

# Condensed matter physics 2013

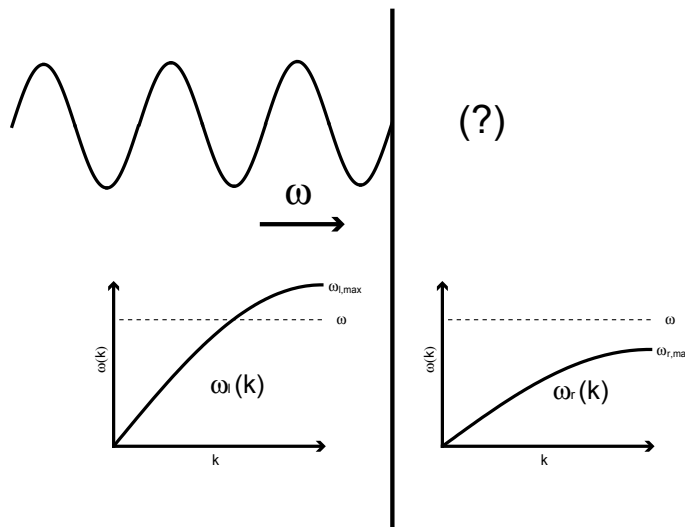
## Exercise 5

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### Problem 19

a) Assume two crystals with different phonon dispersion relations  $\omega_l(k)$  and  $\omega_r(k)$  in perfect contact with each other. Assume a lattice vibration (phonon) with frequency  $\omega$  incident from



the left to the right crystal. What will happen, if  $\omega > \omega_{r,max}$ , meaning that there are no allowed lattice vibrations at the frequency of the incident oscillation in the right crystal ?

b) For a quantitative solution, use a complex Ansatz for the wave vector  $k_r$  in the right crystal. Use the conditions for the reflection and transmission coefficient, that have been derived in the lecture. Use the continuum approximation, and consider only acoustic phonons.

### \* Problem 20 (3)

We consider a linear 1D chain with identical masses  $m$  connected by two different kind of springs to the left and right (force constants  $f_1$  and  $f_2$ ). The distance between two masses is  $a/2$ , the periodicity of the chain is  $a$ . Assume that forces are only exerted between nearest neighbours.

- Find the equation of motion for longitudinal oscillations, and derive the dispersion relation from it. Sketch and discuss the two limits  $k = 0$  and  $k = \pi/a$ . What are the amplitudes at these two points ? Give an expression for the phase- and group velocity  $v_\phi$  and  $v_g$ .
- Discuss the case  $f_1 = f_2 = f$  and show the transition to a linear chain with identical spring constants!
- Compare the above results with the ones derived in the lecture for a linear chain with two different masses  $m_1$  and  $m_2$ .
- Compare the dispersion relation with that of the monatomic linear chain when  $m_1 = m_2$

### \* Problem 21 (1)

Consider a linear KCl-chain with a distance between K and Cl of  $3.1\text{\AA}$  and a spring constant  $f = 8.8\text{ N/m}$ . The atomic weight of K is 39, that of Cl 35.5.

a) How wide are the phonon bands, and how big is the energy gap between the acoustic and the optical branch ?

b) Calculate the wavelength of a photon, that has the same energy as the optical phonons at  $k = 0$  !

### \* Problem 22 (2)

Determine the phonon density of states in k-space  $\rho_k(k)$  for two dimensions! How does the density of states in frequency space  $\rho_\omega(\omega)$  look like for the case of a linear dispersion relation  $\omega \propto k$  (for one, two and three dimensions) ?