Condensed matter physics 2013 Exercise 2

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*Problem 6

Demonstrate that the atom-atom distances, the number of nearest neighbors and the filled volume ratio of the simple cubic, bcc, and fcc-structure given in the lecture notes, are correct.

*Problem 7

Sketch the (001)-, (011)- and (111)-plane of the simple cubic lattice and calculate the respective layer spacing. Determine the rotation symmetries of these planes.

*Problem 8

The following are the primitive basis vectors of a hexagonal lattice:

$$\vec{a}_1 = a \cdot \vec{e}_x, \ \vec{a}_2 = \frac{a}{2} \cdot \vec{e}_x + \frac{\sqrt{3}a}{2} \cdot \vec{e}_y \text{ and } \tilde{a}_3 = c \cdot \tilde{e}_z$$

- a) Show that the volume of the primitive cell is $\frac{\sqrt{3}a^2c}{2}$
- **b)** Show that the basis vectors of the reciprocal lattice form again a hexagonal lattice, with the c-axis rotated with respect to the direct lattice.
- c) Sketch the first Brillouin zone (Wigner-Seitz cell in the reciprocal lattice)) of the hexagonal space lattice.

Problem 9

Consider the plane (h_1, h_2, h_3) in a crystal lattice

a) Prove that the reciprocal lattice vector

$$\vec{q} = \sum_{i} h_i \vec{b}_i$$

is perpendicular to the lattice plane (h_1, h_2, h_3) .

- b) Prove that the distance between two consecutive parallel planes of the lattice is given by $d(hkl) = 2\pi/|\vec{q}|.$
- c) Show that for a simple cubic lattice $d^2 = a^2/(h_1^2 + h_2^2 + h_3^2)$.

Problem 10

Silicon (Si) is the most important material of today's semiconductor industry. Here we investigate wet chemical etching of silicon, which can be used to pattern Si-based computer chips.

Crystalline silicon forms a covalently bonded diamond-cubic-like structure. The $\{111\}$ planes, which have the highest atom-packing density, are etched much slower than the other planes. And the sidewalls of an etched pit in single-crystal silicon (SCS) on a (100) surface will ultimately be bounded by this type of plane.

Give the edge directions of these $\{111\}$ plane-bounded inverted pyramids. What is the top angle α of the pyramid?



Figure 1: Problem 10