Condensed matter physics 2013
Problem series 12

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Problem 45

The transverse mass $m_t^\star$ and longitudinal mass $m_l^\star$ describe an anisotropic dispersion relation $E(\vec{k}) = \hbar^2/2\left[(k_x^2+k_y^2)/m_t^\star + k_z^2/m_l^\star\right]$, e.g. in Silicon. Use the semi-classical equation of motion $d(\hbar\vec{k})/dt = -e\vec{v}\times\vec{B}$ and show that the cyclotron frequency for a static magnetic field in the (x,y)-plane has the form $\omega_c = eB/\sqrt{m_t^\star m_l^\star}$. Note that these effective masses do not enter different measured properties in the same way.

* Problem 46 (1 point)

The crystal structure of GaAs under standard conditions is the Zinkblende structure, see script p. 1.25 (Fig. 1.26). Using a periodic table of the elements to find the atomic electron configuration, try to predict if GaAs is a metal or an insulator. Discuss in few words how good this prediction is and whether this is observed in nature.

* Problem 47 (5 points)

We use the zero-order crystal wave function Ψ of exercise 38 to estimate the $\vec{k}$-dependence of the crystal's energy. For this we want to find the eigen energy $\epsilon(\vec{k})$ in the Schrödinger equation

$$H_{\text{cryst}}|\Psi\rangle = \epsilon|\Psi\rangle \tag{1}$$

Multiplication from the left with $\langle\psi|$ (the atomic orbital), using the linear combination of atomic orbitals and some tedious, but simple explicit writing-down-equations leads to a series of integrals similar to overlap-, Coulomb- and exchange integrals in atomic physics. The tight binding method is mainly of use if the overlap of the wave functions of nearest neighbors is small, and one finds

$$\epsilon(\vec{R}) = \underbrace{\epsilon_n}_{\text{indep. of }\vec{k}} + \underbrace{\sum_{\vec{R}\neq 0}\gamma(\vec{R})e^{i\vec{k}\vec{R}}}_{\vec{k}\text{-dependent}} \tag{2}$$

a) What is $\vec{R}$ and what determines the width of the band, i.e. the amplitude of the $\vec{k}$-dependent part?
b) In a two-dimensional quadratic lattice with spacing a we assume γ_1 =const. for the wave function at one atom with the nearest neighbors and another constant γ_2 for the next-nearest neighbors. All other γ's we set to zero. The wave function overlaps we assume to be weak. Write down the corresponding tight binding energy as a function of k_x and k_y. Draw this dispersion relation for the two directions in k-space $[1,0]$ and $[1,1]$.

Problem 48

In this exercise we study Bloch oscillations in one dimension, see script p. 6.59. We assume a dispersion relation of the form $E(k) = 1/2\epsilon[1-\cos(ka)]$, with the width of the band ϵ, e.g.

figure 6.31 in the manuscript. We consider an electron starting with $k = 0$, i.e. at the bottom of the band. In a semi-classical picture the electron momentum changes due to an externally applied electric field F by $\hbar\dot{k} = -eF$. Integration yields

$$k(t) = -eFt/\hbar \tag{3}$$

This means that without scattering the electron k-vector increases indefinitely. When it reaches the Brillouin zone, it is folded back to the other side of the zone. Inserting into the expression for the electron group velocity yields

$$v = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a\epsilon}{2\hbar}\sin(ka) = -\frac{a\epsilon}{2\hbar}\sin(eFat/\hbar), \tag{4}$$

which is periodic in time. Integration leads to the position

$$s = \frac{\epsilon}{2eF}\cos(eFat/\hbar) \tag{5}$$

1. Draw a series of qualitative dispersion relations with the electron position in (k, E) for several interesting times t.

2. Typical bands have a width of the order of $\epsilon \approx 1\,\mathrm{eV}$ (see for example script p. 6.41), the largest electrical fields in modern nanostructures are roughly $F \approx 1\,\mathrm{V/nm}$ and we may take $a \approx 1\,\text{Å}$. What are the oscillation frequency and the oscillation amplitude of such a Bloch oscillation?

3. Compare the numbers in a) to the typical Drude scattering time and mean free path in a metal. Draw a schematic of what happens in an inelastic scattering event in the dispersion relation.

Comment: A 'hot' topic in modern semiconductor nanophysics are artificially grown crystals with a periodic structure in one direction ('superlattices'). These can have larger mean free paths and a period given by the experimentalist. In such structures a large variety of effects can be studied, among them Bloch oscillations, see for example T.M. Fromhold et al., Nature **428**, 726 (2004).