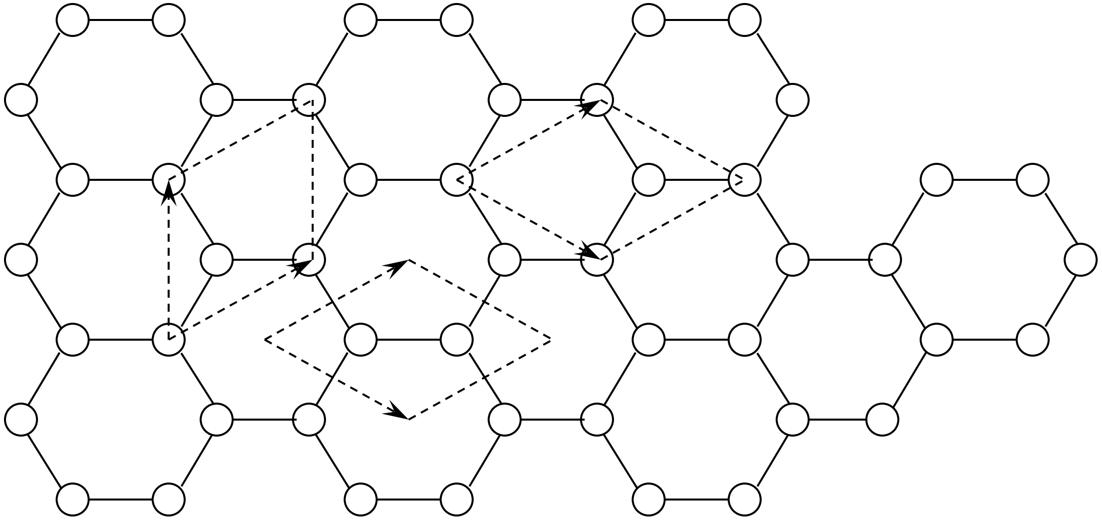
Problem 1

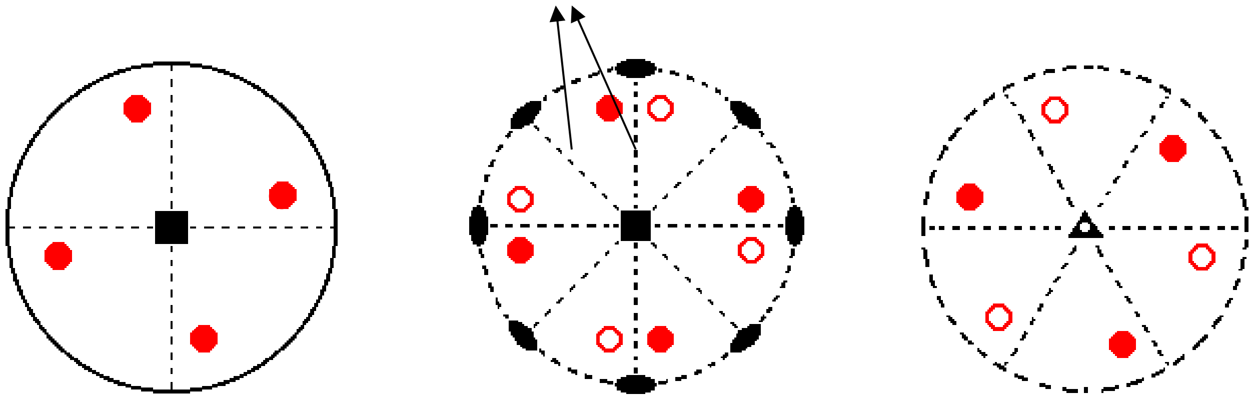


A **primitive basis** is a crystallographic basis of the vector lattice **L** such that every lattice vector **t** of **L** may be obtained as an integral linear combination of the basis vectors, **a**, **b**, **c**.

A **primitive cell** is a unit cell built on the basis vectors of a primitive basis of the direct lattice

Problem 2

The two axis here are different from each other.

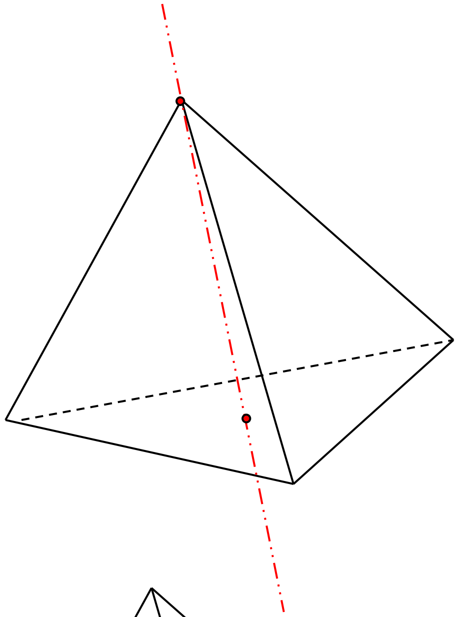


C_4 D_4 C_{3i} (S_6)

D_4 is also called 422 because it is composed of 4-fold rotation and 2 different 2-fold rotation

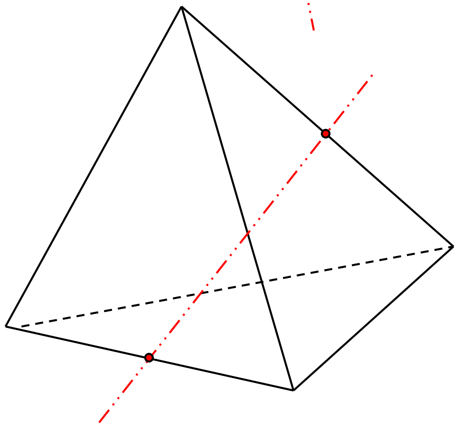
- Central symbol: rotational symmetry around vertical axis
- Symbols on edge: rotational symmetry around horizontal axes
- Lines through centre point: mirror planes
- Inner symbols (red in these diagrams): faces on the crystal
    - filled circle: lower face
    - open circle: upper face

Problem 3a



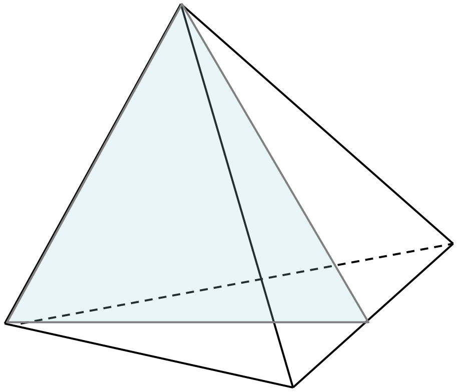
4 x C_3 rotations

Axis from one apex to the centre of the opposite face through the centre of the tetrahedron



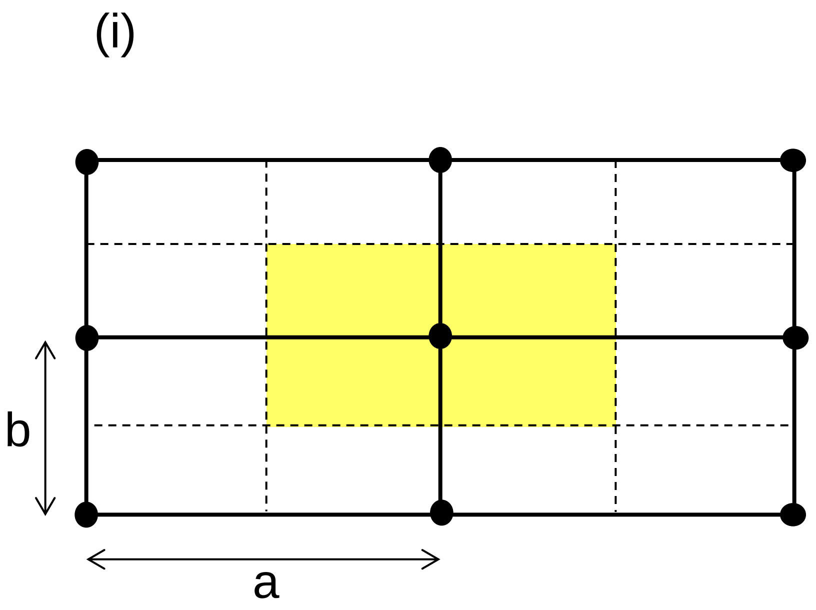
3 x C_2 rotations

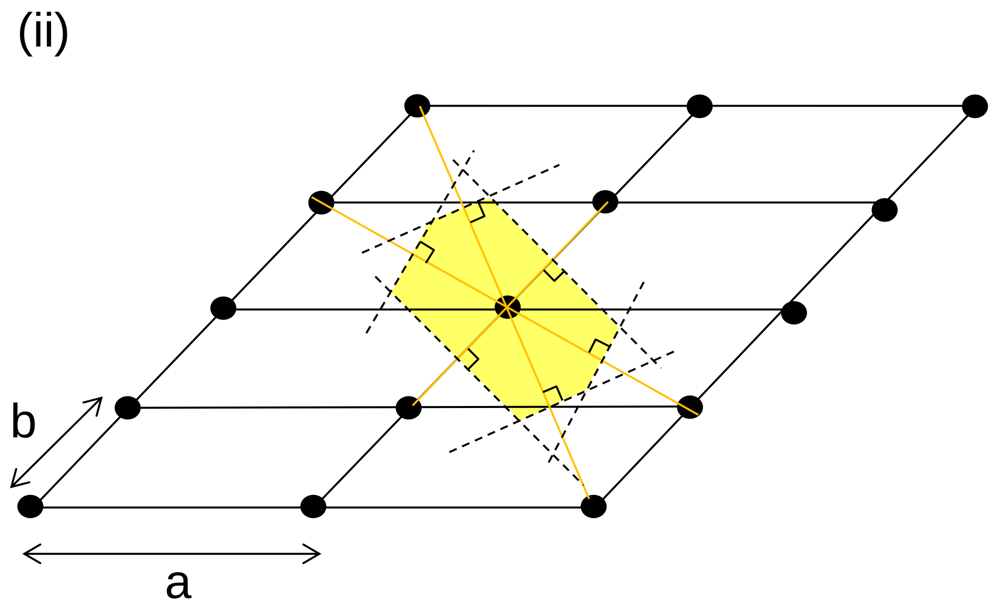
Axis from midpoint on one side to the midpoint of the opposite side



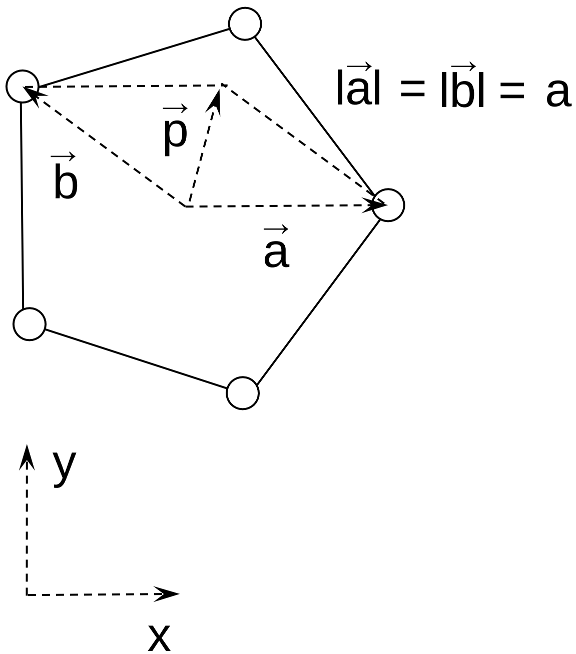
6 x reflections

Problem 3b





Problem 4



Obviously,

$$\vec{a} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \vec{b} = a \begin{pmatrix} \cos\theta_b \\ \sin\theta_b \end{pmatrix} \qquad \theta_b = \sigma * 4\pi/5$$

And,

$\vec{p} = \vec{a} + \vec{b}$ should still be a part of the lattice under translation operation of a Bravais Lattice

However,

$$|\vec{p}| = a\sqrt{\left(1+\cos\frac{4\pi}{5}\right)^2 + \sin^2\frac{4\pi}{5}} = 0.62a < a$$

This means that it is not a part of the lattice, which contradicts with our expectation. So we may come to the conclusion that a 5-fold rotation symmetry is not compatible with the translation properties of a Bravais Lattice.

Problem 5

Some symmetry transformations of the crystal are also shown here:

Rotation by $\theta = 60^{o}$, $C_6 = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}_{\theta = n\times 60^{o}}$

Mirror symmetry : $\sigma_x = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The conductivity tensor is invariant under these symmetry transformations of the crystal:

$$\sigma = C^{-1}\sigma C \Rightarrow \sigma_x^{-1}\sigma\sigma_x = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -\sigma_{11} & \sigma_{12} \\ -\sigma_{21} & \sigma_{22} \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_{11} & -\sigma_{12} \\ -\sigma_{21} & \sigma_{22} \end{pmatrix} \equiv \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

no info about σ_{11}, σ_{22}; but from

$\sigma_{12} = -\sigma_{12} \Rightarrow \sigma_{12} = 0$

$\sigma_{21} = -\sigma_{21} \Rightarrow \sigma_{21} = 0$

$\therefore \sigma_{12} = \sigma_{21} = 0$

$$\therefore \sigma = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}$$

$\theta = 60^{o}$

$$\sigma = C_6^{-1}\sigma C_6 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{\sigma_{11}}{2} & -\frac{\sqrt{3}\sigma_{11}}{2} \\ \frac{\sqrt{3}\sigma_{22}}{2} & \frac{\sigma_{22}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sigma_{11}}{4} + \frac{3\sigma_{22}}{4} & -\frac{\sqrt{3}\sigma_{11}}{4} + \frac{\sqrt{3}\sigma_{22}}{4} \\ -\frac{\sqrt{3}\sigma_{11}}{4} + \frac{\sqrt{3}\sigma_{22}}{4} & \frac{3\sigma_{11}}{4} + \frac{\sigma_{22}}{4} \end{pmatrix} = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}$$

e.g. from first element :

$$\frac{1}{4}(\sigma_{11} + 3\sigma_{22}) = \sigma_{11}$$

$$3\sigma_{22} = 3\sigma_{11}$$

$$\therefore \sigma_{11} = \sigma_{22}$$