentum of the CD can be calculated by:

$$L_{CD} = J\omega = \frac{1}{2}mr^2\cdot 2\pi f = 0.015\,\text{kg}\cdot(0.06\,\text{m})^2\cdot\pi\cdot 100\,\text{Hz} = 0.02\,\frac{\text{kg m}^2}{\text{s}}$$

(d) Because of the conservation of angular momentum, the angular momentum of the CD L_{CD} is equal to that of the player L_{Player}.

$$L_{CD} = L_{Player}$$

This results in:

$$\frac{L_{CD}}{J_{Player}} = 2\pi f_{Player}$$

The moment of inertia of the player can be seen as a cuboid:

$$J_{Player} = \frac{1}{12}\cdot m\cdot(a^2+b^2)$$

Therefore, the frequency of the player is:

$$f_{Player} = \frac{L_{CD}}{2\pi\cdot\frac{1}{12}\cdot 0.5\,\text{kg}\cdot\left[(0.15\text{ m})^2+(0.15\text{ m})^2\right]} = 1.44\,\text{Hz}$$