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Exercises and Complements for the Introduction to Physics II for Students of Biology, Pharmacy and Geoscience

Sheet 6 / 06.04.2022 **Solutions**

Exercise 21.

(a) See Script 313-2.

$$\overline{P} = I_{eff} U_{eff} \cos\varphi$$

$$\cos\varphi = \frac{\overline{P}}{I_{eff} U_{eff}} = 0.68 \qquad \Rightarrow \qquad \varphi = \arccos\left(\frac{\overline{P}}{I_{eff} U_{eff}}\right) = 47.2°$$

(b) For the phase-shift in the RLC-circuit in series:

$$\tan\varphi_2 = \frac{Z_L - Z_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{\omega L}{R} - \frac{1}{\omega RC}$$

Without capacitance in the RL-circuit :

$$\tan\varphi_1 = \frac{Z_L}{R} = \frac{\omega L}{R}$$

follows

$$\tan\varphi_1 - \tan\varphi_2 = \frac{1}{\omega RC} \qquad \Rightarrow \qquad C = \frac{1}{2\pi f R(\tan\varphi - \tan\varphi')}$$

Here $\omega = 2\pi f$, $\varphi_2 = \arccos(0.9) = 25.8°$, and R is resistance. For the resistance R the following holds:

$$P = I_{eff} U_{eff}\ \cos\varphi = I_{eff}^2 R\ \cos\varphi$$

For self-inductance and capacitances the phase shift between current and voltage is $\pm\pi/2$, hence $\overline{P} = 0$. These are therefore dissipationless switching elements. The active power $\overline{P}$ in the RLC-series circuit dissipates only at the ohmic resistance, since there $\varphi = 0(\cos(0) = 1)$, and therefore

$$P = I_{eff} U_{eff}\ \underbrace{\cos\varphi}_{=1} = I_{eff}^2 R\ . \quad \text{And finally using} \quad U_{eff} = R I_{eff},$$

$$C = \frac{I_{eff}^2}{2\pi f \overline{P}(\tan\varphi_1 - \tan\varphi_2)} = 2.43 \text{ mF},$$

Here we plugged in the values $\bar{P} = 22$ kW, or alternatively $R = 2.2\ \Omega$, without using the last substitution.

Exercise 22.

(a) For the oscillation period we get:

$$\begin{aligned}\omega &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L(C_1+C_2)}} \\ &= 8,94 \cdot 10^3\,\mathrm{Hz} \\ T &= \frac{2\pi}{\omega} = 7.02 \cdot 10^{-4}\,\mathrm{s}\end{aligned}$$

(b) For the energy of a capacitor:

$$\begin{aligned}E_C &= \frac{1}{2}CU^2 \\ \Rightarrow E_{C_1} &= \frac{1}{2}C_1 \cdot U = 3.24 \cdot 10^{-2}\,\mathrm{J} \\ E_{C_2} &= \frac{1}{2}C_2 \cdot U = 4.86 \cdot 10^{-2}\,\mathrm{J}\end{aligned}$$

(c) The following applies to the energy of a coil:

$$E_L = \frac{1}{2}LI^2$$

From conservation of energy law, it follows:

$$\begin{aligned}E_L &= E_{C_1} + E_{C_2} \\ \frac{1}{2}LI^2 &= E_{C_1} + E_{C_2} \\ I^2 &= \frac{2 \cdot (E_{C_1} + E_{C_2})}{L} \\ \Rightarrow I &= \sqrt{\frac{2 \cdot (E_{C_1} + E_{C_2})}{L}} \\ &= 8.05\,\mathrm{A}\end{aligned}$$

Exercise 23.

(a) Resonance is an important special case of a driven oscillation. Resonance occurs when an oscillating system is driven by a periodically acting external force, and the frequency of the external force coincides with the natural frequency of the oscillating system. The amplitude of the driven oscillation reaches the highest possible value.
Ohm;s law:

$$I_0 = \frac{U_0}{Z}$$

where Z is impedance. In the resonant case I_0 becomes maximal $\Rightarrow$ Z muss be at its minimum. Impedance in the RLC-circuit

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

becomes minimal when

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1.02 \text{ kHz}$$

(b)

$$I_0 = \frac{U_0}{Z} = \frac{U_0}{\sqrt{R^2 + \underbrace{\left(\omega L - \frac{1}{\omega C}\right)^2}_{=0 \text{ at resonance}}}} = \frac{U_0}{R} = 100 \text{ mA}$$

(c)

$$U_{C,0} = I_0 Z_C = \frac{U_0}{R} Z_C = \frac{U_0}{R}\frac{1}{\omega_0 C} = \frac{U_0}{R}\frac{\sqrt{LC}}{C} = \frac{U_0}{R}\sqrt{\frac{L}{C}} = 156 \text{ V}$$

Exercise 24.

Neglecting losses, the voltages U_1 and U_2 at the primary and secondary coils are directly proportional to the number of turns, and the currents I_1 und I_2 are inversely proportional to the number of turns:

$$\left|\frac{U_1}{U_2}\right| = \left|\frac{I_2}{I_1}\right| = \frac{N_1}{N_2} = 20$$

From which follows:

$$|U_2| = \frac{|U_1|}{20} = 125 \text{ V}$$

$$|I_1| = \frac{|I_2|}{20} = 4 \text{ A}$$

The powers in the primary and secondary sides are:

$$P = U_1 I_1 = U_2 I_2 = 10 \text{ kW}$$