Condensed matter physics 2013
Exercise 9

handed out:: 12. Nov. 2013 discussion: 19. Nov. 2013

Problem 34

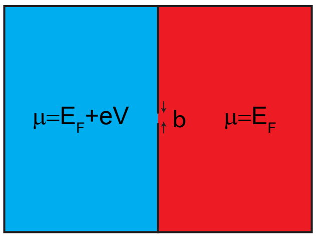
When a metallic cathode with work function Φ is heated to a temperature T, electrons from the metal can overcome the work function barrier and contribute to the so-called thermionic emission current towards the anode. Derive the current density $\mathbf{j}$ for a thermionic emission process.

Hint: Assume that all the electrons moving in the positive x-direction contribute to the current density. In addition, assume that only those electrons with enough thermal energy to overcome the barrier contribute to the thermionic emission current,i.e, we neglect tunnelling of electrons.

Problem *35

A 2D electron gas at T=0 is divided into two parts by a barrier with an opening of width b. The 2D gas is a free electron gas with equal densities on both sides of the barrier. A small potential V is applied on one side.

a)Calculate the electrical conductivity G in units of $G_0 = 2e^2/h$ assuming that $\lambda_F \ll b$ and neglecting reflections in the opening. The inverse of calculated conductance $(1/G)$ is called Sharvin resistance. Hint: Since the wavelength is assumed to be small, the free electron states on both sides and within the opening are characterized with "good quantum numbers" k_x and k_y. You only need to take into account the states which contribute to the conduction from left to right that lie within the energy window $E_F < E < E_F + eV$.



b) could you guess what will happen if $\lambda_F \leq b$

Problem *36

a) Derive a relation for the temperature $(T_0 = f(T_F, \Theta_D, Z))$ above which the phonon contribution to the specific heat exceeds the electronic contribution. T_F is the Fermi temperature, Θ_D is the Debye temperature and Z is the nominal valence of the solid.

b) What is the value of T_0 for silver? Estimate the relative contribution of electrons and the lattice vibrations to the specific heat of silver (at constant volume) at room temperature -300 K (think!) and 4 K. The Fermi energy of silver is 5.5 eV and its Debye temperature is 215 K. The number of valence electrons in silver is 1.

Problem *37

At one point of a 1D conductor with electron gas of density n, a ferromagnetic STM tip approaches and induces a constant spin imbalance, $\Delta n = n_\uparrow - n_\downarrow \neq 0$. The "spin up" and "spin down" electrons are diffusing away at the same rates from this point. At the same time, the spins can also relax from one orientation to the other with some constant probability p which is the same for both orientations. This leads to a decay of spin imbalance with a characteristic relaxation time $\tau = \frac{1}{2p}$. Show that the diffusion equation for the carriers with diffusion constant D is given by

$$\frac{\partial n_i}{\partial t} = D\frac{\partial^2 n_i}{\partial x^2} - p(n_i - n_j) \tag{1}$$

where $(i, j) = (\uparrow, \downarrow), (\downarrow, \uparrow)$. (Hint: Use the continuity equation.)

Solve the above equations in the steady state and find the typical length, λ, of the spin relaxation.