Condensed matter physics 2013
Exercise 8

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*Problem 30

In graphene, a two dimensional (2D) sheet of hexagonally packed carbon atoms, the dispersion relation is given by $E = \hbar v_F |\vec{k}|$.

a) Draw this dispersion relation in the space (k_x, k_y, E).

b) Calculate the density of states $\rho(E)$ of graphene. Compare your result with the density of states of a 2D electron gas with quadratic dispersion relation (See script 4.14-4.16)

*Problem 31

Consider a 3-dimensional metal with $n = N/V$ free electrons per volume unit. We will consider free electron theory and the temperature $T = 0$.

a) Calculate the Fermi energy E_F expressed by $n, \hbar, m_e$ (the electron mass).

b) Show that the total energy E of the electrons is given by $E = 3/5NE_F$. Aluminum has a density of 2.70×10^3 kg/m^3, and its molar mass is 26.98 g.

c) Calculate the number of aluminum atoms per unit volume (i.e. cubic nanometer)

d) Use the fact that the Fermi energy is 11.63 eV for Al and the electron mass is $m_e = 9.11 \times 10^{-31}$ kg to find the number density of free electrons.

e) Combine your results to estimate the number of conduction electrons per atom.

f) For Aluminum, the length of the side of the face centered cube is 4.05×10^{-10} m. At zero temperature $T = 0$, the inverse compressibility is defined as $B = -V \left(\frac{\partial p}{\partial V} \right)_N$, where the pressure $p = -\left(\frac{\partial E}{\partial V} \right)_N$. Calculate the electronic contribution to compressibility for the aluminum in [m^2/N].

Problem 32

Look into the Sommerfeld expansion to derive the temperature dependence of the chemical potential μ of a Fermi gas. Use standard textbooks (eg. Ashcroft & Mermin).

Problem *33

Consider two Fermi gases at respective potential μ_L and μ_R with constant density of states which are at the same temperature T and separated by a tunnel barrier. Taking into account the Pauli principle, show that the total current will follow this equation:

$$I \propto \int [f_L(E) - f_R(E)]\, dE \tag{1}$$