

Exercises and Complements for the Introduction to Physics II
 for Students
 of Biology, Pharmacy and Geoscience

Sheet 3 / 18.03.2021

Solutions

Exercise 9.

(a) The capacity:

$$C_1 = \varepsilon_0 \frac{A}{d_1} = 1.1 \text{ nF}$$

(b) The charge stays constant, thus:

$$Q = U_1 C_1 = U_2 C_2$$

$$C_1 = \varepsilon_0 \frac{A}{d_1} \quad \text{and} \quad C_2 = \varepsilon_0 \frac{A}{d_2}$$

Consequently:

$$U_2 = \frac{U_1 d_2}{d_1} = 100 \text{ V}$$

(c) For a serial connection the following applies:

$$\frac{1}{C_1} = \frac{1}{C_x} + \frac{1}{\varepsilon C_1}$$

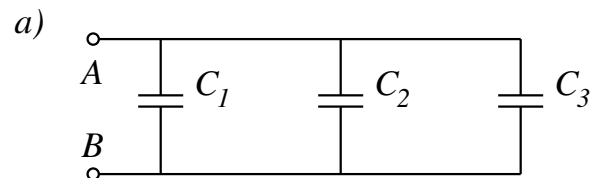
And therefore:

$$C_x = \frac{\varepsilon C_1}{\varepsilon - 1} = 2.1 \text{ nF}$$

Exercise 10.

(a) The circuit can be simplified as shown in the figure. Therefore, for the parallel connection the following applies:

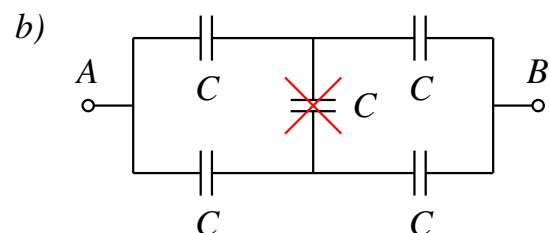
$$C_{AB} = C_1 + C_2 + C_3$$



(b) Similar to (a) the circuit can be simplified (see Figure). Then we realize that there is no potential difference at the capacitor in the middle. Thus, this capacitor can be left out for the calculation.

For the individual serial connections (top and bottom) we get:

$$\frac{1}{C_{t,b}} = \frac{1}{C} + \frac{1}{C}$$



Hence, for the entire circuit we have:

$$C_{AB} = \frac{C}{2} + \frac{C}{2} = C$$

Aufgabe 11.

a) The 20-pF-capacitor will be labelled with index 1 in the following while the 50-pF-capacitor will be labelled with index 2.

Because no charge is lost when connecting the two capacitors, the charge of the first capacitor is equal to the sum of the charges of the connected capacitors.

$$Q_{1,disconn} = Q_{1,conn} + Q_{2,conn} = C_1 U_{conn} + C_2 U_{conn}$$

$$U_{conn} = \frac{C_1}{C_1 + C_2} U_{1,disconn}$$

Thus, for the charges on the capacitors we obtain:

$$Q_{1,conn} = C_1 U_{conn} = \frac{C_1^2}{C_1 + C_2} U_{1,disconn} = \frac{(20 \text{ pF})^2}{20 \text{ pF} + 50 \text{ pF}} \cdot 3 \text{ kV} = 17.14 \text{ nC}$$

$$Q_{2,conn} = C_2 U_{conn} = \frac{C_1 C_2}{C_1 + C_2} U_{1,disconn} = \frac{20 \text{ pF} \cdot 50 \text{ pF}}{20 \text{ pF} + 50 \text{ pF}} \cdot 3 \text{ kV} = 42.86 \text{ nC}$$

b) For the electric energy it follows:

$$E_{el,1,disconn} = \frac{1}{2} C_1 U_{1,disconn}^2 = \frac{1}{2} \cdot 20 \text{ pF} \cdot (3 \text{ kV})^2 = 90 \text{ } \mu\text{J}$$

c) After being connected the capacitors have the following energy:

$$E_{el,1,conn} + E_{el,2,conn} = \frac{1}{2} C_1 U_{1,conn}^2 + \frac{1}{2} C_2 U_{2,conn}^2 = \frac{Q_{1,conn}^2}{2 \cdot C_1} + \frac{Q_{2,conn}^2}{2 \cdot C_2} = \frac{(17.14 \text{ nC})^2}{2 \cdot 20 \text{ pF}} + \frac{(42.86 \text{ nC})^2}{2 \cdot 50 \text{ pF}} = 25.7 \text{ } \mu\text{J}$$

Exercise 12.

The case shown in the left schematic can be regarded as a parallel connection of two capacitors. The total capacity is then obtained by:

$$C_{\text{left}} = C_{\text{air}} + C_{\text{diel.}} = \varepsilon_0 \frac{A}{d} + \varepsilon_0 \varepsilon_r \frac{A}{d} = \varepsilon_0 \frac{0.075 \text{ m}^2}{0.03 \text{ m}} + \varepsilon_0 \cdot 2.1 \cdot \frac{0.075 \text{ m}^2}{0.03 \text{ m}} = 68.6 \text{ pF}$$

The case shown in the right schematic can be regarded as a serial connection of two capacitors. Here, the total capacity is obtained by:

$$C_{\text{right}} = \left(\frac{1}{C_{\text{air}}} + \frac{1}{C_{\text{diel.}}} \right)^{-1} = \left(\frac{d}{\varepsilon_0 \cdot A} + \frac{d}{\varepsilon_0 \varepsilon_r \cdot A} \right)^{-1} = \left(\frac{0.015 \text{ m}}{\varepsilon_0 \cdot 0.15 \text{ m}^2} + \frac{0.015 \text{ m}}{\varepsilon_0 \cdot 2.1 \cdot 0.15 \text{ m}^2} \right)^{-1} = 60.0 \text{ pF}$$