Condensed matter physics 2013
Problem series 10

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* Problem 38 – 2P

The quantum Hall effect measurements of a 2D electron gas are shown in Figure (b). The longitudinal R_{xx}, and the Hall (transverse) resistance R_{xy} are measured as a function of the magnetic field in a Hall-bar shaped sample. For fields greater than $B = 2T$, Hall plateaus (labeled with 2 and 4) are clearly visible. The small field ($B < 2T$) measurements are shown in the inset, where the oscillation of R_{xx}, known as Shubnikov de Haas (SdH) oscillations, are visible. Using the periodicity of SdH oscillations, calculate the 2D-carrier density from the available data. The carrier density can also be estimated from the classical Hall effect data (linear low field part of R_{xy} in the inset). Hint: The filling factors (ν) label the corresponding R_{xx} minima, plot ν^{-1} vs B and using the slope calculate the electron density n.

* Problem 39

A 2D electron gas in the shape of a Hall bar (Figure (a)), is placed in a strong perpendicular magnetic field which leads to the quantum Hall effect. We assume a completely filled lowest spin-resolved Landau level and all others are empty, where only one edge channel is present. A quantum point-contact with transmission T is placed in the middle of the Hall bar. Using the four terminal geometry where the current is passed through leads a and b (here: 1 and 4) and voltage is measured between α and β (e.g. 2,3) we define the resistance $R_{ab,\alpha\beta} = U_{\alpha,\beta}/I_{a,b}$. Calculate: a) $R_{14,23}$ and b) $R_{14,63}$ as function of transmission T.

* Problem 40

Consider, at a vanishing temperature, a quadratic 2DEG of both length and width L in the $x - y$ plane to which a magnetic field is applied in z direction. Neglecting the electron spin, show that for

$$B > \frac{h}{e}\frac{N}{L^2}$$

all electrons occupy the lowest Landau level. Plot $\mu(B)$ for a fixed number of electrons.

Hint: Compare the area per state in $\boldsymbol{k}$ space (i. e., the $k_x - k_y$ plane) with and without a magnetic field, respectively. You may use approximations for free electrons where necessary.

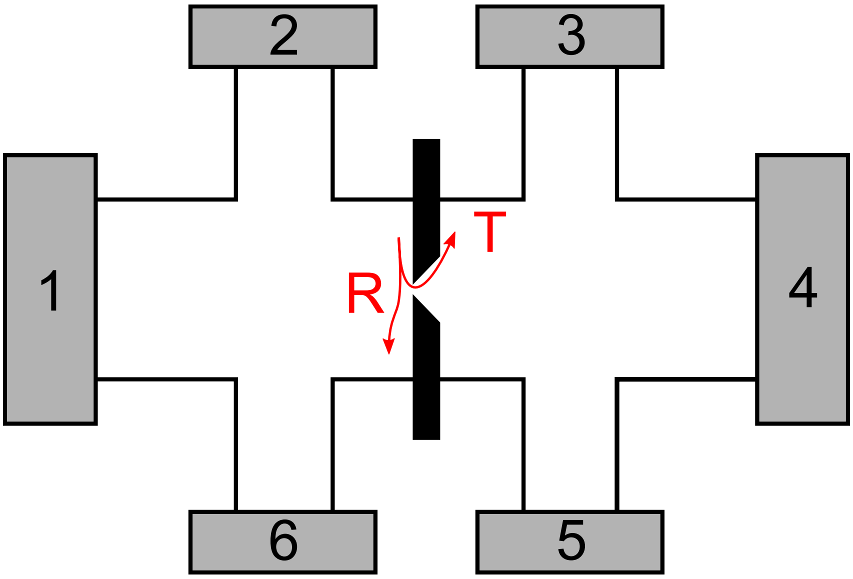

Figure (a). Problem 39

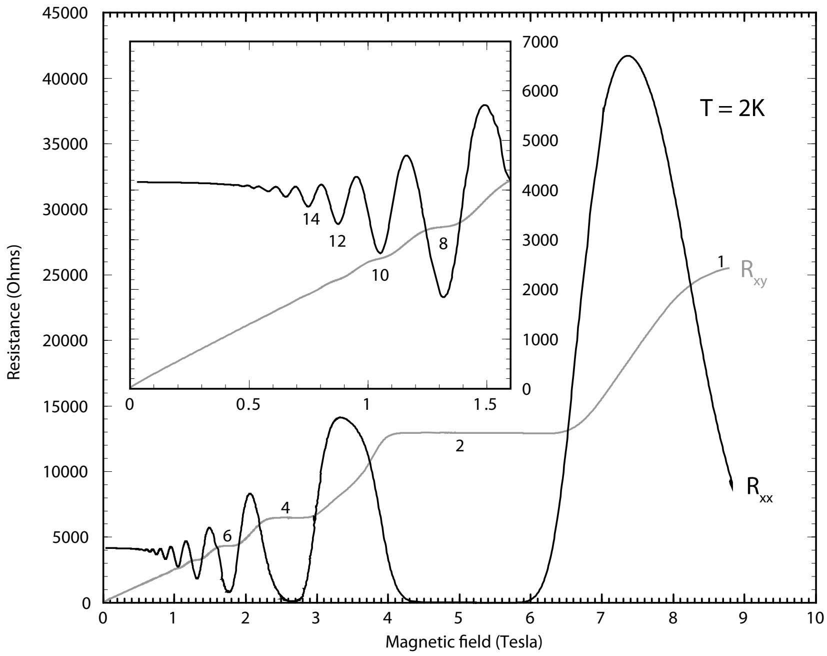

Figure (b). Problem 38