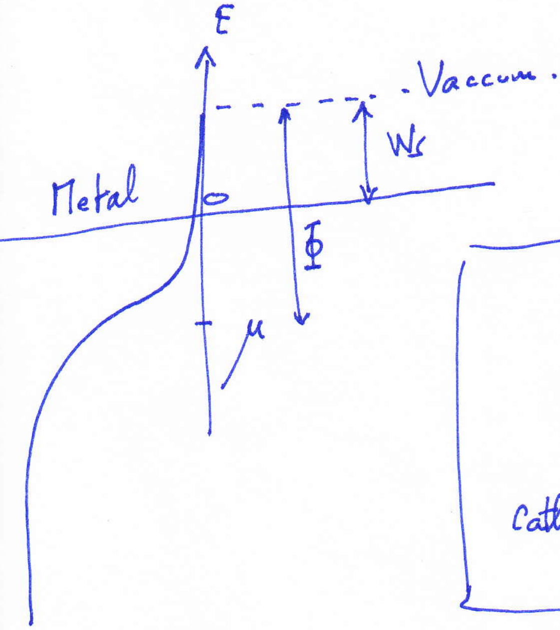
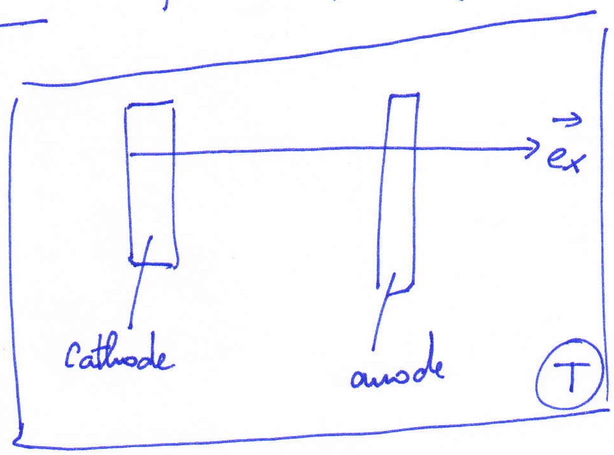
2011 série 9

Probleme 33.

Consider a metallic cathode at T with a work function of Φ.



definition of work function: see Ashcroft/Mermin (p358)



velocity component along $\vec{e}_x$.

$$\boxed{v_x = \frac{\hbar k_x}{m}}$$

Then we can write the current density:

$$j = -e \int_{k_x > 0} \frac{1}{4\pi^3} v_x f(\vec{k}) \, d\vec{k}.$$

By definition $f(\vec{k})$ is the Fermi-Dirac distribution function

$$f(\vec{k}) = \frac{1}{1 + e^{\beta[\varepsilon_n(\vec{k}) - \mu]}} \qquad \text{with } \beta = \frac{1}{k_B T}.$$

in the vacuum between the cathode & the anode we have

$$\varepsilon_n(\vec{k}) = \text{kinetic energy} + \text{potential energy}.$$

$$= \frac{\hbar^2 k^2}{2m} + W_s \quad (\text{see drawing}) \quad W_s = \mu + \Phi$$

$$= \frac{\hbar^2 k^2}{2m} + \mu + \Phi$$

finally $f(\vec{k}) = \dfrac{1}{1 + e^{\beta\left(\frac{\hbar^2 k^2}{2m} + \Phi\right)}}$ at physical temperature for the experiment $T < 10^4\,K$.

we get $\Phi > k_B T$

then we can simplify $f(\vec{k})$ as $e^{-\beta\left(\frac{\hbar^2 k^2}{2m} + \Phi\right)}$.

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then $j = -e \int \frac{\hbar k_x}{4\pi^3 m} e^{-\beta\left(\frac{\hbar^2 k^2}{2m} + \Phi\right)} \vec{dk}$ after we integrate over all possible $\vec{k}$.

$$= -\frac{e\hbar}{4\pi^3 m} \int_{-\infty}^{+\infty} e^{-\beta \frac{\hbar^2 k_y^2}{2m}} dk_y \int_{-\infty}^{+\infty} e^{-\beta \frac{\hbar^2 k_z^2}{2m}} dk_z \int_{k_x>0}^{\infty} k_x e^{-\beta\frac{\hbar^2 k_x^2}{2m}} dk_x \times e^{-\Phi/k_B T}$$

using $\int e^{-u^2} du = \sqrt{\pi}$

we finally get $j = -\frac{e m}{2\pi^2 \hbar^3} (k_B T)^2 e^{-\Phi/k_B T}$.

Probleme 34.

a) We have to calculate the conductance $G = I/V$.

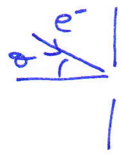
let's consider the current density $j = \frac{I}{b}$

then one can write $\vec{j}_x = e \langle v_x \rangle \underbrace{n_{eV}}_{\text{number of } e^- \text{ between } E_F \text{ and } E_F + eV.}$

$$n_{eV} = \int_{E_F}^{E_F+eV} \rho_{2D}(E)\, dE = \int_{E_F}^{E_F+eV} \frac{m}{\pi \hbar^2}\, dE = \boxed{\frac{m e V}{\pi \hbar^2}}$$

$\hookrightarrow$ script 4.10.

$eV << E_F$ then $v_x = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} v_F \cos\theta \, d\theta = \frac{v_F}{\pi}$



all e^- has $\vec{v}_F$ since $eV << E_F$.

finally we write $I = b \times j_x = \frac{e^2}{\pi^2 \hbar^2} m b v_F V$.

which leads to $G = I/V = \frac{2 m b v_F}{h} G_0$ $\qquad G_0 = \frac{2e^2}{h}$ quantum of conductance

using $\lambda_F = \frac{h}{m v_F}$ $\qquad \boxed{G = \frac{2b}{\lambda_F} G_0}$

b) if $\lambda_F \geq b$ e^- will see the hole as a scattering center. The wave behavior of e^- should be taken into account.

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Problème ~~34~~ 35

a)

in the Fermi gas description:

$$C_V^{el} = \frac{\pi^2}{2}\left(\frac{k_B T}{E_F}\right) n_{el} k_B \quad \text{and for } T < \theta_D \quad C_V^{ph} = \frac{12\pi^4}{5}\left(\frac{T}{\theta_D}\right)^3 n_i k_B$$

Z is the valence of the solid, then we can write $n_{el} = Z n_i$

We are looking for T_0 such that $C_V^{el}(T_0) = C_V^{ph}(T_0)$

$$\Rightarrow \frac{\pi^2}{2}\left(\frac{k_B T_0}{E_F}\right) Z n_i k_B = \frac{12\pi^4}{5}\left(\frac{T_0}{\theta_D}\right)^3 n_i k_B$$

$$\Rightarrow \frac{Z}{2}\left(\frac{k_B T_0}{E_F}\right) = \frac{12\pi^2}{5}\left(\frac{T_0}{\theta_D}\right)^3$$

$$\Rightarrow \frac{T_0}{T_0^3} = \frac{24\pi^2}{Z5} \times \left(\frac{1}{\theta_D}\right)^3 \times \frac{E_F}{k_B} = \frac{1}{T_0^2} = \frac{24\pi^2 T_F}{5 Z \theta_D^3} \qquad \text{with } T_F = \frac{E_F}{k_B}.$$

finally we get $\boxed{T_0 = \left(\frac{5Z}{24\pi^2} \times \frac{\theta_D}{T_F}\right)^{1/2} \theta_D}$

b) For silver $T_F = 63800\,K$ with $E_F = 5.5\,eV$.

$\theta_D = 215\,K \Rightarrow$ we get $\underline{T_0 = 1,81\,K}$

$Z = 1$

Moreover we know that $E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2 n_{el}\right)^{2/3}$ see serie 8 exercice 31-a).

$$\Rightarrow n_{el} = 5,86 \times 10^{28}\ e^-/m^3.$$

<u>At 300K:</u> $C_V^{el} = 18746,25\ J.m^{-3}K^{-1}$

$\theta_D = 215\,K$ then Debye model is not valid anymore

finally we use the law of Dulong and Petit $C_V^{ph} = 3 n_i k_B = 2426040\,J.m^{-3}K^{-1}$

<u>At 4K:</u> $C_V^{el} = 249,95\ J.m^{-3}.K^{-1}$ and $C_V^{ph} = 1218,59\,J.m^{-3}.K^{-1}$.

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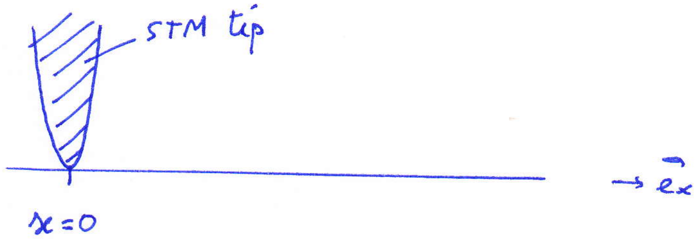
Exercice 36.

Consider a 1D conductor with electron gas density n.

A ferromagnetic STM tip induces a constant spin imbalance at one point of the conductor.

Let's make a nice drawing



Constant spin imbalance $\Delta n = n_\uparrow - n_\downarrow = \boxed{\Delta n_0}$

Generalités:

* First we can write the equation of particules conservation in general (3D)

$$\underbrace{\frac{\partial n(x,t)}{\partial t} dV}_{\substack{\text{variation of } n \\ \text{in the volume } dV}} = \underbrace{j(x,t)S}_{\substack{\text{~~entering~~ incoming} \\ \text{particules}}} - \underbrace{j(x+dx,t)S}_{\text{outgoing particules}}.$$

$$= -\frac{dj}{dx} S$$

* Finally in 1D we get $\boxed{\frac{\partial n}{\partial t} = -\frac{\partial j}{\partial x}}$ (1) when zero particule are created.

* We can also remind the Fick Law in 1D. (Diffusion Law).

$$\boxed{\vec{j} = -D\frac{\partial n}{\partial x}} \text{ (2)}$$

Let's come back to our problem:

with equation (2) we can write $\vec{j}_\uparrow = -D\frac{\partial n_\uparrow}{\partial x}$ spin up diffusion

$\vec{j}_\downarrow = -D\frac{\partial n_\downarrow}{\partial x}$ spin ~~up~~ down diffusion

Equation (1) needs an other term since we have spin-flip process.

$$* \ \frac{\partial n_\uparrow}{\partial t} = -\frac{\partial j_\uparrow}{\partial x} + \underbrace{p n_\downarrow}_{\text{you gain spin }\uparrow\text{ with spin-flip.}} - \underbrace{p n_\uparrow}_{\text{you loose }\uparrow\text{ spin with spin-flip}}$$

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finally we can combine Fick law and particule conservation for the 2 spins.

$$\begin{cases} \dfrac{\partial n_\uparrow}{\partial t} = D\dfrac{\partial^2 n_\uparrow}{\partial x^2} - p(n_\uparrow - n_\downarrow) \\ \dfrac{\partial n_\downarrow}{\partial t} = D\dfrac{\partial^2 n_\downarrow}{\partial x^2} - p(n_\downarrow - n_\uparrow). \end{cases}$$

in the steady state we have $\dfrac{\partial n_\uparrow}{\partial t} = 0$ and $\dfrac{\partial n_\downarrow}{\partial t} = 0$.

By combining the 2 équations one can write:

$$D\left[\frac{\partial^2 n_\uparrow}{\partial x^2} - \frac{\partial^2 n_\downarrow}{\partial x^2}\right] = 2p\left(n_\uparrow - n_\downarrow\right) \Rightarrow \boxed{\frac{\partial^2 \Delta n_{\uparrow,\downarrow}}{\partial x^2} = \frac{1}{\tau D}\Delta n_{\uparrow,\downarrow}.}$$

at $x=0$ $\Delta n = \Delta n_0$.

then $\underline{\Delta n(x) = \Delta n_0 \exp\left(-\dfrac{x}{\sqrt{\tau D}}\right)}$ which leads to a typical

spin relaxation time $\underline{\lambda = \sqrt{\tau D}}$.