

## Physik III Atom- und Quantenphysik

### Kapitel 3: Atome – Teilchen oder Welle



Prof. Dr. Christian Schönenberger  
[www.nanoelectronics.ch](http://www.nanoelectronics.ch)

basierend auf der Vorlesung von  
Prof. Dr. Philipp Treutlein  
<http://atom.physik.unibas.ch>

# Louis de Broglie

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



de Broglie (1892-1987), Nobelpreis 1929



Louis deBroglie

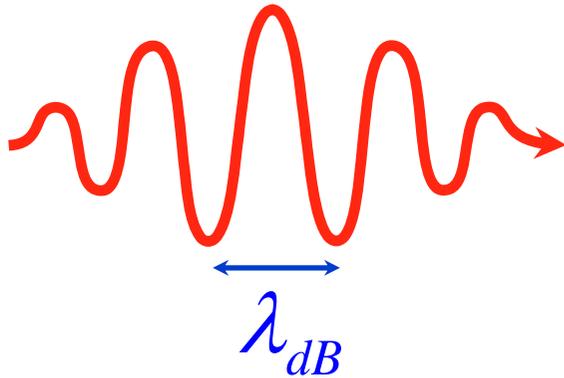
Doktorarbeit 1924

## deBroglie-Wellenlänge

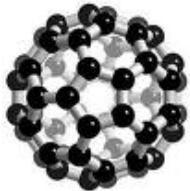
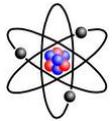
$$\lambda_{dB} = \frac{h}{p} = \frac{h}{m \cdot v}$$

# Louis de Broglie Wellenlängen

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$$\lambda_{dB} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$



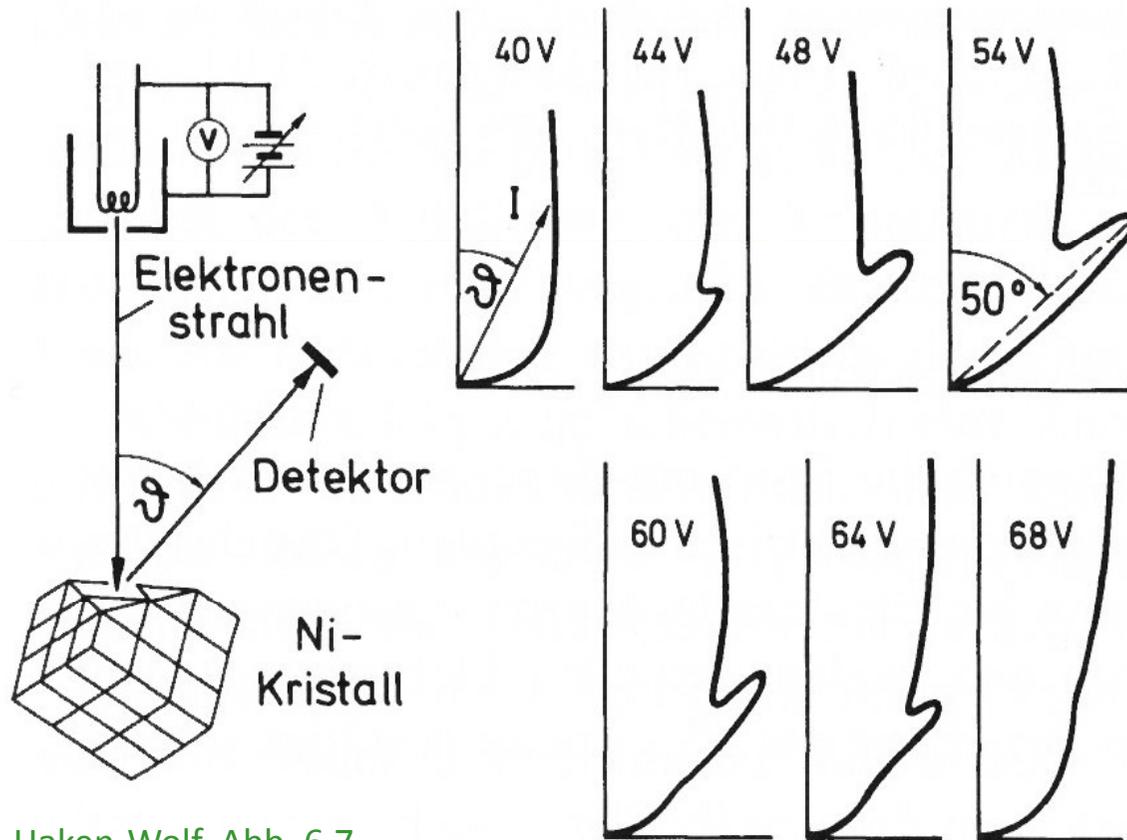
Objekt	m [kg]	v [m/s]	$\lambda_{dB}$ [m]
Elektron	$9.1 \times 10^{-31}$	$6 \times 10^6$	$1 \times 10^{-10}$
$^{87}\text{Rb}$ Atom	$1.4 \times 10^{-25}$	270	$2 \times 10^{-11}$
$\text{C}_{60}$ Molekül	$1.2 \times 10^{-24}$	210	$3 \times 10^{-12}$
Fussball	0.5	20	$7 \times 10^{-35}$

# Elektronenbeugung am Kristall



Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

## Davisson + Germer 1927

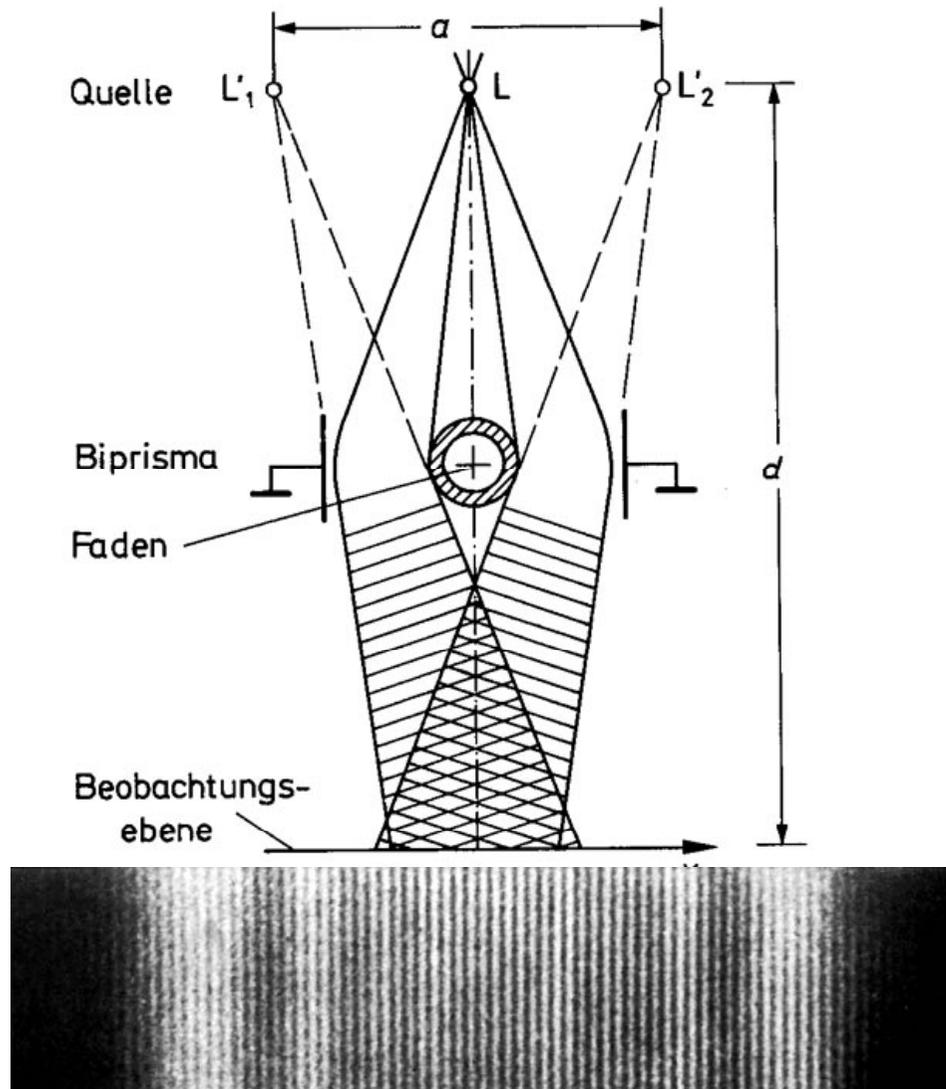


Haken-Wolf, Abb. 6.7

Intensitätsmaxima für bestimmte Winkel analog zur Bragg-Streuung (X-rays = Photonen).  
Hinweis, dass auch Elektronen Wellencharakter haben müssen

# Möllenstedt Prisma

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

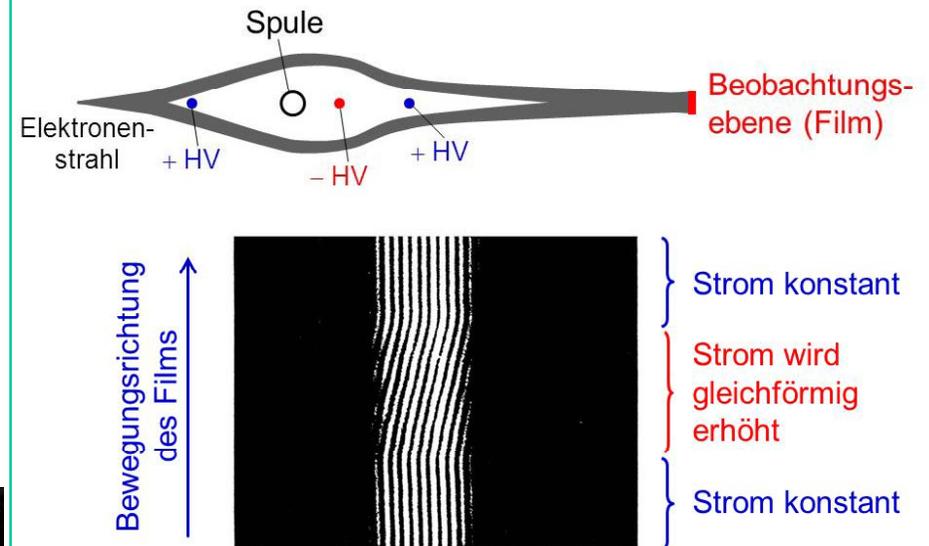


(Möllenstedt)

## Möllenstedt 1956

zwischen Faden und Gegenelektroden liegt eine positive elektrische Spannung an

### Der Aharonov-Bohm-Effekt:

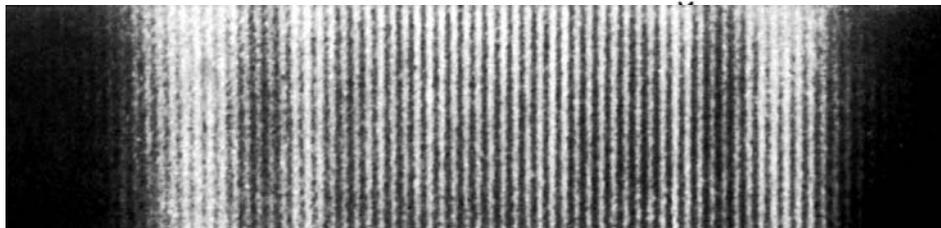
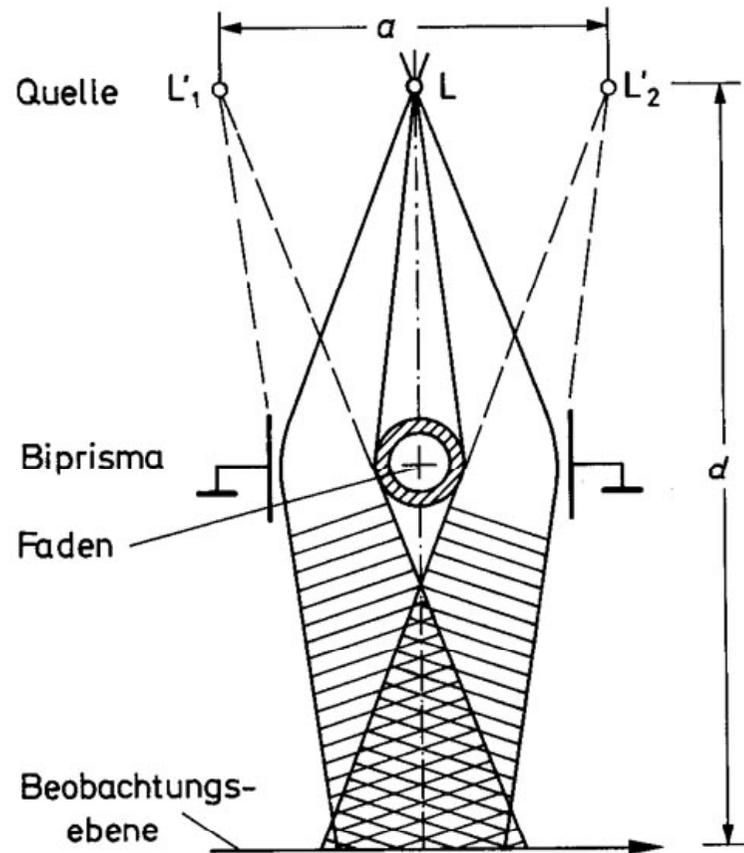


Experiment: Möllenstedt und Bayh, 1962

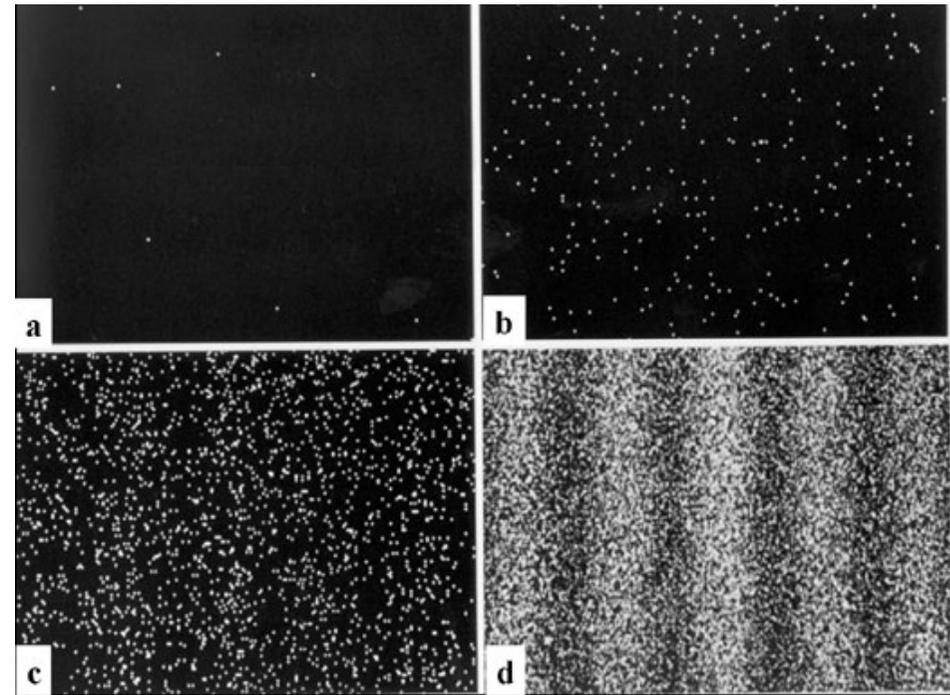
magnetischer Fluss ergibt Phasenverschiebung -> Aharonov-Bohm Effekt

# Möllenstedt Prisma

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



Wie bei den Photonen, das Interferenzmuster erscheint auch bei sehr niedriger Intensität, wenn immer nur ein Elektron in der Apparatur ist



(Moellenstedt)

# Elektronen am Doppelspalt

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



CH

$\hbar v$

## Jönsson, Uni Tübingen (1961)

Doppelspalt aus freistehenden Cu-Drähten,  
300 nm dünn (Pionierarbeit der Nanophysik)

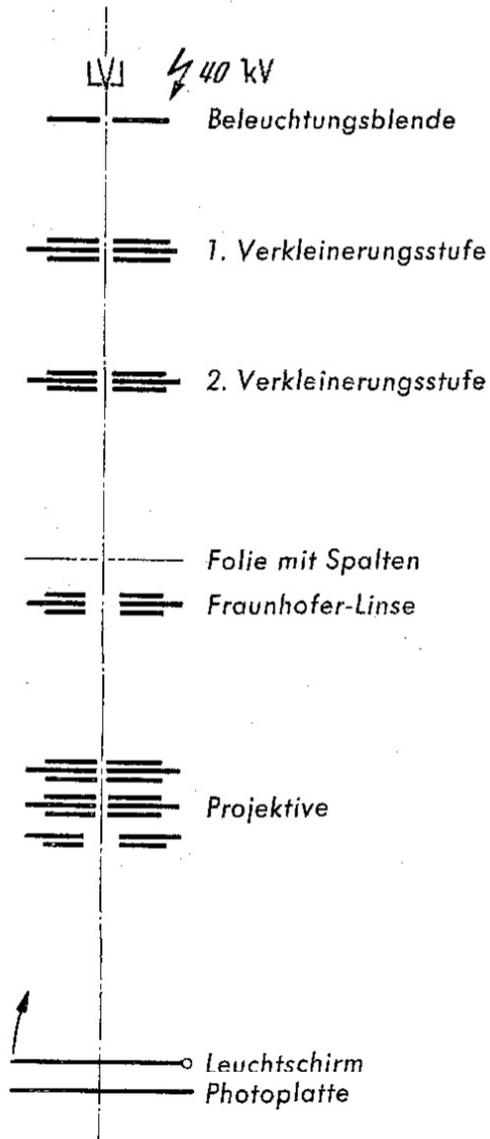
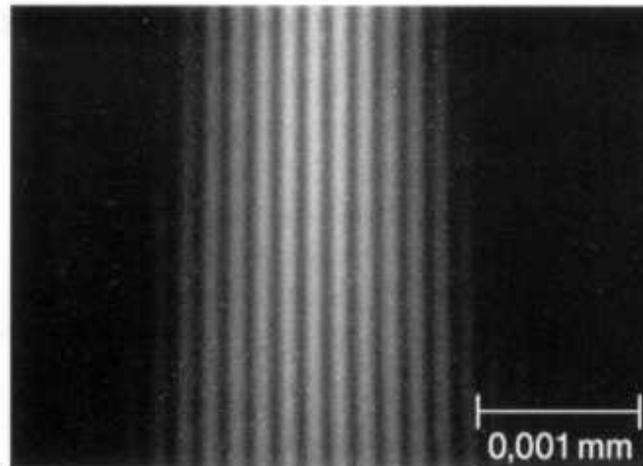


Fig. 6. Schema der Elektronenbeugungsanlage



Umfrage von "Physics World", 2002:

Die Top Fünf der schönsten Experimente

1. Jönssons Doppelspaltexperiment mit Elektronen (1961)
2. Galileis Experiment über den freien Fall (um 1620)
3. Millikans Öltropfenversuch (1909)
4. Newtons Spektralzerlegung des Lichts (1665-66)
5. Youngs Experiment zur Interferenz des Lichts (1801)

# Elektronen am Doppelspalt

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

bei niedrigen Energien (Elektronen mit 100eV)

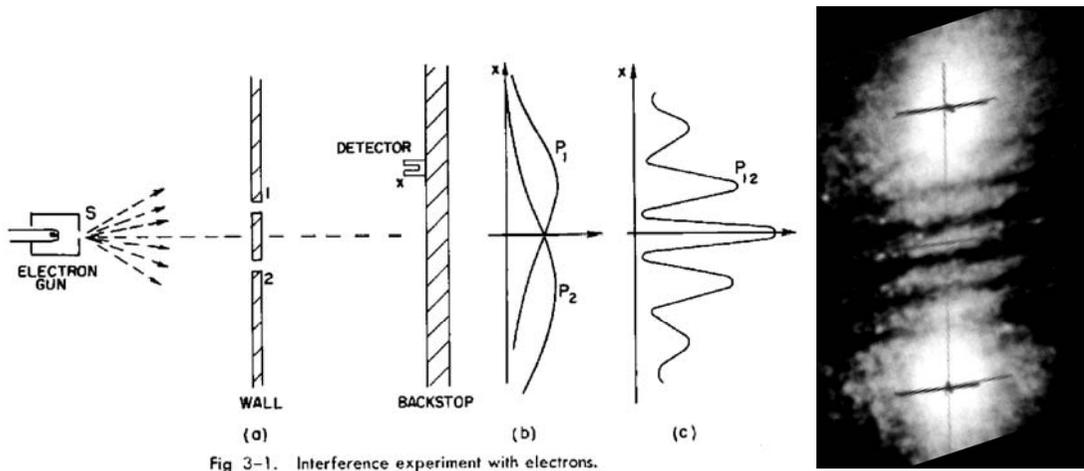
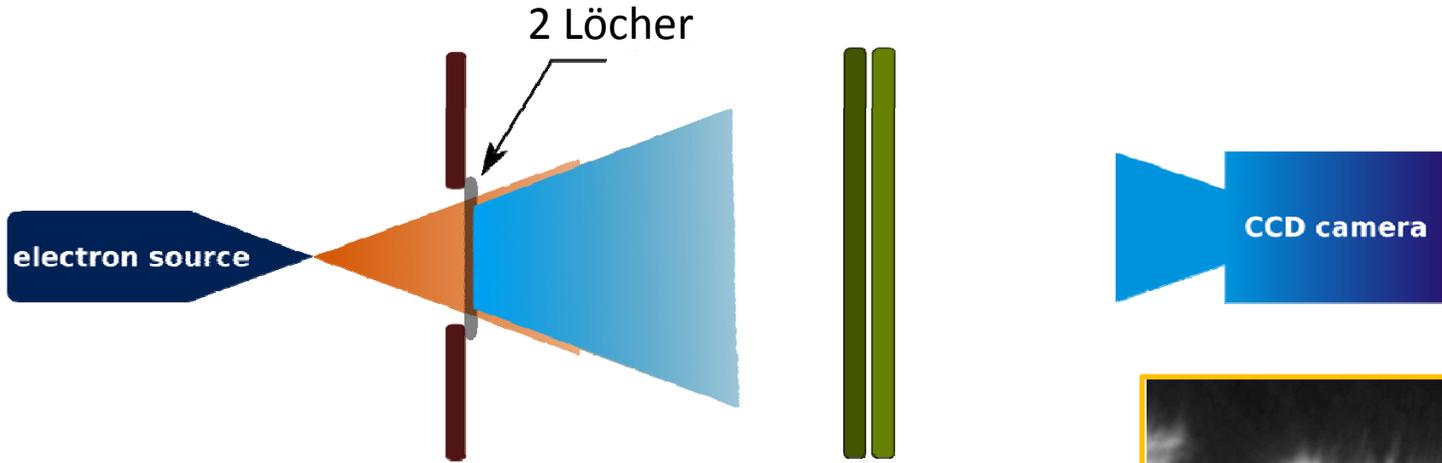
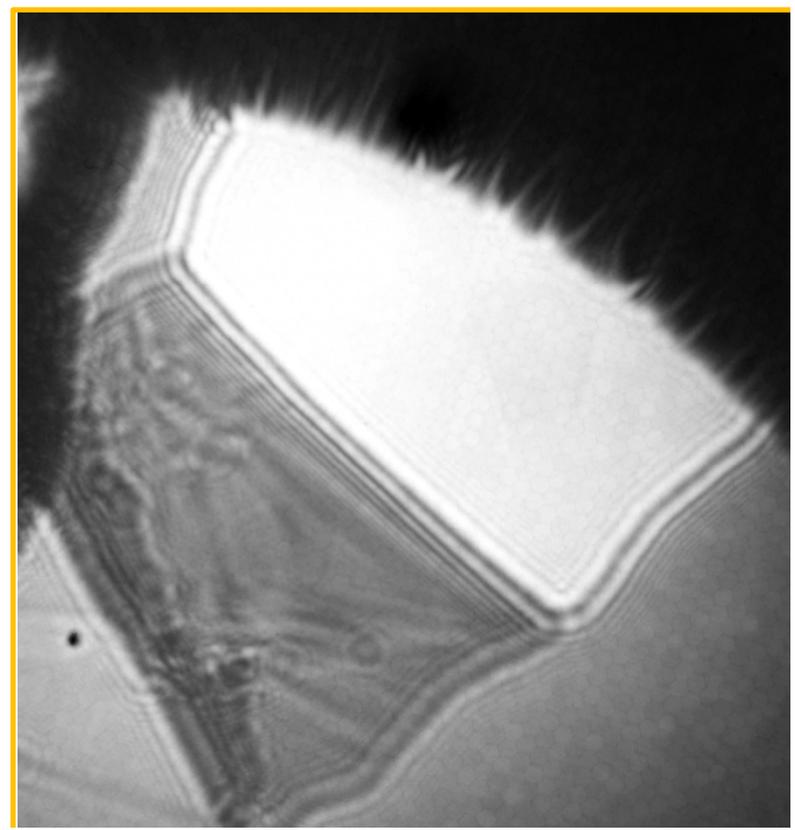


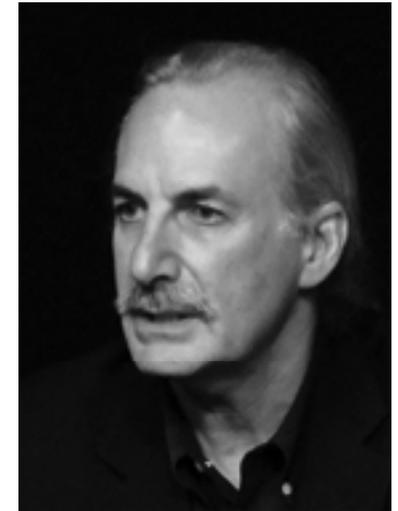
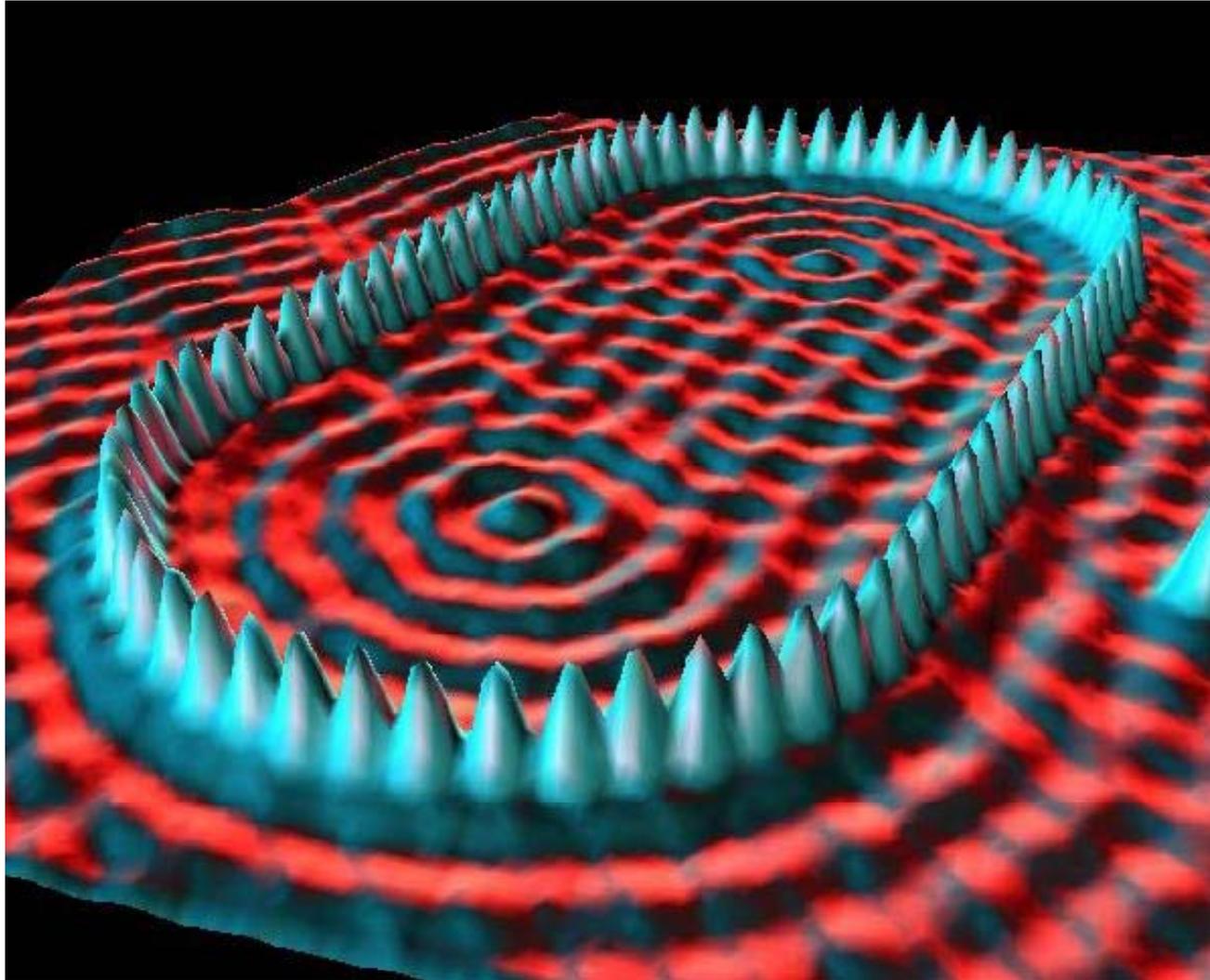
Fig 3-1. Interference experiment with electrons.



lens-less, low-energy electrons (H.-W. Fink)

# Elektroneninterferenz

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

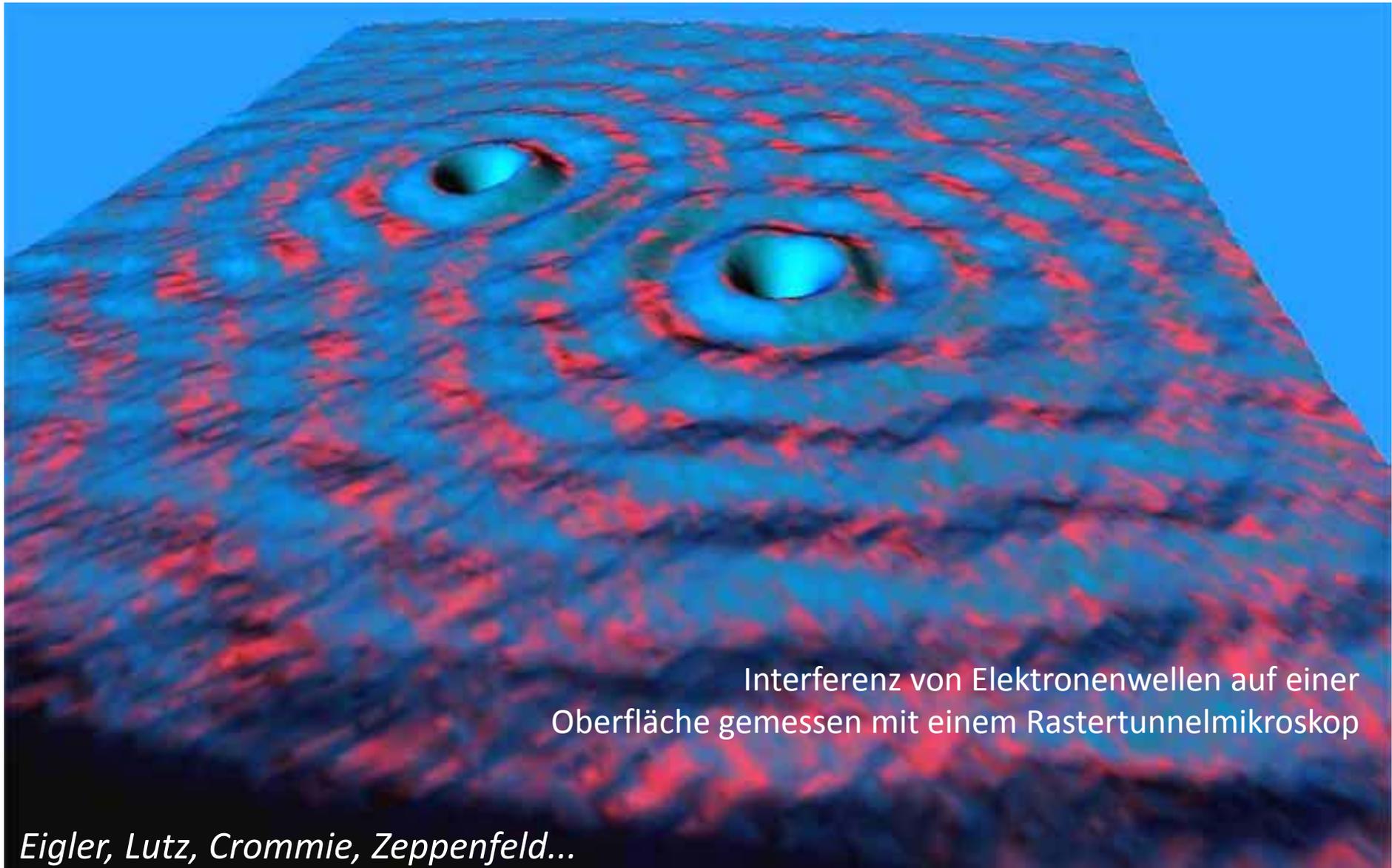


**the quantum corral**

Interferenz von Elektronenwellen auf einer Oberfläche gemessen mit einem Rastertunnelmikroskop

# Elektroneninterferenz

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

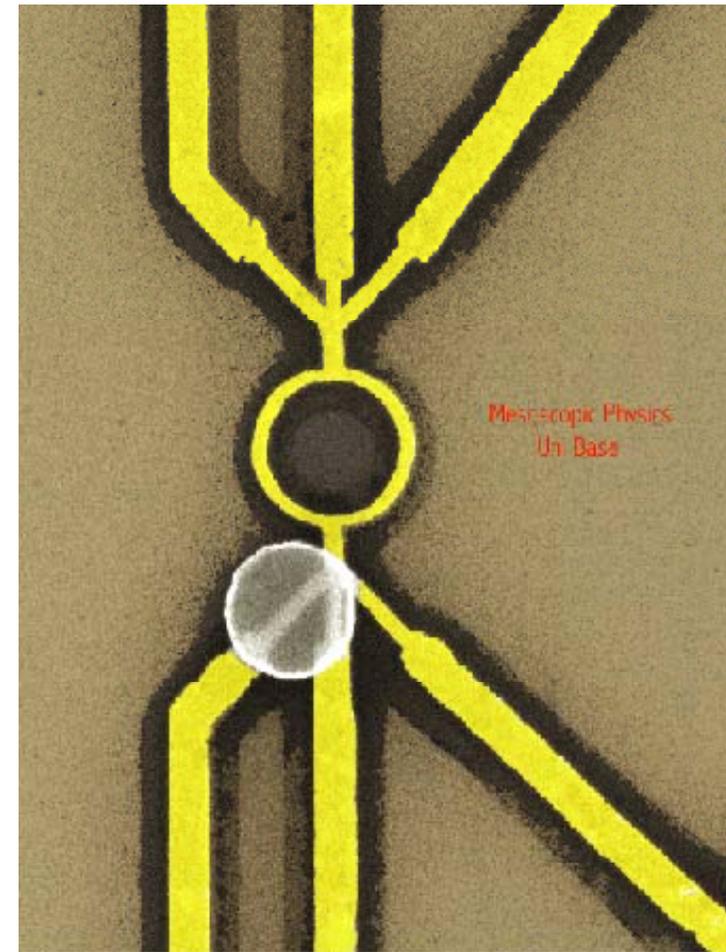
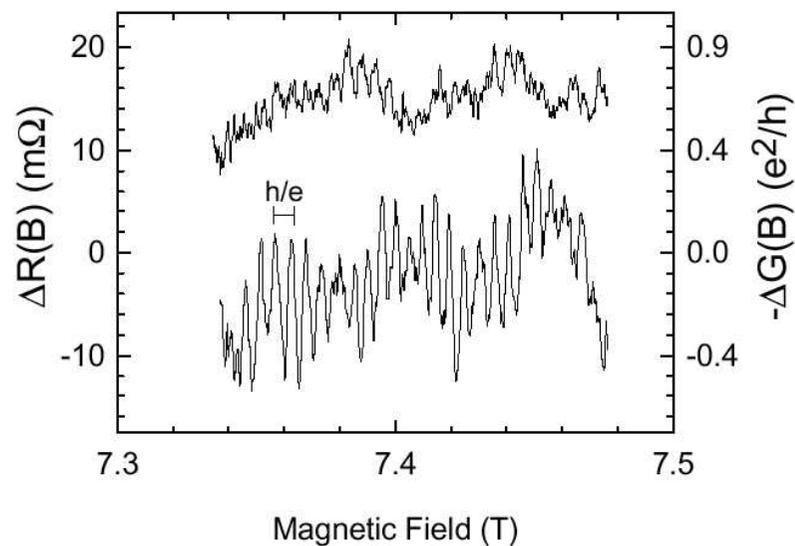
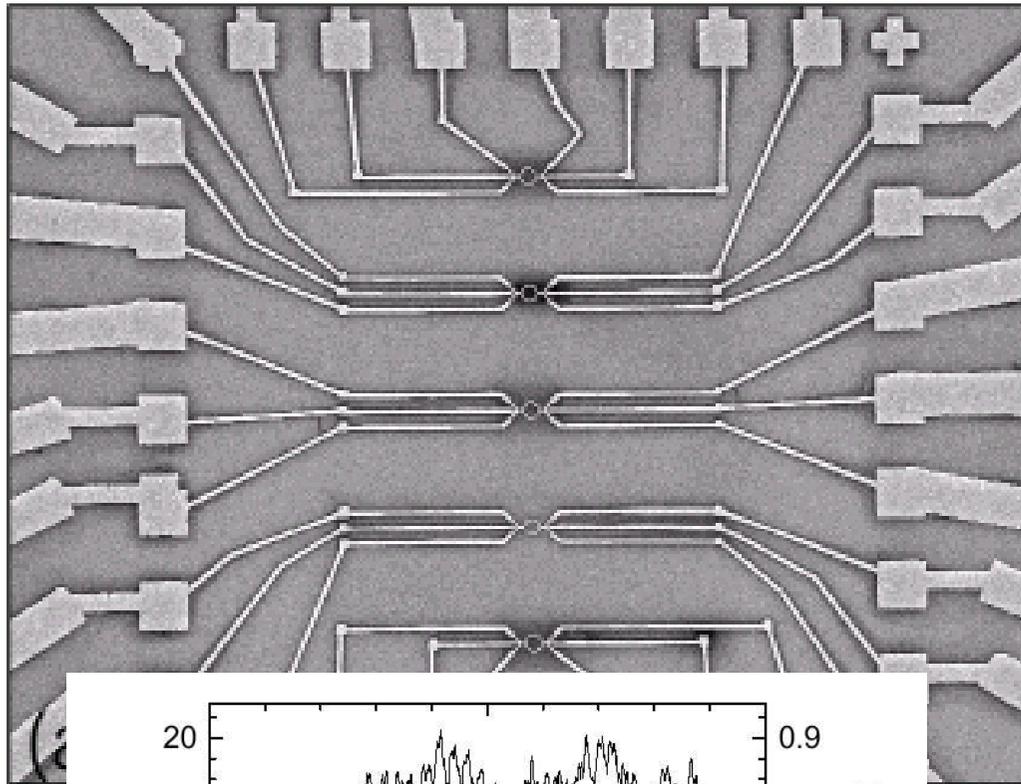


Interferenz von Elektronenwellen auf einer  
Oberfläche gemessen mit einem Rastertunnelmikroskop

*Eigler, Lutz, Crommie, Zeppenfeld...*

# Elektroneninterferenz

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

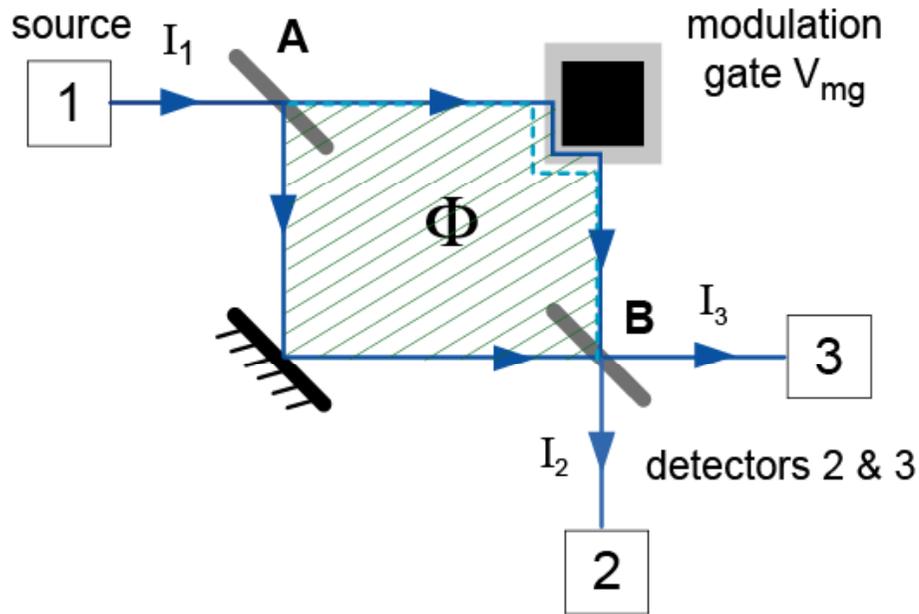


1  $\mu\text{m}$  diameter rings for studying quantum interference effects (Christophe Terrier et al., Inst. of Physics, Univ. Basel)

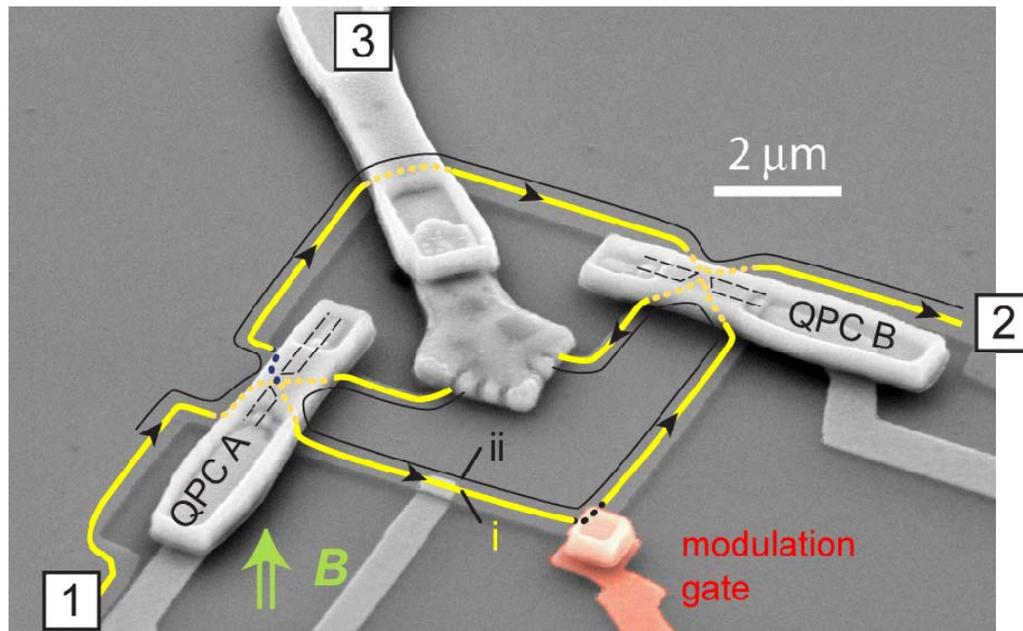
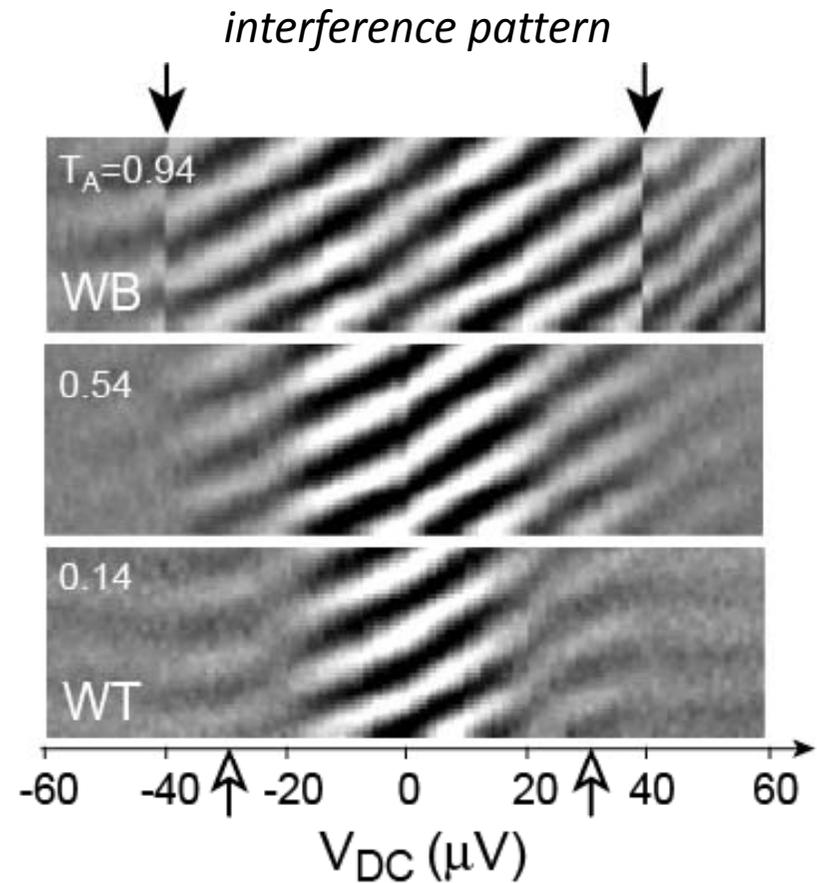
# Elektroneninterferenz



Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



Mach-Zehnder in einem Schaltkreis



*E. Bieri et al. Uni Basel*

# Elektroneninterferenz

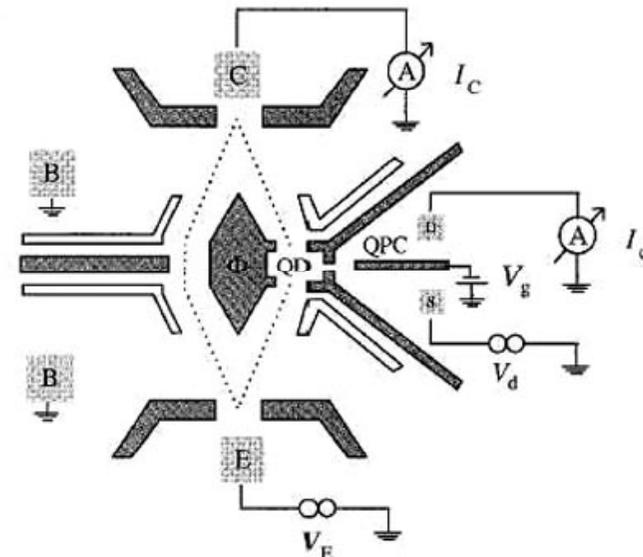
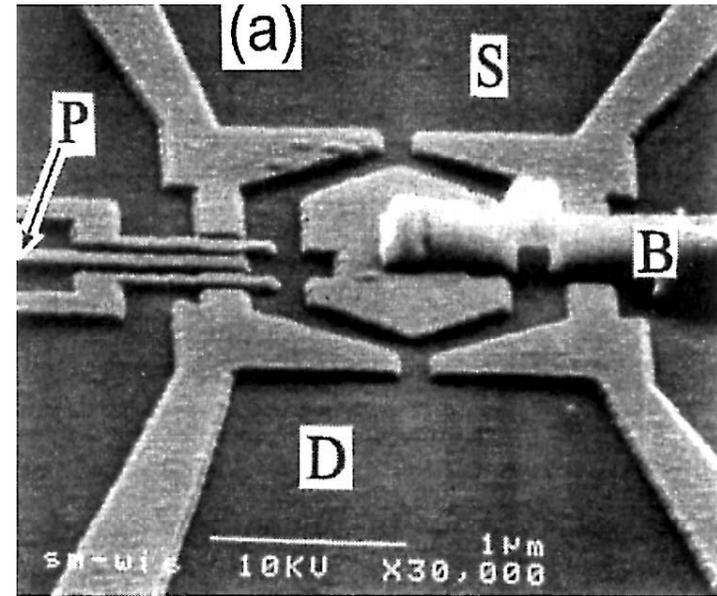
Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



$$\psi = \frac{1}{\sqrt{2}}(|left\rangle + |right\rangle)$$



*which path experiment*



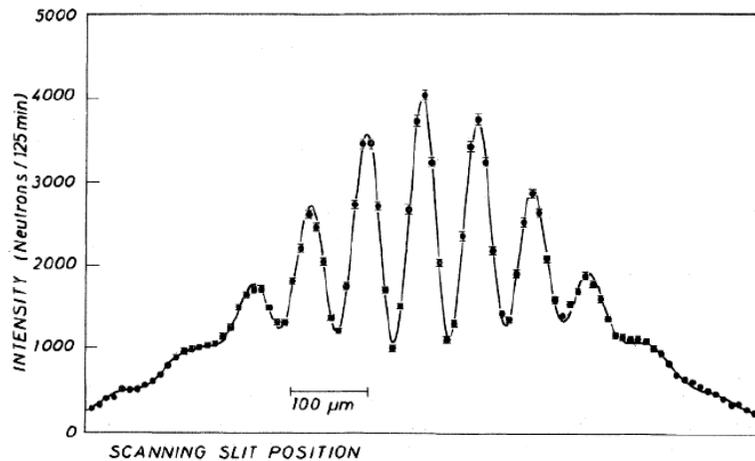
*(Heiblum, Yacoby, Schuster et al. Weizmann)*

# Neutroneninterferometer

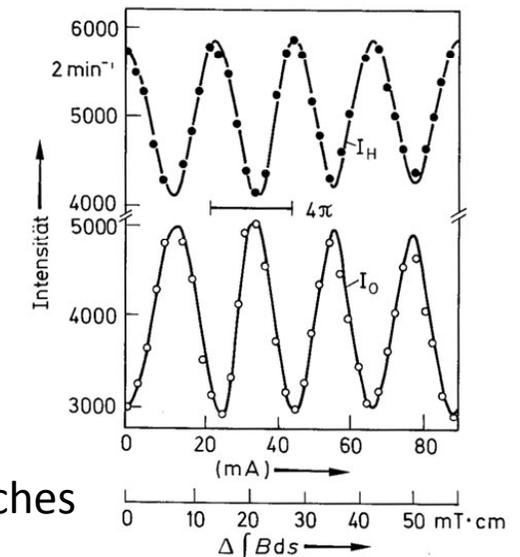
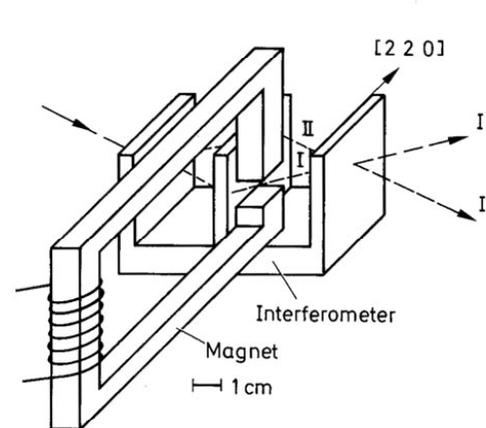
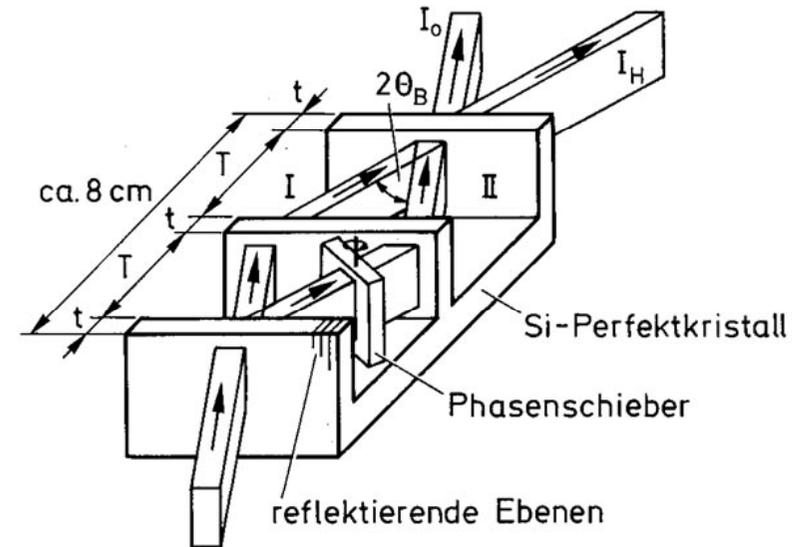
Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



## Neutronen Interferometer (Rauch et al.)



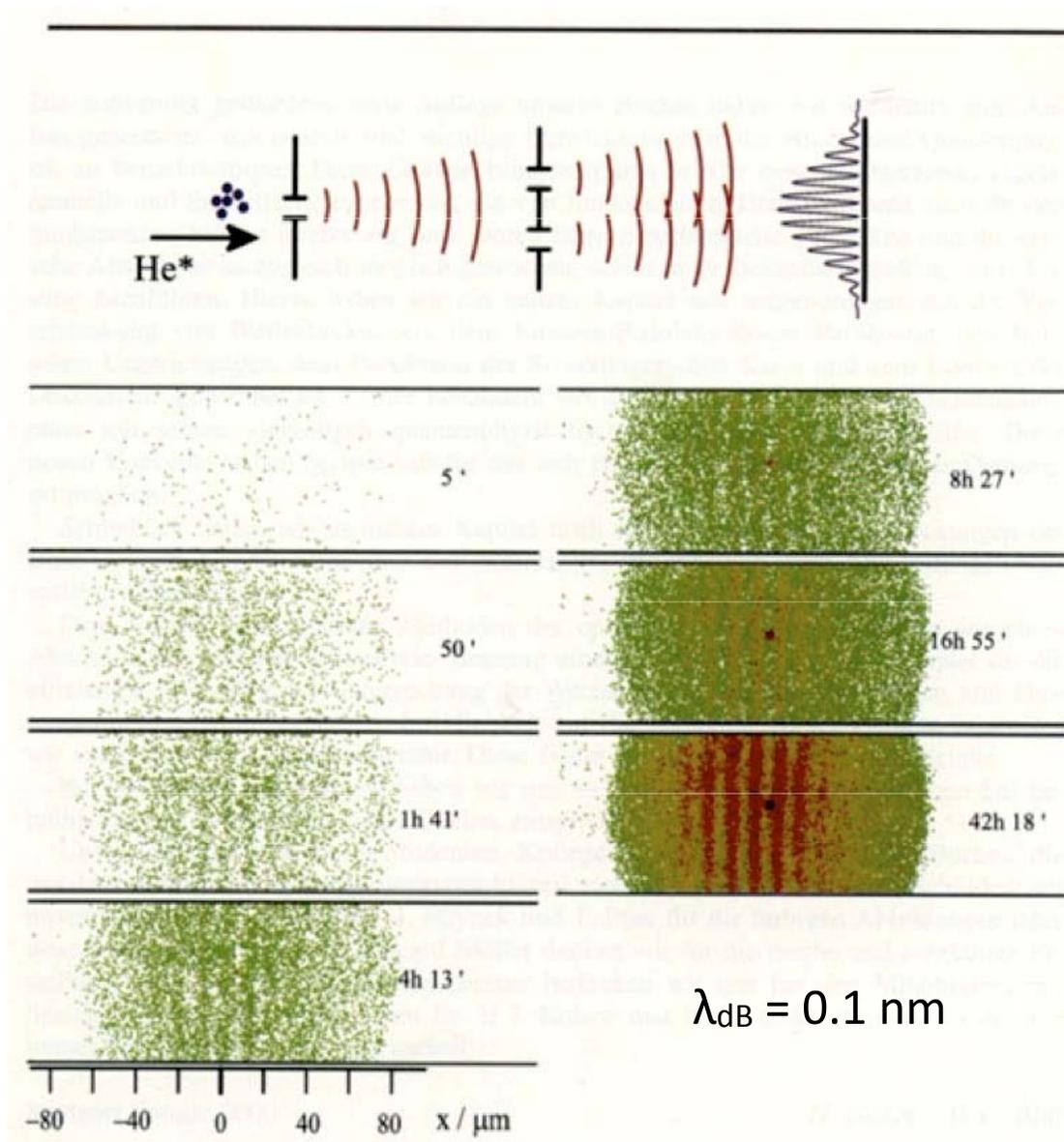
Doppelspaltexperiment  
mit thermischen Neutronen  
Anton Zeilinger 1988



Phasenshift durch magnetisches  
Moment des Neutrons

# Doppelspaltexp. mit He-Atomen

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

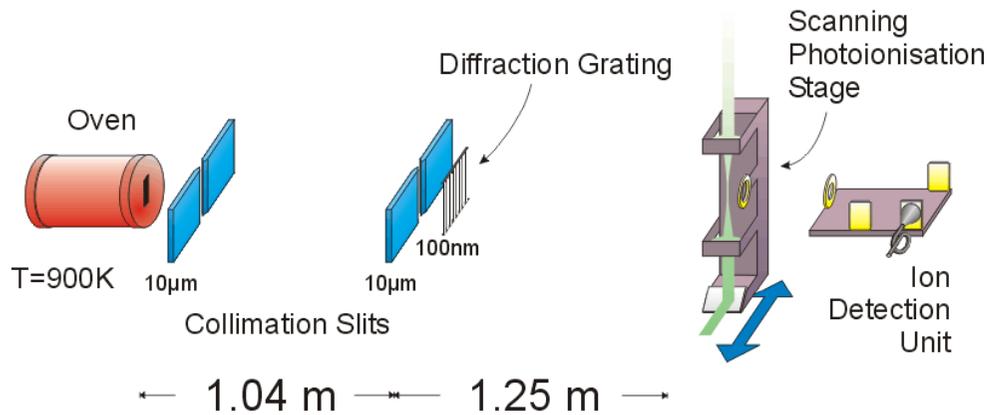


Interferenzmuster  
baut sich Atom für  
Atom auf

Mlynek et al., Konstanz  
(Haken-Wolf, Bild im Vorwort)

# mit grossen Objekten

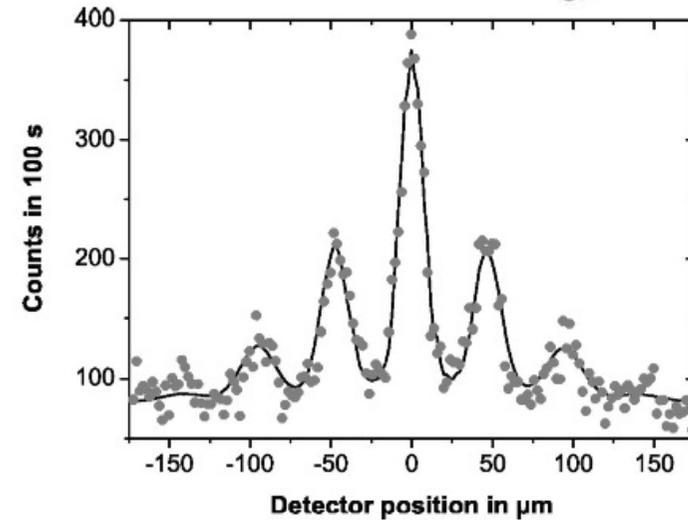
Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



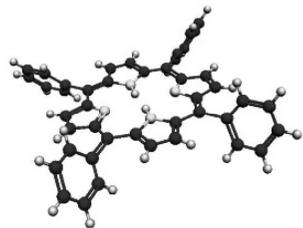
Pressure  $\sim 5 \cdot 10^{-7}$  mbar

M. Arndt et al., Wien

## C<sub>60</sub>-Interferenz



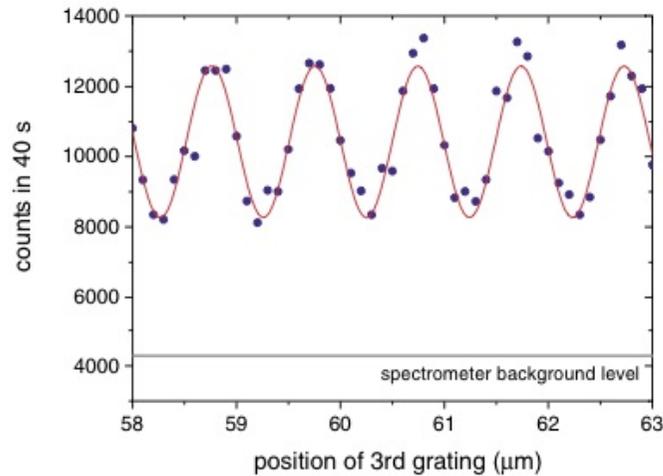
## Interferenz von Biomolekülen



$\text{C}_{44}\text{H}_{30}\text{N}_4$   
( $m = 614 \text{ u}$ )



$\text{C}_{60}\text{F}_{48}$   
( $m = 1632 \text{ u}$ )



Talbot-Lau-  
Interferometer  
mit drei Gittern

# mit grossen Objekten

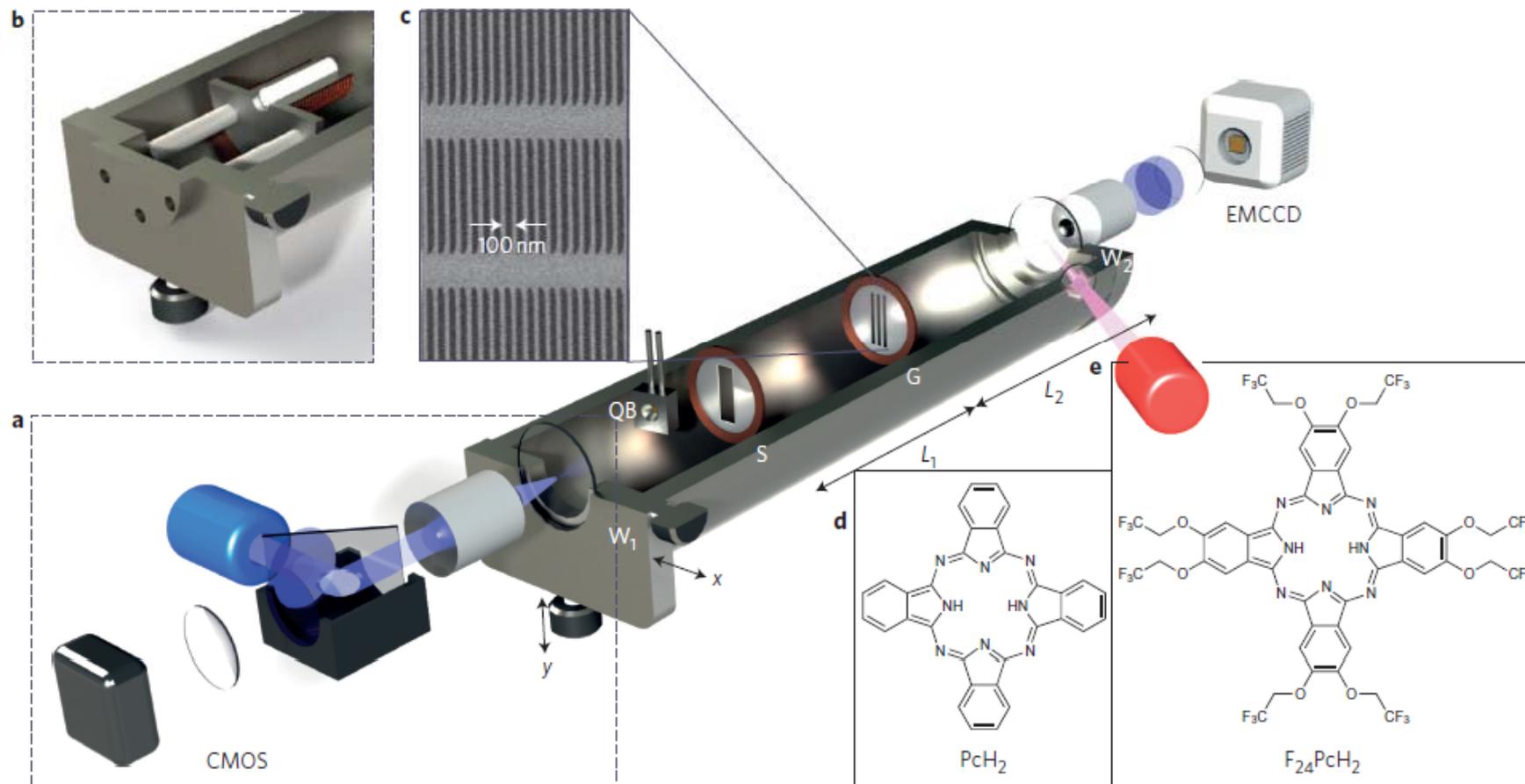
Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



## Real-time single-molecule imaging of quantum interference

Nature Nano 2012

Thomas Juffmann<sup>1</sup>, Adriana Milic<sup>1</sup>, Michael Müllneritsch<sup>1</sup>, Peter Asenbaum<sup>1</sup>, Alexander Tsukernik<sup>2</sup>, Jens Tüxen<sup>3</sup>, Marcel Mayor<sup>3,4</sup>, Ori Cheshnovsky<sup>2,5</sup> and Markus Arndt<sup>1\*</sup>



# mit grossen Objekten

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



## Real-time single-molecule imaging of

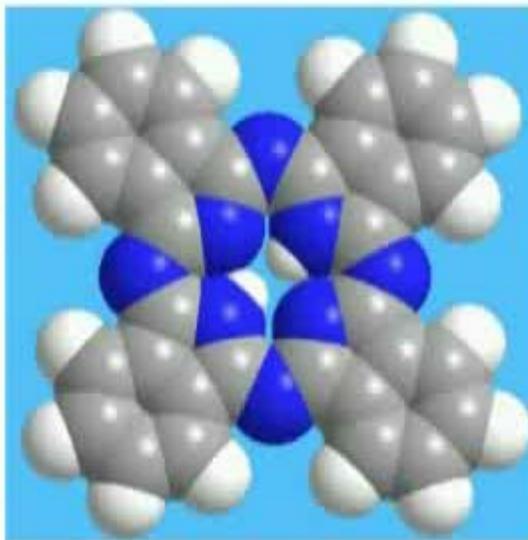
Nature Nano 2012

quan

Thomas J  
Jens Tüxe

## The quantum molecular movie

The wave-particle duality of phthalocyanine



Thomas Juffmann  
Adriana Milic  
Michael Müllneritsch  
Peter Asenbaum  
Alexander Tsukernik  
Jens Tüxen  
Marcel Mayor  
Ori Cheshnovsky and  
Markus Arndt



# mit grossen Objekten

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



Wenn die Temperatur sinkt, kann Materie einen **Phasenübergang** erfahren!

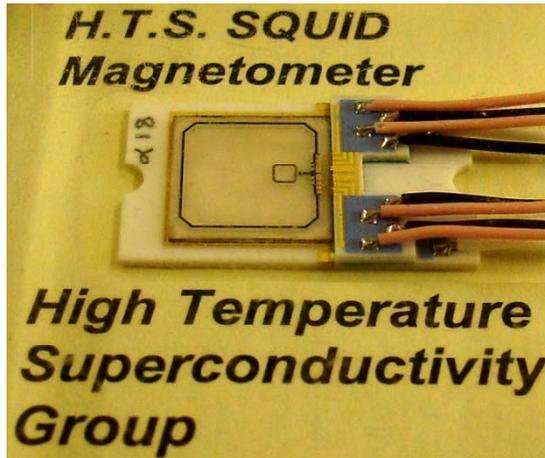
- Gas  $\Rightarrow$  Flüssigkeit
- Flüssigkeit  $\Rightarrow$  Festkörper
- Normalleiter  $\Rightarrow$  Supraleiter
- normale Flüssigkeit  $\Rightarrow$  Supraflüssigkeit
- klassisches Gas  $\Rightarrow$  Bose-Einstein-Kondensat



**Supraleiter, Supraflüssigkeit und Bose-Einstein Kondensat** sind **makroskopische Quantenobjekte**, welche eine **makroskopische Phase** besitzen. Somit kann man Interferenzexperimente durchführen

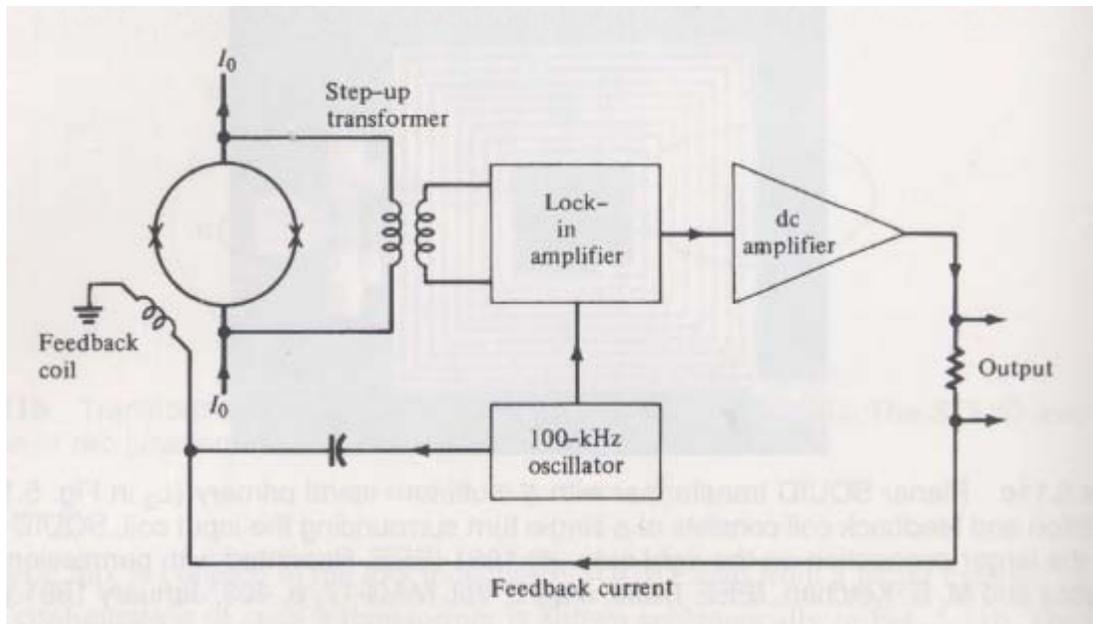
# mit grossen Objekten

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SQUID = Superconducting Quantum Interference Device

Messung von Magnetfeld



kommerzielles SQUID Magnetometer

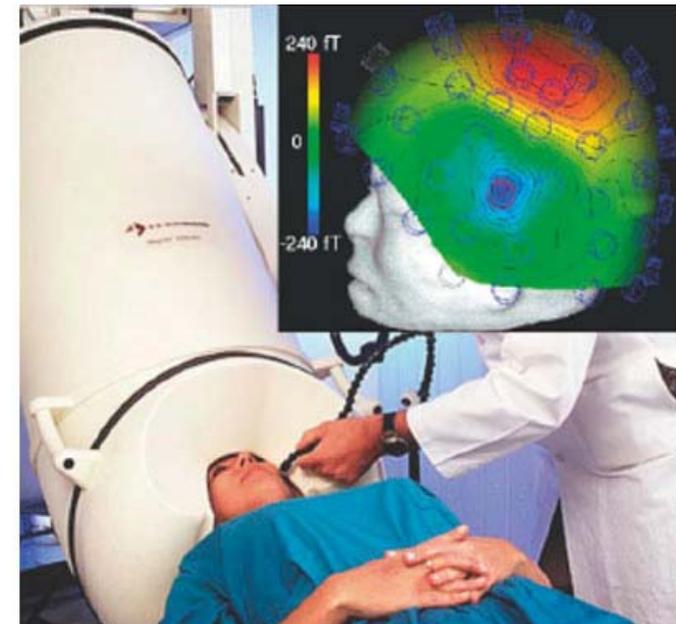
# mit grossen Objekten

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



## Magnetoenzephalographie

Gehirnaktivität erzeugt  
über Gehirnströme  
Magnetfeld  $\sim 100$  fT



# mit grossen Objekten

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

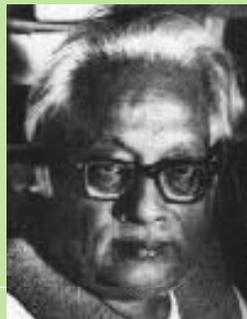
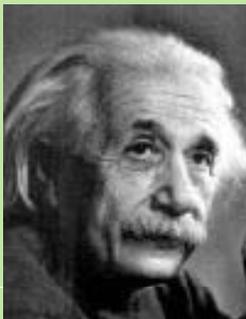


mit Bose-Einstein-Kondensat (Nobelpreis 2001)

 University of Colorado at Boulder	 EPFL / PRE	 University of Colorado at Boulder
Eric Cornell	Wolfgang Ketterle	Carl Wieman

Experiment 1995

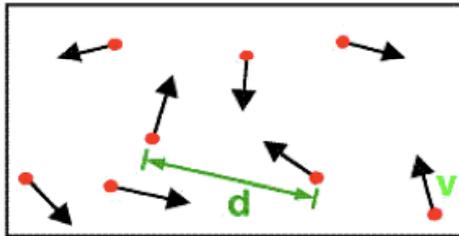
Nobelpreis 2001

	
Satyendranath Bose	Albert Einstein

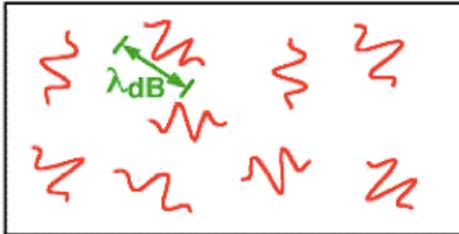
Vorhersage 1925

# Bose-Einstein Kondensat

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

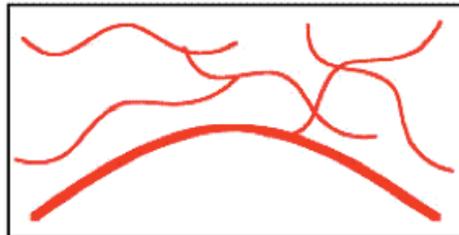


hohe Temperatur:  
Atome = Teilchen



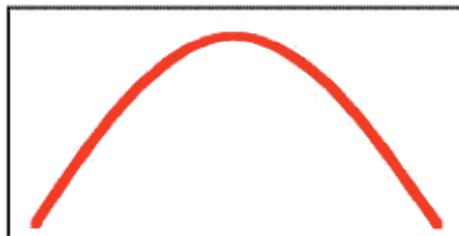
niedrigere Temperatur:  
Atome = Wellenpakete

$$\lambda_{dB} \ll d$$



$T = T_c$   
Wellenpakete überlappen  $\Rightarrow$  BEC

$$\lambda_{dB} \approx d$$



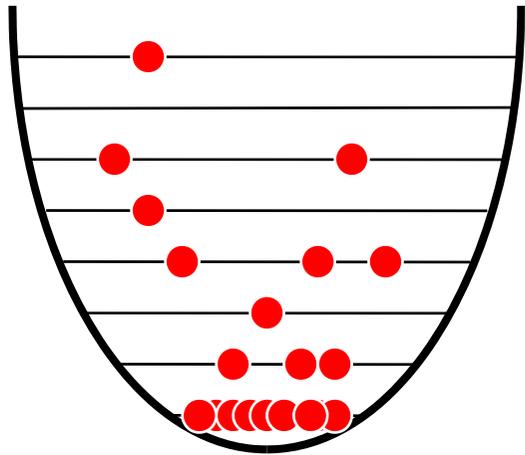
$T \ll T_c$   
reines BEC = riesige Materiewelle

thermische deBroglie-Wellenlänge

$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m k_B T}}$$

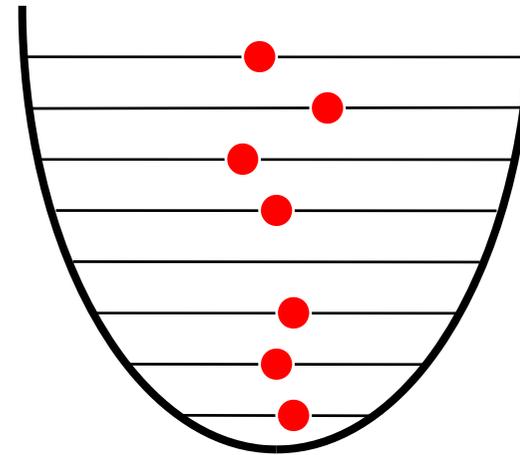
# Bose-Einstein Kondensat

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



**Bosonen**

Spin 0, 1, 2, ...



**Fermionen**

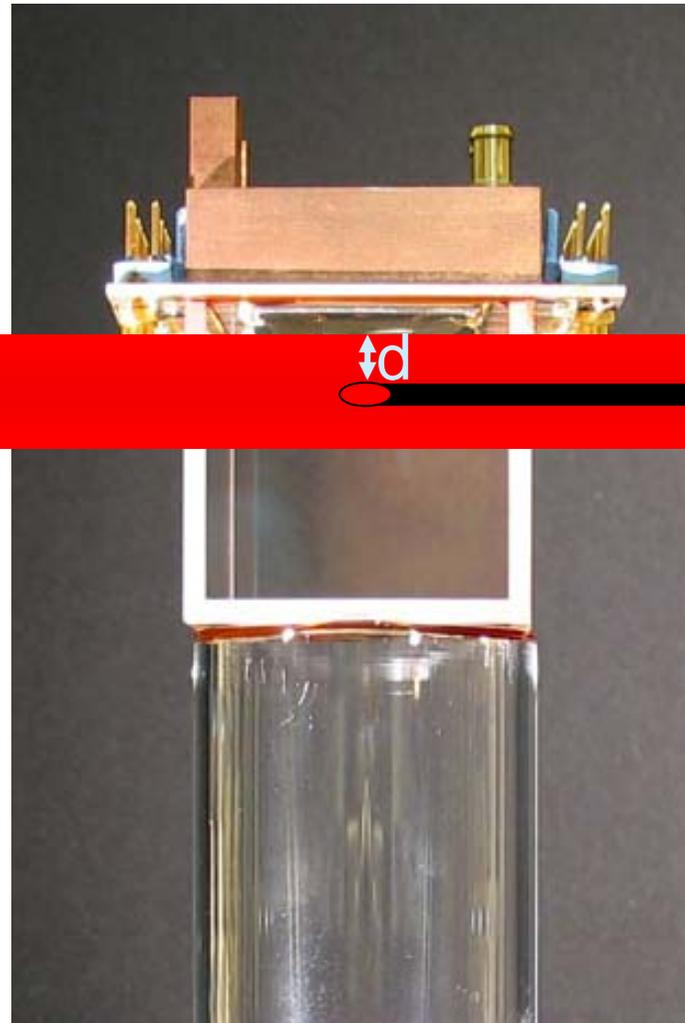
Spin 1/2, 3/2, ...

**Bosonen sind "sozial":**  
bevorzugen mehrfache Besetzung  
des selben Zustands

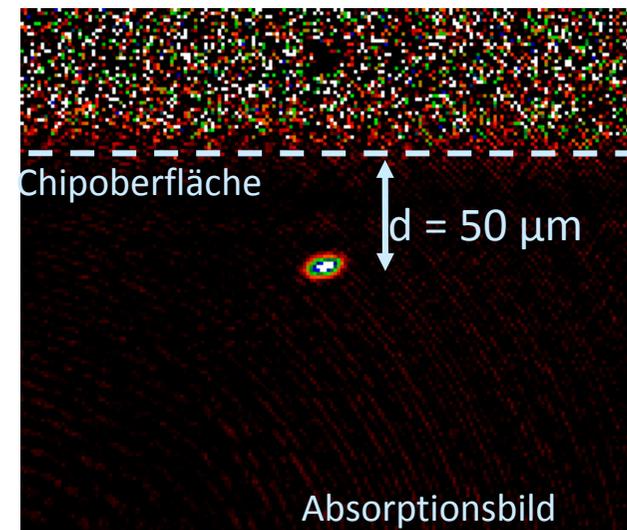
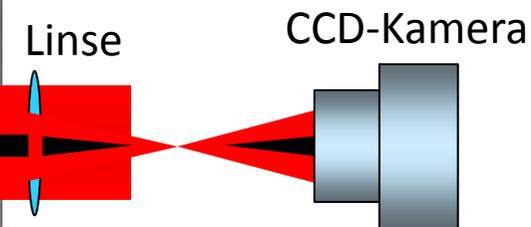
**Fermionen sind "Einzelgänger":**  
zwei Fermionen besetzen nie den  
selben Zustand (**Pauli-Prinzip**)

# Bose-Einstein Kondensat

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



Schatten der Atome wird auf eine CCD-Kamera abgebildet



Philipp Treutlein (Uni Basel)

# Bose-Einstein Kondensat

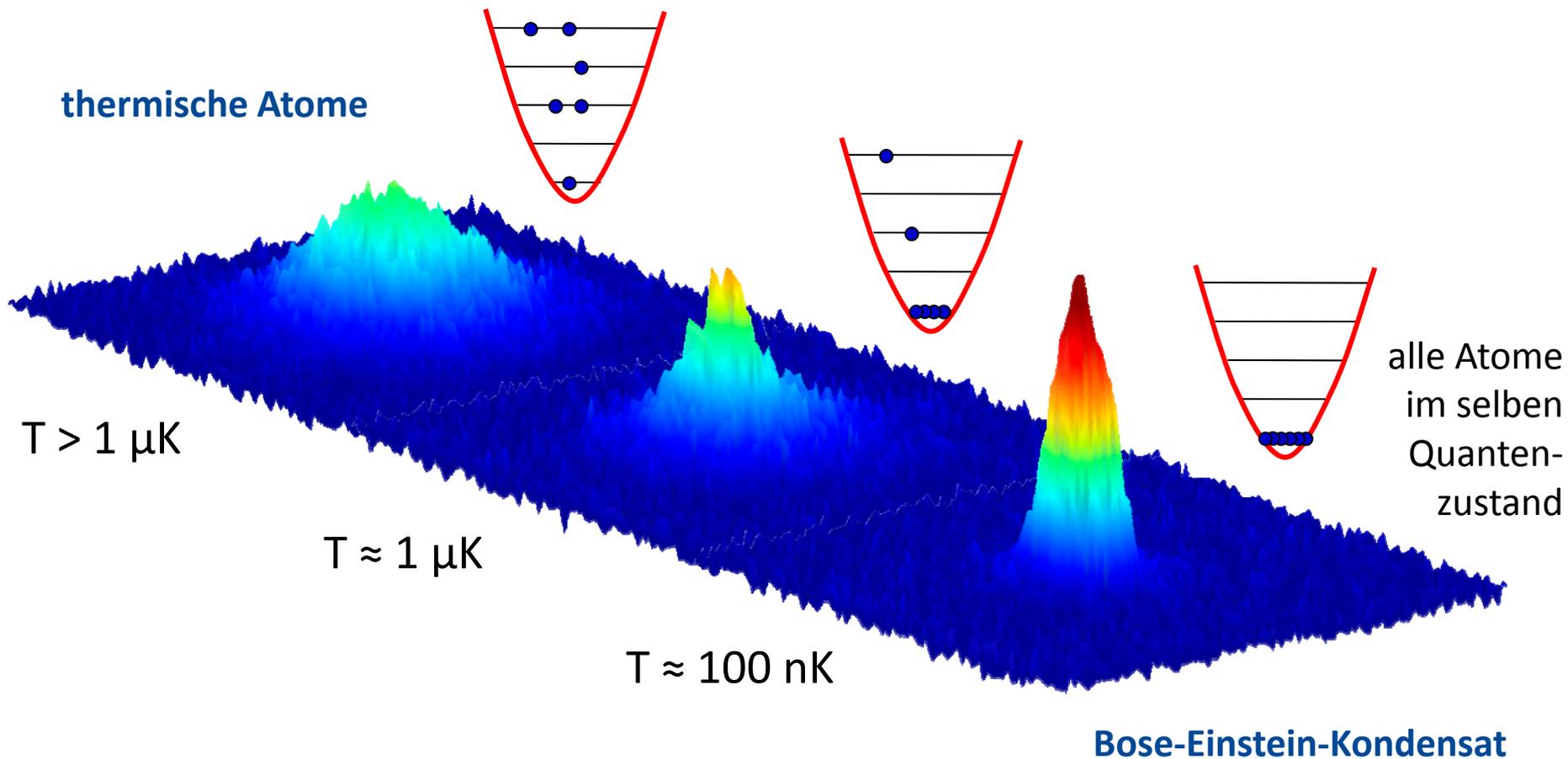
Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



CH



$\hbar\nu$

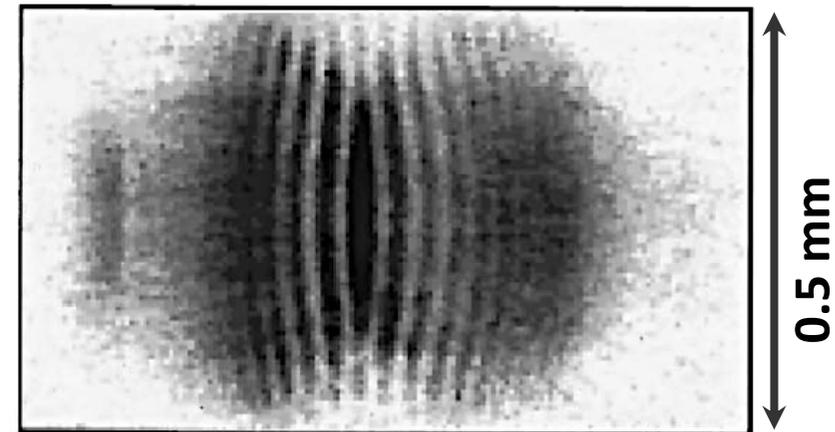
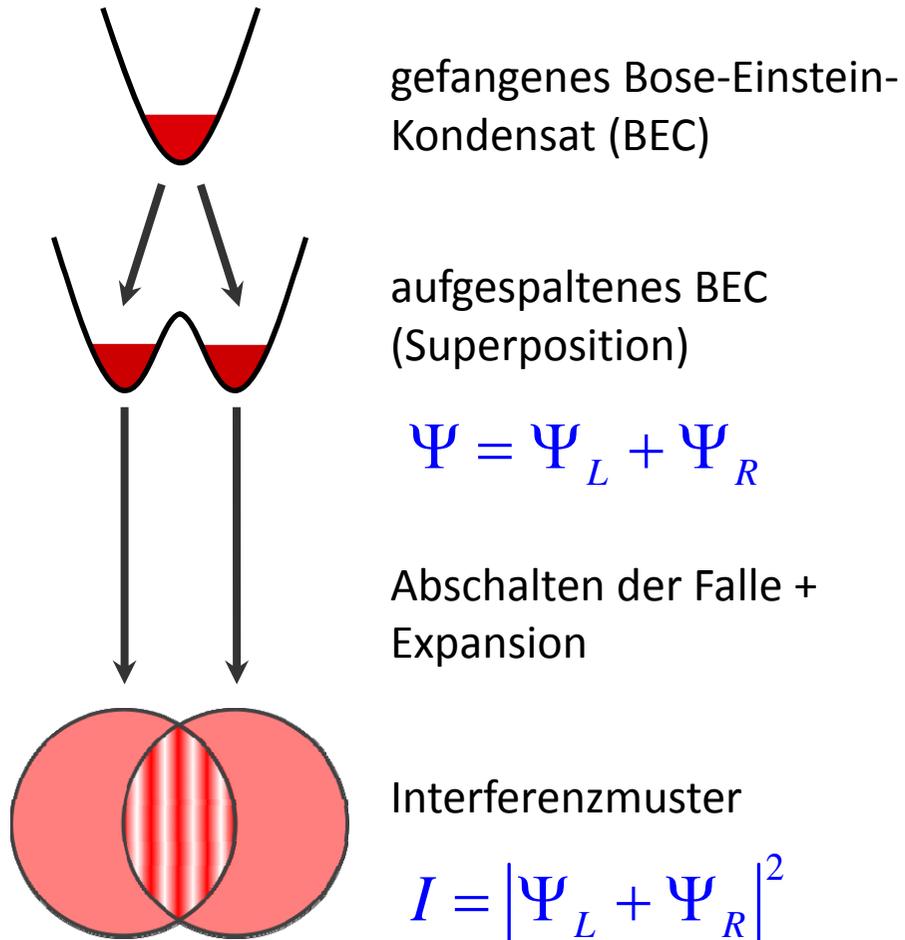


# Bose-Einstein Kondensat

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)



## Doppelspaltexperiment mit Bose-Einstein-Kondensat



Ketterle et al., MIT

# Heisenberg Unschärferelation

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CH



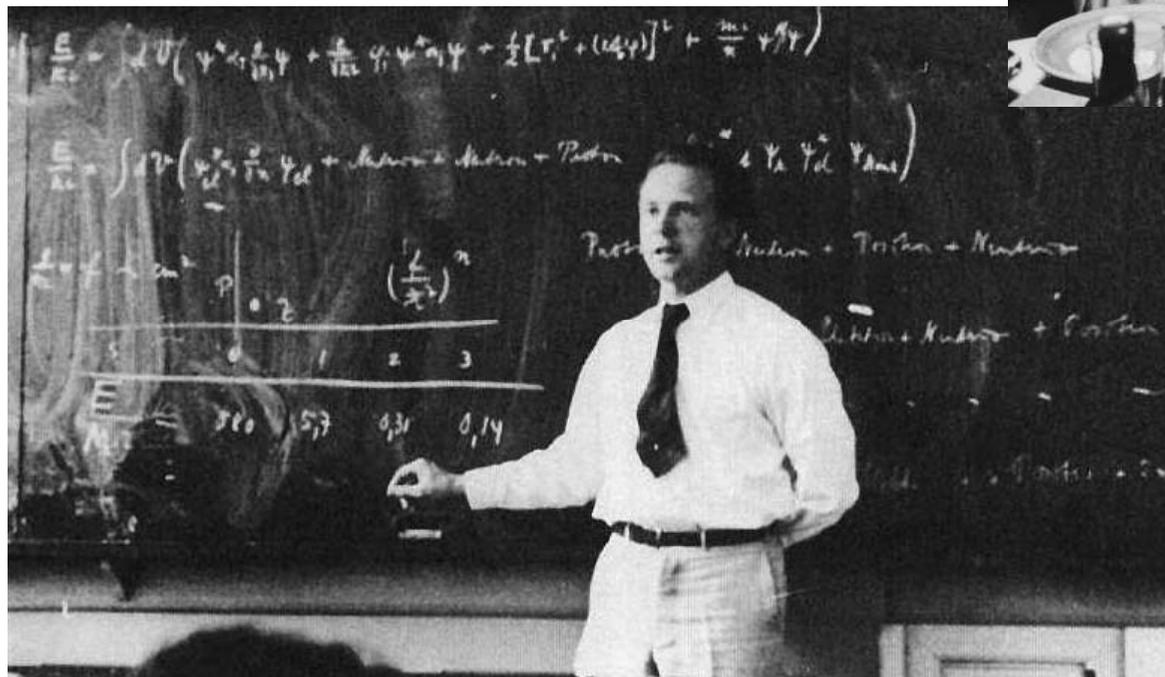
hv



Werner Heisenberg (1901-1976)  
Nobelpreis 1932

$$\Delta x \cdot \Delta p \geq \hbar / 2$$

Heisenberg and Bohr  
in Copenhagen

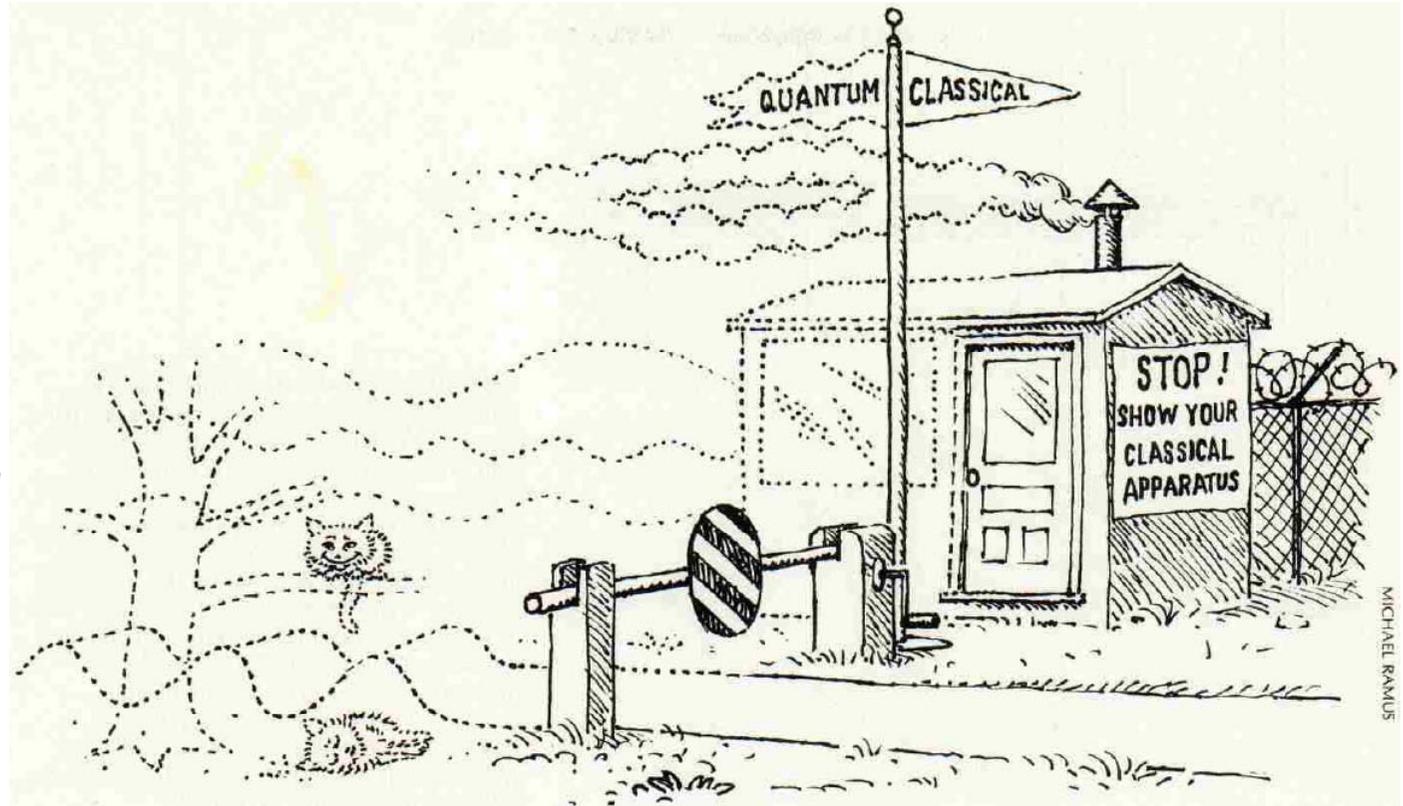
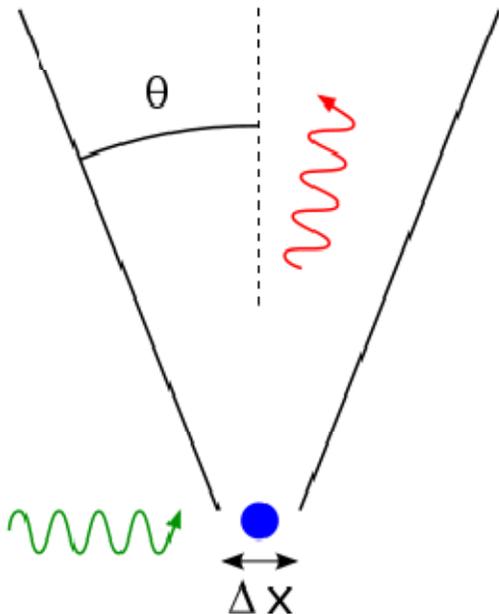
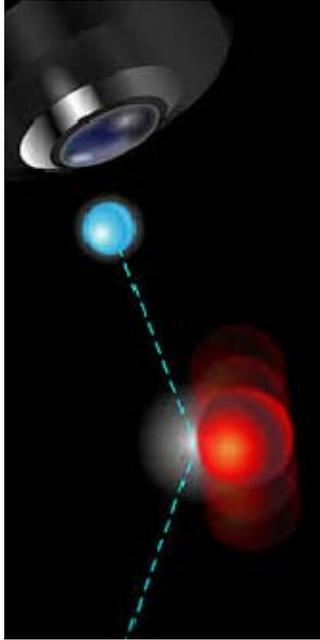


# Heisenberg Unschärferelation

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CH



MICHAEL RAMUS

in the quantum world “only” half of the world can be known with certainty. But more is not possible

# freies Wellenpaket

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wegen der Unschärfe im Impuls, fließt ein Wellenpaket mit der Zeit auseinander, d.h. die Ortsinformation geht verloren



$$\Delta x(t) = \left( (\Delta x_0)^2 + \left( \frac{\hbar}{2\Delta x_0} \right)^2 \right)$$

# Energie-Zeit Unschärfe

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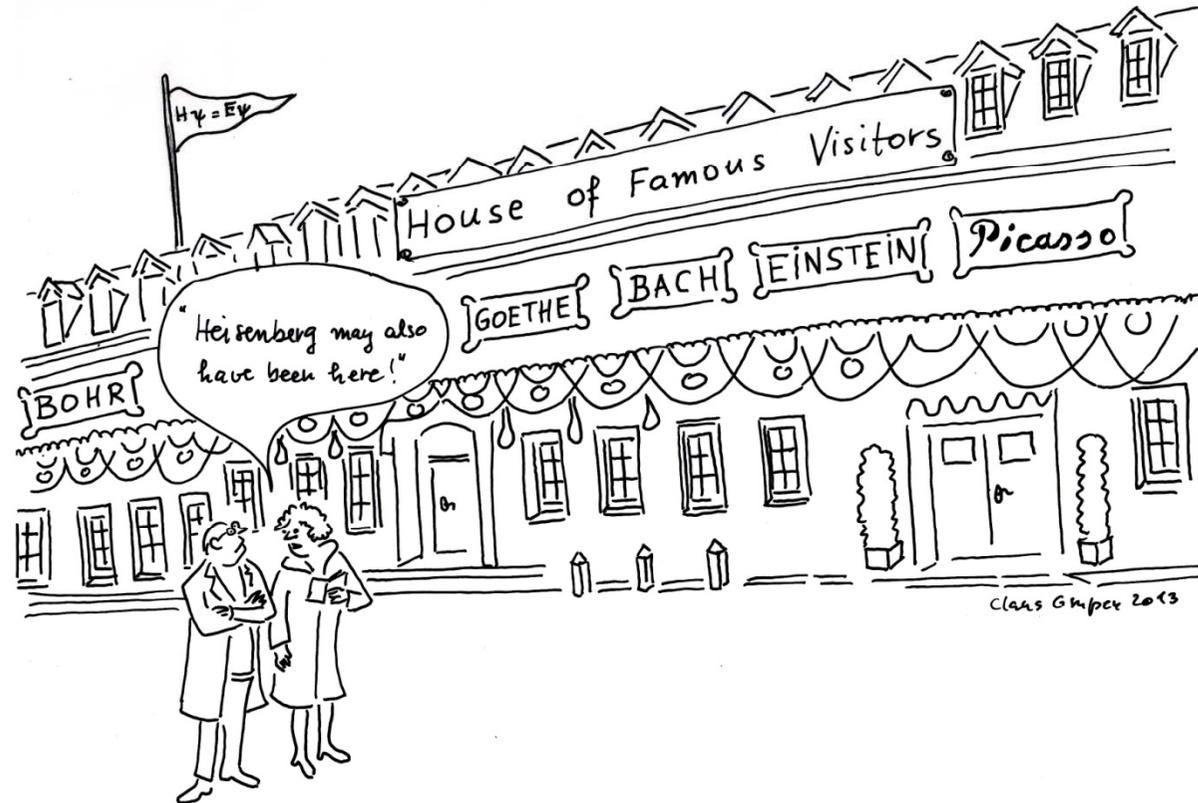


CH



$\hbar\nu$

$$\Delta E \cdot \Delta t \geq \hbar / 2$$



- a long-lived excited state is a narrow band photon source
- short-lived resonance states (like some in particle physics) have a relatively undefined mass
- it takes time to measure the frequency accurately
- it takes time to know the energy

# rf Spectroscopy in an Atomic Fountain

Mark A. Kasevich, Erling Riis, and Steven Chu

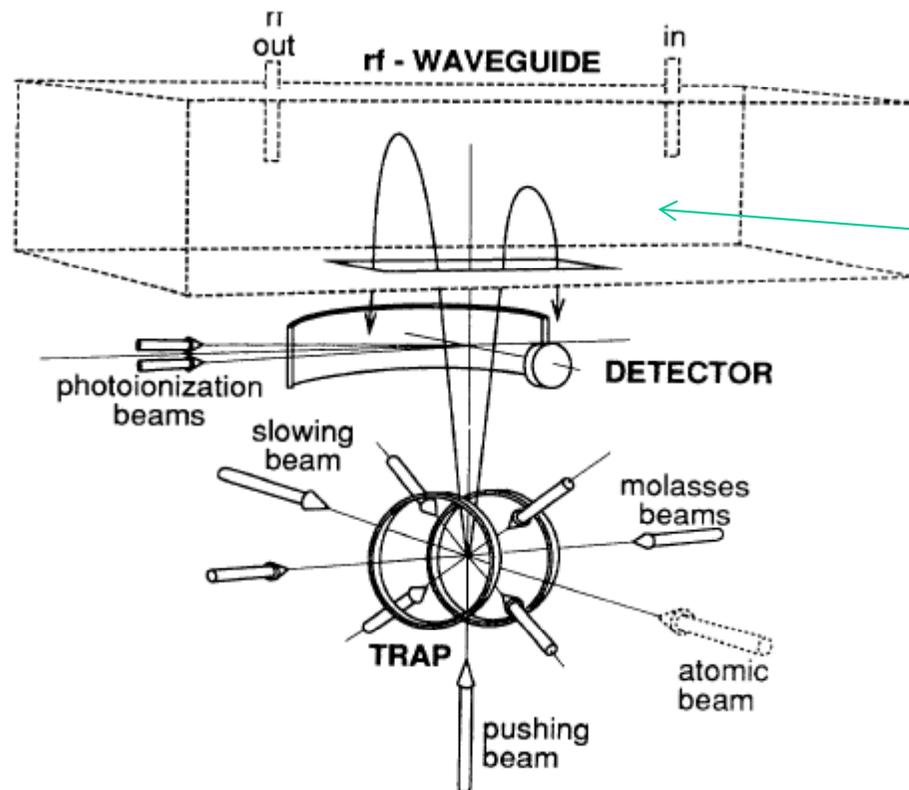
*Department of Physics, Stanford University, Stanford, California 94305*

Ralph G. DeVoe

*IBM Almaden Research Center, San Jose, California 95120*

(Received 15 May 1989)

Laser-cooled sodium atoms pushed up on a vertical trajectory by radiation pressure are observed to turn around due to gravity. The relatively long time the atoms spent freely falling in this “atomic fountain” allowed the ground-state hyperfine splitting to be measured with a linewidth of 2 Hz. After a 1000-sec integration time, the center of the line was resolved to  $\pm 10$  mHz. The absolute splitting was measured to be 1 771 626 129(2) Hz.

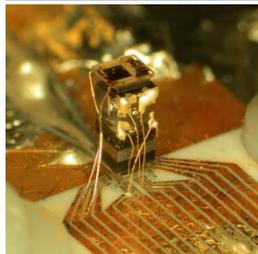
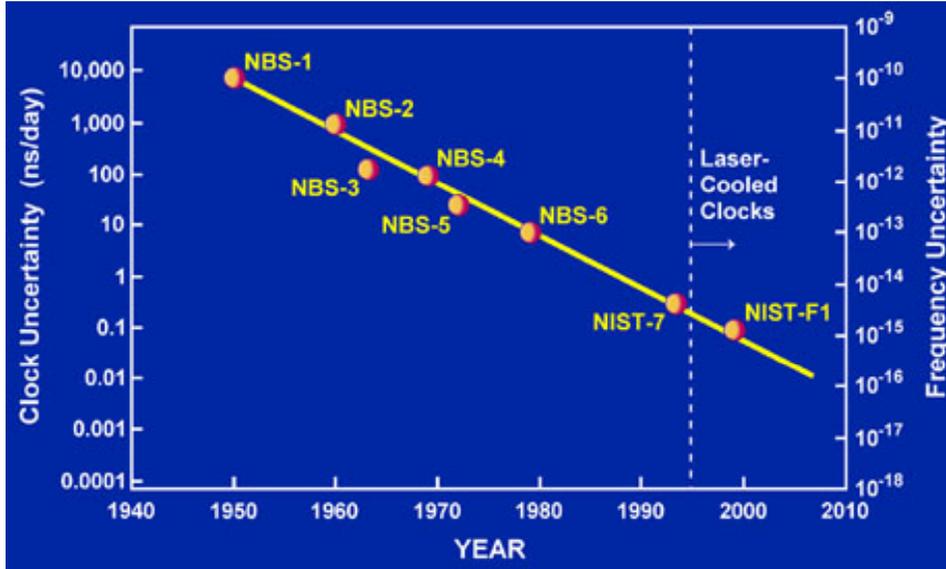


Ziel: am Umkehrpunkt sind die Teilchen langsam, so dass sie für lange Zeit mit der Hochfrequenzkavität wechselwirken können. Dadurch wird die Energiemessung, resp. Frequenzmessung genauer:

Energie-Zeit Unschärferelation!

# Atom clock

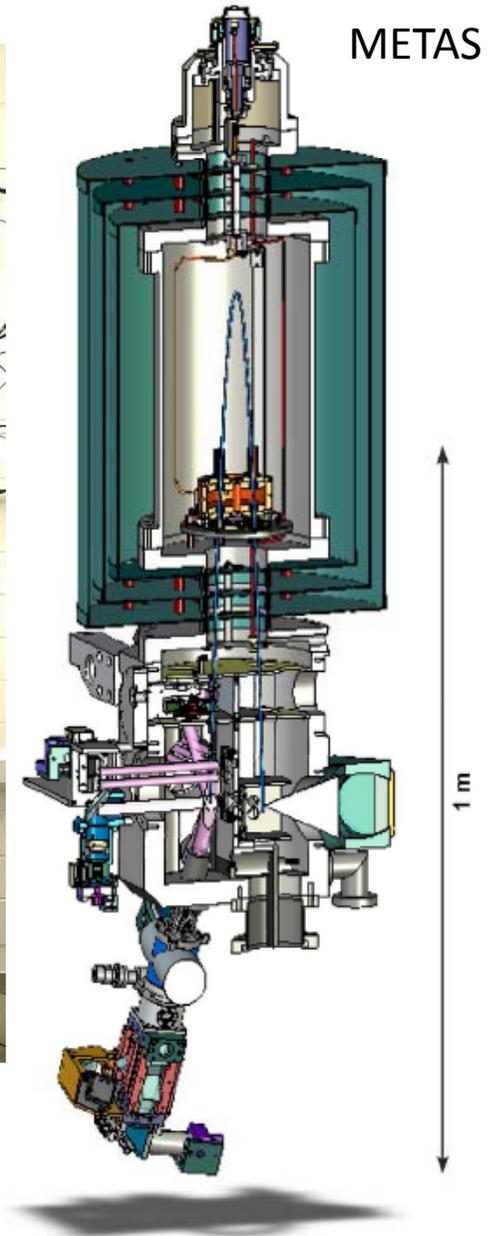
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Cs oven



fountain



# Atom clock

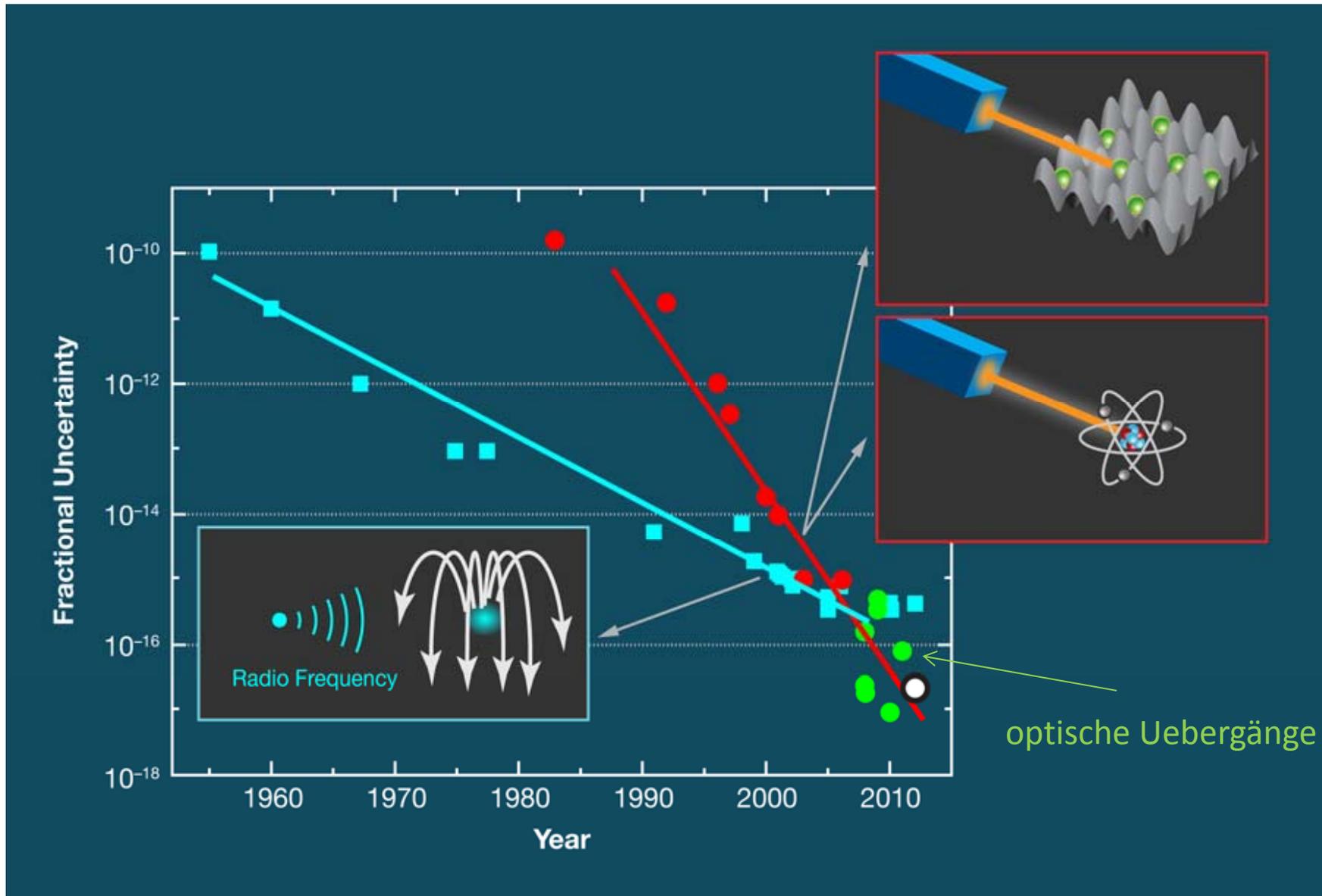
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CH



$\hbar\nu$



# Solvay Conferences

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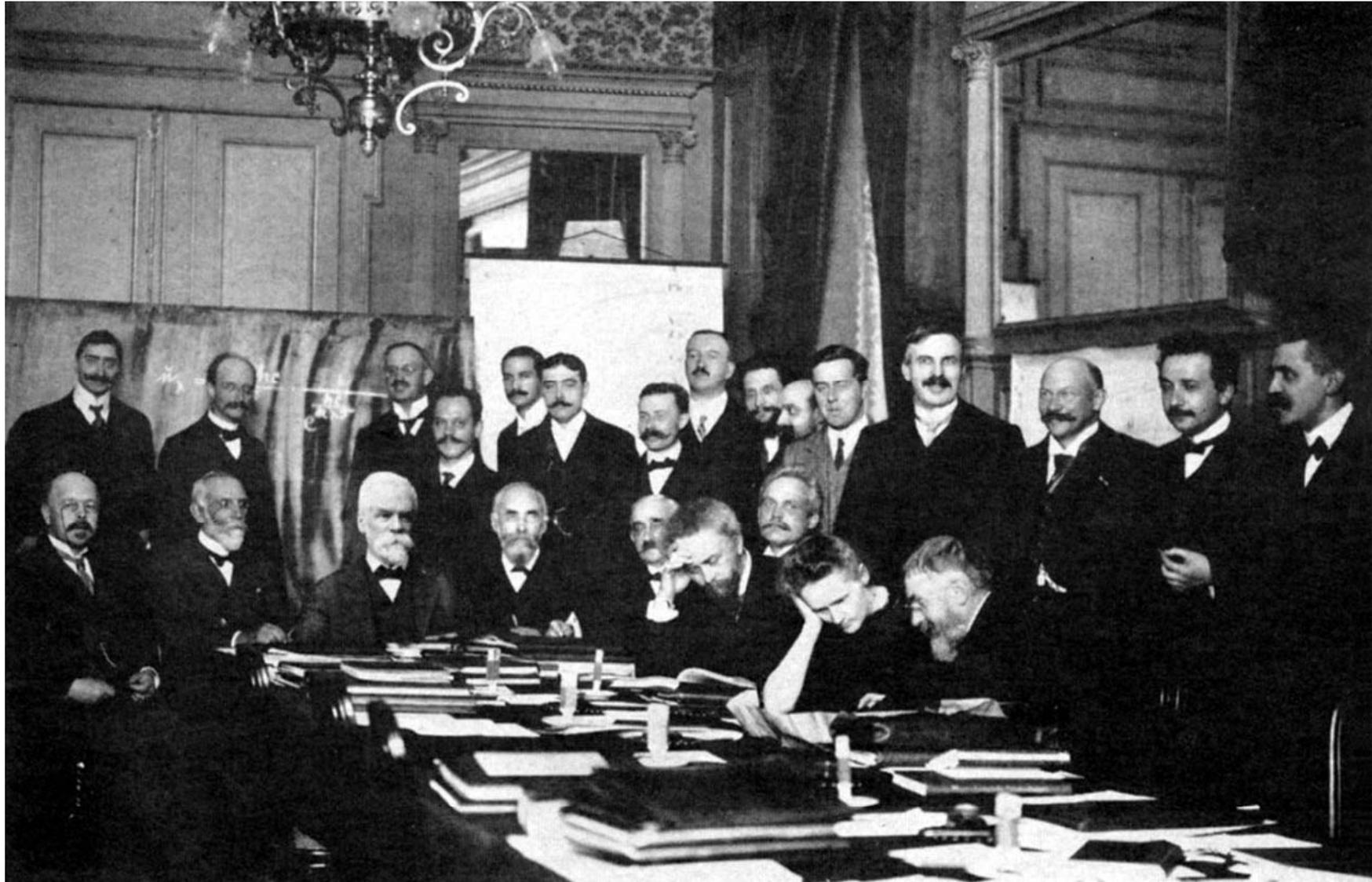


CH

UNI  
BASEL



h<sub>v</sub>

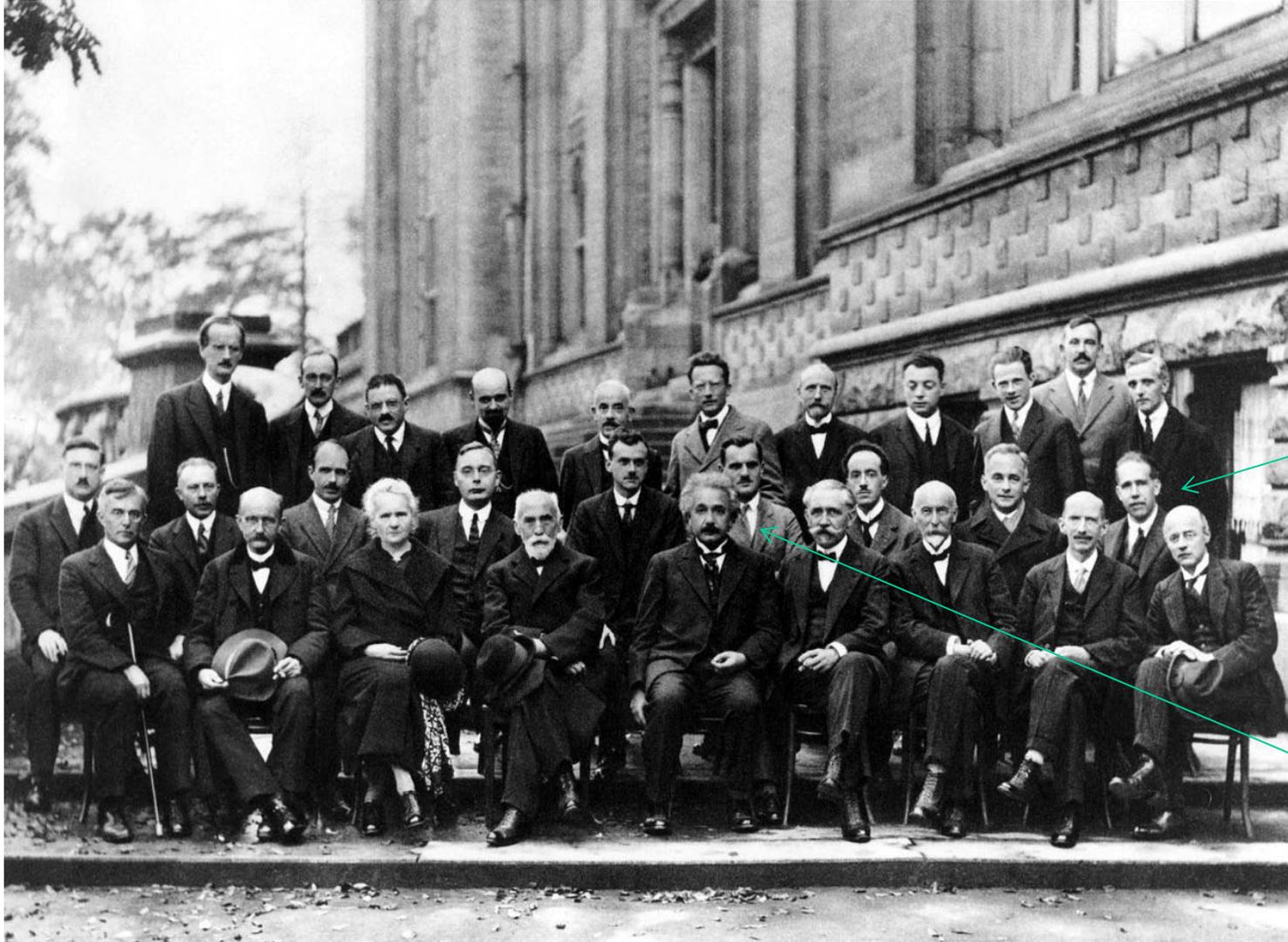


# Solvay Conferences

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5<sup>th</sup> Solvay conference  
Brussels 1927  
on the Electron  
and Photon



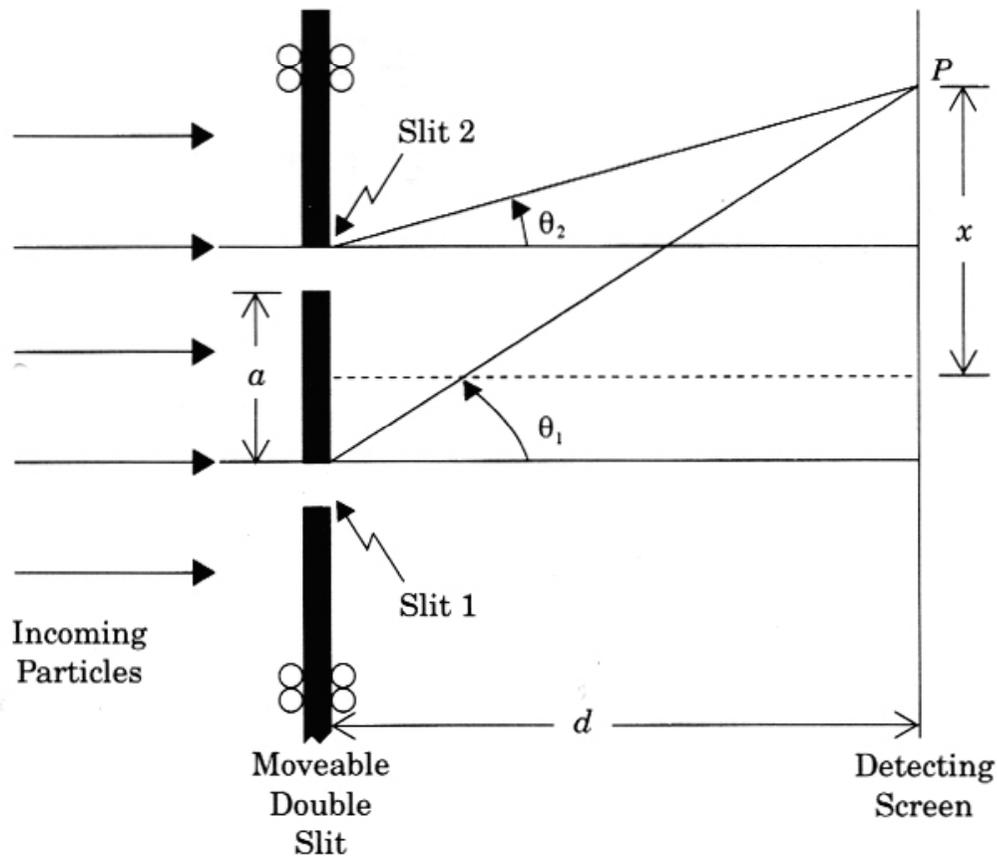
Bohr

Einstein

Left-to right: Top row: A. Piccard, E. Henriot, P. Ehrenfest, Ed. Herzen, Th. De Donder, E. Schrödinger, E. Verschaffelt, W. Pauli, W. Heisenberg, R.H. Fowler, Léon Brillouin. Middle: P. Debye, M. Knudsen, W.L. Bragg, H.A. Kramers, P.A.M. Dirac, A.H. Compton, Louis de Broglie, Max Born, Niel Bohr. Front row: I. Langmuir, Max Planck, Marie Curie, H.A. Lorentz, A. Einstein, P. Langevin, Ch. E. Guye, C.T.R. Wilson, O.W. Richardson.

# Einstein-Bohr Debatte

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

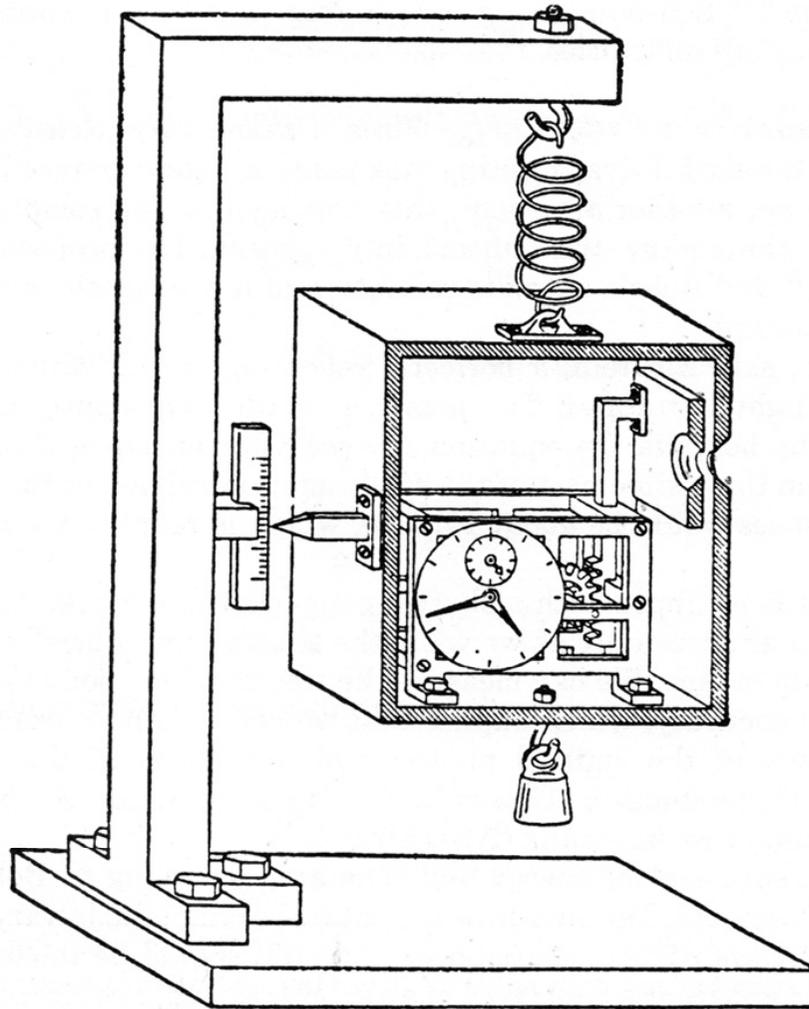


aus: "The Quantum Challenge"  
by G. Greenstein & A.G. Zajonc

Bohr kann zeigen, dass die Interferenz verschwindet, wenn man in der Lage ist, die Impulsänderungen im Schirm genau genug messen zu können

**Figure 4-1** *Einstein's First Attack on Complementarity.* The two slits of the classic double-slit experiment are free to move. By measuring their recoil, Einstein argued, we can find out which path the particle took.

aus: "The Quantum Challenge"  
by G. Greenstein & A.G. Zajonc



Einstein: open slit and detect when photon leaves by noting down the time, the later measure the change of mass with arbitrary accuracy, hence energy with  $\Delta E \rightarrow 0$ .

Bohr uses Einstein's principle from general relativity to show that time cannot be measured accurately and that energy is uncertain due to position measurement of mass pointer

**Figure 4-2** *Einstein's Second Attack on Complementarity.* The box contains light; at a predetermined time, known as accurately as we wish, the mechanism opens the shutter and allows a single photon to escape. We then measure the new mass of the box and so, according to Einstein, determine the escaped photon's energy to arbitrary accuracy. SOURCE: Reprinted by permission of Open Court Trade & Academic Books, a division of Carus Publishing Company, Peru, IL, from *Albert Einstein: Philosopher-Scientist*, P.A. Schilpp (ed.) (Evanston, IL: Library of Living Philosophers, 1949), p. 227.

# Einstein-Bohr Debatte

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When champions meet:  
Rethinking the Bohr–Einstein debate

N.P. Landsman

‘The belief in an external world independent of the perceiving subject is the basis of all natural science.’ (Einstein, 1954, p. 266.)<sup>34</sup>

‘However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. (...) The argument is simply that by the word *experiment* we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangements and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.’ (Bohr, 1949, p. 209.)

‘One may call these uncertainties objective, in that they are simply a consequence of the fact that we describe the experiment in terms of classical physics; they do not depend in detail on the observer. One may call them subjective, in that they reflect our incomplete knowledge of the world.’ (Heisenberg, 1958, pp. 53–54.)

- *Einstein’s realism is the objectivity of spatially separated systems;*
- *Bohr’s realism is the objectivity of classical physics.*

# Schrödinger Gleichung

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de Broglie (1892-1987), Nobelpreis 1929



Louis deBroglie

Doktorarbeit 1924

deBroglie-Wellenlänge

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{m \cdot v}$$



Erwin Schrödinger

1926

Schrödingergleichung

$$i\hbar \frac{\partial}{\partial t} \psi = H \psi$$