QUANTUM TRANSPORT IN NANO–STRUCTURED SEMICONDUCTORS

A Survey

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Abstract. The quantum transport effects in semiconductor nano–structures discovered during the past two decades are summarized. Brief physical arguments for their explanation are provided. Possible directions of future research are outlined.

Due to their unique adjustability of charge carrier density by external means, semiconductor inversion layers have been proven to provide an outstanding laboratory for the investigation of quantum mechanical phenomena in condensed matter. During the past two decades, a great variety of hitherto unforeseen quantization and coherence effects in their electrical transport properties have been discovered. The most prominent example is the quantum Hall effect. The finding of the quantization of the Hall conductivity of MOSFETs in integer multiples of e^2/h at low temperatures and sufficiently strong magnetic fields initiated an "industry" of experimental and theoretical research. The *Integer Quantum Hall* effect established a completely new tool for the investigation of localization phenomena. The subsequent discovery of the *Fractional Quantum Hall* effect gave rise to totally unexpected developments concerning the effects of the Coulomb interaction. Novel phases of the interacting two dimensional electronic system, like the "incompressible electron fluid", were found. New routes to well known concepts like the Wigner crystal suddenly became experimentally accessible.

With refined preparation techniques, it became possible to prepare inversion layers that are laterally structured. Quasi–one dimensional inversion layers exhibit unique quantization and fluctuation phenomena. Systems of

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two–dimensional point contacts were designed to form islands of electrons, quantum dots, which showed characteristic oscillatory transport behavior — signature of the Coulomb repulsion between the electrons. Arrays of quantum dots were discovered to allow for the systematic experimental study of signatures of chaos in quantum systems. The long–standing theoretical prediction of persistent currents in normally conducting metallic systems was experimentally verified by using a structured inversion layer imbedded in a AlGaAs/GaAs–heterostructure. Even nowadays the field is still rapidly evolving. No saturation of the activities is yet in sight. Practically every year a new effect is reported in the literature.

In the following, a brief survey of the quantum transport effects in nano–structured *semiconductors* which were discovered during the past two decades is given [1]. Emphasis will be on those aspects which are not discussed in the articles. Topics which are explained in detail in the later chapters will only be briefly addressed.

1. The Mesoscopic Regime

1.1. FROM DIFFUSIVE TO QUANTUM TRANSPORT

The classical charge transport in metals is described by the Drude theory [2]. The basic result is that the DC–conductivity of a metal is

$$\sigma = \frac{ne^2\tau}{m}, \tag{1}$$

with the density of the electrons (charge $-e$) n, the effective mass m and the *mean free time* τ. The latter incorporates all of the scattering processes the electrons suffer from static impurities, vacancies and dislocations, and also from other elementary processes like electron–phonon and electron–electron scattering. The basic assumption behind the Drude theory is that scatterings are *incoherent*: the electrons, "after having suffered a collision, do not remember that they existed before". Subject to the influence of the electric field, they move *diffusively* through the lattice of the metal ions. One of the consequences of this is *Matthiesen's rule*, stating that the contributions of different scattering processes are independent and additive, i. e. the total scattering rate is given by the sum of the corresponding rates.

At sufficiently low temperatures this assumption breaks down. The quantum mechanical nature of the electrons comes into play. Incoherent processes that destroy the "phase memory" of the electrons, as electron–phonon scattering, are more or less frozen out. What remains is scattering at the impurities which is *not* incoherent. The quantum mechanical state of an electron depends on the configuration of *all* of the imperfections. This important fact, which is the backbone of the physics of almost all of the mesoscopic transport phenomena, became obvious only about twenty years ago when at temperatures close to absolute zero the weak localization correction to metallic conduction in thin metallic films — quasi–two dimensional metallic systems — was discovered [3].

The thickness of the inversion layers in semiconductor hetero–structures is of the order of 5nm. Therefore, they can be considered as almost ideally two dimensional. They are perfect laboratories for the investigation of quantum coherent transport phenomena because it is possible to change the electronic properties by doping, and the electron density by applying an external gate voltage, in contrast to metallic systems. In addition, the lateral structure of the inversion layers can be systematically influenced by voltages at external gates. This enables us to construct single point contacts and also small islands of confined electrons — "artificial atoms" — which show transport quantization properties that are not at all predicted by the semi–classical theory, and they are *externally tunable* [4]. The arsenal of tools for the systematic investigation of quantum transport effects in structured semiconductors is completed by externally applied magnetic fields. This causes a number of additional, most surprising effects which are also not foreseen when using the semi–classical theory of electron transport.

1.2. MESOSCOPIC LENGTHS SCALES

There are several lengths scales which can be used to characterize the mesoscopic transport regime. The presence of imperfections in a metallic system gives rise to the *elastic mean free path*

$$\ell = v_F \tau \tag{2}$$

with the Fermi velocity v_F. It is the only limiting length for transport at $T = 0$ and is independent of the temperature. The mean free time τ has to be determined by quantum mechanical theory. If the perturbation introduced by the impurities is only weak one can use perturbation theory. In lowest order, $\tau^{-1} \propto \overline{V^2}$ where V is the random impurity potential. It is very important to note here that *the elastic mean free path has nothing to do with the destruction of phase coherence.* In principle, the underlying impurity scattering can be exactly taken into account by diagonalizing the Hamiltonian of the electron in the presence of the impurity potential. In metallic systems, ℓ is usually of the order of nanometers. In very pure semiconductor hetero–structures the mean free path can be much longer than 10μm, several orders of magnitude larger than the interatomic distance!

At finite temperatures, there are basically two additional limiting influences on the transport. First of all, the conductivity is an average over the states within an interval $\Delta E \propto kT$ near the Fermi level, as one can easily

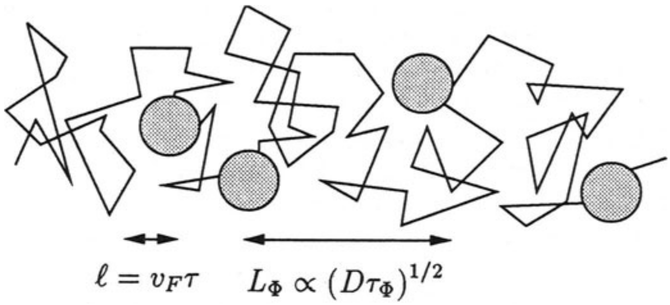

Figure 1. **Diffusive motion of a particle in an impure metal at a temperature close to absolute zero under the influence of impurity scattering (mean free path $\ell = v_F\tau$), and rare phase randomizing scattering processes (shaded circles, phase coherence length L_φ).**

see by using the Kubo formula. Since the phases of different eigenstates are completely independent, we expect a decay of the average correlation function on a time scale

$$\tau_T(T) \propto (kT)^{-1}. \tag{3}$$

This is usually interpreted as a *temperature–induced phase coherence time.* On the other hand, interactions with other elementary excitations as phonons or the Boson–like pair excitations of the electrons, lead to mixing of the one–electron states. These scatterings are in general inelastic and therefore lead to *phase incoherence* with a temperature dependent characteristic time $\tau_i(T)$ which is the mean free time between *inelastic* scattering events. If one assumes that at low temperatures phase randomizing processes are sufficiently rare in comparison with the mean free time due to the impurities one can determine a *phase coherence length* by assuming diffusive transport — due to the impurity scattering — between two phase destroying scattering events (Fig. 1),

$$L_\varphi = \sqrt{D\tau_\varphi(T)}. \tag{4}$$

The phase coherence time τ_φ is the mean free time between two successive phase randomizing events. The diffusion constant D contains only the impurity scattering which does not destroy quantum coherence. It is related to the residual conductivity via the Einstein relation

$$\sigma = e^2\rho D \tag{5}$$

where ρ is the state density at the Fermi energy. In general, the relation between τ_φ and L_φ is more complicated. For instance, in the hopping region, where $D = 0$, L_φ is given be the mean hopping distance.

The temperature dependence of L_φ is determined by the nature of the contributing scattering processes and is presently a subject of the research world–wide. The understanding is far from complete. Generally, one assumes for the phase coherence time in a metal ($D \neq 0$)

$$\tau_\varphi(T) \propto T^{-p} \qquad (6)$$

with $1 \leq p \leq 5$ depending on the nature of the scattering, the temperature and other parameters. At low temperature, the smallest of the phase coherence times limits the transport. If $p > 1$, the phase coherence at very low temperatures is eventually given by $\tau_T \propto (kT)^{-1}$ such that $L_\varphi \propto T^{-1/2}$.

It is now easy to provide a *criterium for mesoscopic transport*: the temperature has to be so low that $L_\varphi(T) > L$, the geometrical diameter of the sample. Typically, in metals $L_\varphi(1K) = O(1\mu m)$. In semiconductor systems, especially when a magnetic field is applied, L_φ can be considerably larger. We can also specifiy now what we mean by a d–dimensional mesoscopic system: if the thickness of the system in, say, the z–direction is smaller than L_φ we have a two dimensional system. When in addition L_φ is larger than the extensions in the x and y–directions, the dimensionality will be further reduced to $d = 1$ and $d = 0$, respectively.

2. Mesoscopic Transport Phenomena

2.1. THE INTEGER QUANTUM HALL EFFECT

The Quantum Hall Effect was discovered in 1980 by Klaus von Klitzing when he investigated the magneto–transport properties of the inversion layer in a Silicon MOSFET at low temperature ($T \approx 1$K) and at high magnetic field ($B \approx 20$T) [5]. He found that when the (negative) voltage at the gate of the transistor was increased, the Hall voltage did not decrease monotonically. Such a decrease is indeed expected according to the classical theory of the Hall effect, when assuming that the charge density in the inversion layer decreases monotonically with increasing gate voltage. Instead, the Hall voltage was found to remain constant in certain regions. Here, the voltage parallel to the source–drain current turned out to be unmeasurably small. The corresponding values of the Hall resistance R_H were precisely given by integer fractions of $R_K = h/e^2$,

$$R_H = \frac{1}{i} R_K \qquad (i = 1, 2, 3, \cdots). \qquad (7)$$

The Hall conductance $\Gamma_H \equiv 1/R_H$ is then quantized in units of e^2/h, the Sommerfeld constant.

While the relative accuracy of the quantization in first experiment was only of the order of e few 10^{-6}, later experiments, done at lower temperatures, $T \approx 50$mK, and different samples, AlGaAs/GaAs hetero–structures, showed a dramatic increase in precision. Nowadays, the reproducibility of the plateaus is better than 10^{-8} such that the Quantum Hall Effect is used as a standard for the electrical resistance.

A number of fundamental questions emerged as a result of the discovery of this first of the quantization effects in electrical transport. One of the conclusions of the weak localization theory of transport was that two dimensional disordered quantum systems at zero temperature cannot conduct the electrical current due to strong enhancement of quantum backscattering. All of the quantum states are localized. Under these conditions, it was hard to believe that such a precise, material–independent quantization effect could exist. The only way out was the assumption that the strong magnetic field delocalized at least a few of the states [6].

This hypothesis could be confirmed by later numerical calculations [7]. The results showed that indeed all of the states in two dimensional disordered systems in a strong magnetic field are localized. However, the localization length was found to diverge in the centers of the Landau bands, $E = 0$, with a power law

$$\lambda_0 |E|^{-\nu}, \qquad (\lambda_0 = \text{ constant}). \tag{8}$$

The critical exponent was quantitatively determined, $\nu = 2.34 \pm 0.04$, and shown to be universal, i. e. independent of the nature of the randomness, and the Landau band index. By using this divergent behavior and assuming that the largest possible localization length in the system was the temperature dependent phase coherence length it turned out to be possible to determine, for instance, the temperature dependence of the widths of the Hall plateaus. The results were consistent with the experimental findings. Further experiments done on samples with different geometrical sizes yielded even a value for the exponent that was consistent with the above result [8].

Basically, the existence of the singularities of the localization length in the centers of the Landau bands my be qualitatively understood by considering the percolation limit: for an extremely high magnetic field the magnetic length $\ell_B \equiv (h/eB)^{1/2}$ is small compared with the spatial correlation length of the random potential. Then one can show that only the Landau states centered at the positions corresponding to the randomly percolating equi–potential lines defined by $V(r_\mathrm{E}) = E$ contribute to the eigenstates at energy E in the presence of disorder [11, 12]. The localization problem is reduced to a percolation problem: the "landscape" of the random potential is filled with water up to a given level — the energy of the state. The shore lines correspond to the equi–potential lines. For low water level, there are only isolated lakes. All shore lines are closed. The states are localized. Correspondingly, for high water levels, there are isolated mountains in a sea of water. Again all of the shore lines are closed, and the corresponding states localized. It is intuitively clear that there must be exactly one water level at which one can reach two different edges of the system by travelling along the shore lines. This corresponds to the percolation threshold, and represents the energy where the localization length diverges.

In this way, the integer Quantum Hall Effect was identified as a *degenerate metal–insulator transition.* Although there is no predictive theory up to now which explains why the plateaus in the Hall resistance are practically *exactly* given by integer fractions of the von Klitzing constant R_K, a large number of new quantum properties were discovered when attempting to find such a theory. A most important discovery of the past years was that the states at the critical point have multifractal properties [9, 10].

A further important discovery was that in two dimensional systems in a strong magnetic field quantum coherent edge states play an important role for the understanding of magneto–transport [13, 14]. In the above picture of the landscape filled with water they can be vizualized by considering a landscape with boundaries represented by infintely high walls. Then one of the shore lines goes around the whole system. In the semi–classical picture of magneto–transport edge states correspond to the so–called "skipping orbits" which are essentially cyclotron orbits travelling along the edges.

Edge states can have coherence lengths even of several hundred micrometers due to the absence of backscattering induced by the magnetic field. They might play an important role for the explanation of the precision of the Quantum Hall Effect.

2.2. FRACTIONAL QUANTUM HALL EFFECT

The integer Quantum Hall Effect initiated numerous experimental and theoretical investigations of the two dimensional electron systems in semiconductor hetero–structures. A very important discovery only a few years later [15] was the fractional Quantum Hall Effect. In highly pure AlGaAs/GaAs samples with electron mobilities higher than, say 100000 $\mathrm{Vcm/s^2}$, the Hall conductance was found to be quantized at certain *rational* multiples of e^2/h,

$$\frac{1}{R_H} = \frac{p}{q}\frac{e^2}{h} \qquad (p, q \text{ integers}). \tag{9}$$

First attempts to explain the additional plateaus which appeared at the rational filling factors $\nu \equiv nh/eB = p/q$ within the one–electron approximation failed. Very rapidly, it became clear that the Fractional Quantum Hall Effect was a direct manifestation of the electron–electron interaction in the two dimensional system subject to the strong magnetic field. There have been several attempts to construct the many particle states for this system [16, 17]. Numerical diagonalizations of several interacting particles provided interesting information about their properties [18]. An important feature is their *incompressibility* which is thought to be crucial for the explanation of the Quantum Hall Effect. The *incompressible electron fluid* is presently a subject of extremely active research. For the first time, there are experimental possibilities to prepare externally controlled correlated many–particle states and to perform systematic experiments in order to investigate their nature. Important fundamental questions, for instance, whether or not, and under what conditions, a Wigner solid is formed in the two dimensional electron system in semiconductor inversion layers, can now be investigated in great detail.

The physics of the Quantum Hall Effect is reviewed extensively in the articles by R. Haug and A. H. MacDonald in the second chapter.

There are also predictions that edge states exist in the fractional quantum Hall regime. They are of particular importance since they can possibly be used to study experimentally [19] the effect of the interactions on the properties of one dimensional electron systems. The latter seem to be paradigms of non–Fermi liquid behavior. The articles on the *Luttinger liquid* in the chapter on "Interactions and Correlations" provide a panorama of many aspects of this presently very active field.

2.3. CONDUCTANCE OF POINT CONTACTS AND QUANTUM WIRES

In contrast to their classical counterparts, properties in mesoscopic physics can be quantized. The above example shows that this can also be the case for non–equilibrium properties such as transport. In the Quantum Hall Effect, the quantization was induced by a magnetic field. In this section, I briefly discuss the quantization of low–temperature transport caused by geometrical confinement.

Experimentally, this was observed for the first time in 1988 [20, 21]. In these experiments, the two dimensional inversion layer in AlGaAs/GaAs hetero–structures were structured laterally by applying a negative voltage to a metallic "split gate" above the electron gas. The applied gate voltage, if it is sufficiently high, eventually leads to a depletion of the electron density below the electrodes. Only below the opening between the two gate electrodes the electron density can be non–zero. The electrical conductance of this "point contact" changes discontinuously with the electron concentration or the width of the opening: it is quantized in units of e^2/h.

It is comparatively simple to understand this effect qualitatively. Consider a gas of non–interacting electrons confined within a strip of length $L(\to \infty)$ (periodic boundary conditions) and width W. The geometrical constriction within the strip leads to an energy spectrum which consists essentially of one dimensional energy bands $E_\mu(k) \propto E_\mu + \hbar^2k^2/2m$. Here,

$E_\mu \propto W^{-2}$ is the quantization of the energy induced by the constriction. Assume that at $T = 0$ only $\mu_{\rm occ}$ of the subbands are occupied. The Fermi velocity in each of the bands then depends on the band index μ

$$v_\mu = \sqrt{2(E_F - E_\mu)/m}. \qquad (10)$$

Further, we assume that there are no scattering processes within the system length L (the length of the constriction in the split gate geometry). Indeed, the experiments were performed using samples with very high mobility with mean free paths of the order of 10μm, whereas the geometrical dimensions of the constriction were about $1 \times 0.25\mu\text{m}^2$. The time needed by an electron with velocity v_μ to pass the interval of the length L is

$$\tau_\mu = L v_\mu^{-1}. \qquad (11)$$

By inserting into the Drude formula for the conductivity, eq. (1), and remembering that the electron number density in the μ–th subband is

$$n_\mu = \frac{1}{h}\sqrt{2m(E_F - E_\mu)}, \qquad (12)$$

one obtains for the corresponding contribution to the conductivity $\sigma_\mu = e^2 L/h$. The total conductance of the constriction, $\Gamma \equiv \sigma/L$ is then

$$\Gamma = \sum_\mu \sigma_\mu = \mu_{\rm occ}\frac{e^2}{h}. \qquad (13)$$

If the subbands are spin–degenerate this expression has to be multiplied by a factor of two. A magnetic field lifts the degeneracy.

It is obvious that the complete theory is more difficult. For instance, the above assumption of ideal one dimensional energy bands is certainly not valid for a constriction of a finite length of the order of a few micrometer. One can, however, show that sufficiently smooth constrictions, i. e. when the radius of curvature is large compared with the Fermi wavelength, the corrections to the quantized values are very small [22]. Also the presence of disorder [23] or the interaction with phonons (at finite temperatures) [24] do not lead to an immediate breakdown of the quantization of conductance. All of these effects can be treated by standard quantum theory essentially quantitatively. The results are consistent with experimental findings. One can summarize that the quantization of the conductance of point contacts in semiconductor inversion layers seems to be presently a relatively well understood phenomenon. An open question, which is the subject of present theoretical and experimental work, is whether or not electron–electron interaction leads to a renormalization of the quantization, as predicted by the theory of transport in the *Luttinger liquid* (cf. chapter 3).

Less complete is the understanding of the behavior of the conductance when several gates are used to laterally confine the electrons within a finite region in the plane. Two point contacts in series, for instance, can be used to confine the electrons between them. In such a way, an electron island — an "artificial atom" or "quantum dot" — is formed, with rather peculiar transport properties. They will be discussed below in more detail. One can also fabricate small electron cavities of a shape which would lead in the classical limit to *chaotic behavior* [25]. Investigation of the quantum transport in such systems provides in principle valuable insight into the connection between classical chaos and quantum behavior, one of the subjects that are now intensively studied theoretically as well as experimentally.

A paradigmatic system which is very suitable for the study of "quantum chaotic behavior" is a regular array of quantum dots in a perpendicular magnetic field. The article of D. Weiss introduces into this subject.

2.4. QUANTUM BLOCKADES

Transport experiments on "artificial atoms" at low temperatures exhibit also distinct and strong signatures of electron–electron interaction. This can be used to manipulate single electrons.

The Coulomb blockade effect was first seen in experiments that were done on tunnel contacts between metallic systems [26, 27]. In semiconductors, "artificial atoms" fabricated by using two split gates in series on top of an AlGaAs/GaAs hetero–structure [28] were used to demonstrate the drastic effect of the Coulomb repulsion on the tunneling of electrons. The current through the two point contacts was measured at millikelvin temperatures as a function of the voltage applied to a gate at the back of the structure for fixed bias voltage. The back gate serves to change the density of the electrons in the island between the point contacts. The current, which is then directly proportional to the conductance, shows characteristic resonance–like peaks that are equidistant.

The analysis of the experimental data showed that each peak in the current corresponds to exactly one electron passing the island. There was no Zeeman splitting of the peaks when a magnetic field was applied. The energetic distance between the peaks turned out to be given by the Coulomb energy of the electrons in the island.

The condition for zero–temperature linear transport through such a structure is that the difference between the ground state energies of $N+1$ and N electrons within the island lies within the interval between the Fermi energy E_F and $E_F + eV$ (V bias voltage),

$$E_F + eV \geq E(N+1) - E(N) \geq E_F. \tag{14}$$

Assuming that the ground state energy is given by the electrostatic Coulomb energy ($N \gg 1$)

$$E(N) \approx \frac{e^2 N^2}{2C} \tag{15}$$

where C is the capacity of the island, one obtains for $V \ll e/C$

$$E_F(N) \approx \frac{Ne^2}{C}. \tag{16}$$

Assuming further that the Fermi energy is proportional to the voltage at the back gate, $E_F = aV_g$ (a constant), the voltage difference between successive peaks in the current should be

$$\Delta V_g = a\frac{e^2}{C}. \tag{17}$$

Between the peaks linear transport is not possible, due to the Coulomb repulsion.

The Coulomb blockade effect can be used to construct standard for the electrical current in which electrons are transported one–by–one through the island by applying an AC–voltage to the island via a gate. The current through this "turnstile device" [29] is given by the electron charge multiplied by the frequency of the AC voltage — typically of the order of a few MHz — which can be very accurately calibrated. Unfortunately, the currents that can be produced by this device are presently rather small, of the order of pA, and the accuracy is limited by several inherent physical effects as, for instance, simultaneous tunneling through both of the point contacts, to approximately 10^{-4}. Another application of the Coulomb blockade effect is the "single electron transistor" [30] (see Chapter 3 of this volume).

The Coulomb blockade of linear transport can be considered to be a consequence of the quantization of the charge, and the "selection rule" that the minimum charge that can enter or leave the electron island is e,

$$\Delta Q_{\text{island}} = \pm e. \tag{18}$$

This point of view leads immediately to a generalization: the blocking of transport processes by selection rules corresponding to other quantum numbers.

For instance, since each electron which enters or leaves the island carries exactly the spin $1/2$ the total spin of the electrons on the island can change only by $\pm 1/2$. Obviously, there is a "spin selection rule"

$$\Delta S = \pm\frac{1}{2}. \tag{19}$$

If the two successive ground states with N and $N+1$ particles differ in their spins by more than 1/2 the corresponding peak in the linear conductance should be suppressed.

In non–linear transport the electron spin can even lead to a negative differential conductance [31, 32]. When the total spin $S(N)$ of N electrons in the island is $N/2$, an electron can leave the dot only by simultaneously lowering the total spin by 1/2, in contrast to the general case, $1/2 < S(N) < N/2$ where the total spin can be increased or decreased by 1/2 depending on the polarization of the spin of the leaving electron. This reduces the possibilities for decreasing the number of electrons in the island. Thus, when *increasing* the voltage such that a totally spin–polarized excited state starts to contribute to the transport, the current through the quantum dot may be *decreased* [33, 34]. The "spin blockade effect" is discussed in the article by D. Weinmann in chapter 4.

2.5. QUANTUM INTERFERENCE

The direct experimental confirmation of the quantum interference mechanism underlying theory of weak localization was the detection of the Aharonov–Bohm like oscillations of the magneto–resistance of thin metallic cylinders [35, 36, 37]. They have diameters of about 1-2μm such that the electron states at low temperature are coherent around the whole circumference. If such a sample is placed into a magnetic field directed along the axis of the cylinder, electrons which travel clockwise around the cylinder (amplitude A_1) experience a phase shift relative to those travelling counter–clockwise (amplitude A_2). The phase shift is given by the magnetic flux Φ enclosed by the paths of the electrons,

$$\Delta\varphi = \frac{4\pi\Phi}{\Phi_0}, \qquad \Phi_0 \equiv \frac{h}{e}. \tag{20}$$

The total backscattering probability is

$$|A_1 + A_2|^2 \equiv 2|A|^2 \left[1 + \cos\left(4\pi\Phi/\Phi_0\right)\right]. \tag{21}$$

When the magnetic field is changed, the interference term leads to maxima in the backscattering probability at fluxes $n\Phi_0/2$ ($n = 0, \pm 1, \pm 2, \cdots$) corresponding to constructive interference. This is reflected in maxima in the magneto–resistance.

The important point is that the scattering at the impurities, which are in any case present in the sample, does not destroy the quantum mechanical coherence of the states. Indeed, from the magnitude of the residual resistance the electronic mean free path due to impurity scattering was estimated to be orders of magnitudes smaller than the sample diameter.

Classically, the electrons diffuse around the cylinder. Nevertheless, one observes effects which prove that the electrons are extended coherently around the whole circumference!

A further, most direct and impressive proof of the wave nature of the electron are the so–called *persistent currents* in small *normally* conducting rings subject to a penetrating magnetic flux. They have been predicted decades ago [38]. It has been only in 1990 that they have been experimentally detected [39, 40, 41].

The basic physics can be understood by considering an ideal, one dimensional ring penetrated by an Aharonov–Bohm flux Φ. The states of the (non–interacting) electrons are plane waves with wave vectors

$$k_n(\Phi) = \frac{2\pi}{L}\left(n - \frac{\Phi}{\Phi_0}\right), \qquad n = 0, \pm 1, \pm 2, \ldots. \tag{22}$$

The corresponding energy, $E_k(\Phi) = \hbar^2 k^2/2m$, is a function of the flux. It is very similar to that of a one dimensional empty lattice as function of the quasi–wave vector. Apparently, the spectrum is periodic in Φ/Φ_0 with the period 1. Therefore, one needs to consider only the interval $-1/2 < \Phi/\Phi_0 \leq 1/2$. Due to the presence of the flux the states carry a diamagnetic current. It is easily calculated to be

$$I_k(\Phi) = -\frac{dE_k(\Phi)}{d\Phi}. \tag{23}$$

The total current is then given by the sum of the contributions of all of the occupied bands, weighted by the Fermi function corresponding to the temperature T.

The presence of a time–independent (impurity) potential does not change the above situation, except that the amplitude of the current is reduced by a factor which depends on $\ell/L \ll 1$. One has to emphasize here that the persistent current is qualitatively different from the current in a transport experiment since it is an *equilibrium property* of the system. Although the latter has a finite resistance, due to the presence of the impurities, the persistent current does not decay (at $T = 0$).

Except for the most recent experiment which was done on a ring in an AlGaAs/GaAs hetero–structure [41] the experimental results indicate that in metallic rings are at least an order of magnitude larger than predicted by the current theories [42]. This reflects the limitations of present days' understanding of the interplay between disorder and interaction in metallic mesoscopic systems.

The recent efforts to improve the experiment — which is definitively at the borderline of measurement technology, since it involves *very* low temperatures and extremely sensitive detection of magnetization in addition to highly advanced nano–fabrication technology — are described in the paper by A. Benoit in chapter 4.

3. Theory of Mesoscopic Transport

The fundamental feature of the physics of nano–structures at low temperatures, namely the coherence of the quantum states over distances much longer than the mean free path, is also one of the main obstacles for the formulation of the theory of transport. Due to the absence of phase–breaking, inelastic processes in the sample the basic assumption of the Drude–Boltzmann theory breaks down, as mentioned before. As a result, it is not only the microscopic properties of the sample which determine the transport, but also its geometry, and in particular *how* a measurement is done. It is, for instance, no longer possible to define a macroscopic, constant parameter "conductivity" which is independent of the sample's geometry, and depends only on microscopic features as the nature of the atoms, their distances and the effective mass. Inherently, as a result of the coherence, the relation between current density and electric field becomes non–local

$$j(x) = \int dx' \sigma(x, x'; \omega) E(x'). \tag{24}$$

The non–local conductivity σ has to be calculated microscopically by linear response theory. It is only for an infinite system with incoherent scattering that one can replace $\sigma(x, x'; \omega)$ by an average value. Then, the relation between current density and voltage becomes local.

Unfortunately, the spatial distribution of the internal electric field $E(x)$ is not known. Its shape depends in a complicated way on the external potentials and incorporates also the influence of the interacting electrons. On the other hand, in the experiment, one measures the current through a sample as a result of an external voltage. Therefore, it is desirable to have a theory which connects not the local current density with the local electric field but the *total current* with the *external voltages.* In other words, one has to construct a theory for the *conductance* Γ which depends on the sample geometry, in order to characterize the linear transport.

One of the first to note this striking difference between classical and quantum transport was Landauer [43]. Already in 1971, he established the close connection between current transport in the quantum coherent regime and the transmission probability $\mathcal{T}$ for a quantum mechanical particle through a potential. The basic conjecture, namely

$$\Gamma = \frac{e^2}{\pi \hbar} \mathcal{T}, \tag{25}$$

is now widely accepted, and describes many of the experimental findings. It can also be derived from the quantum mechanical linear response theory, applied to a system which is connected to infinitely long ideal leads [44, 45, 46]. One of the striking findings of this approach, which is also true in the presence of interactions [47], is that the total current is only determined by the integral over the electric field, i. e. the external voltage. The current status of the Landauer approach to mesoscopic transport and its later generalizations by Büttiker, is summarized in chapter 5.

Unfortunately, it is not straightforward to modify the Landauer–Büttiker theory of mesoscopic transport to the frequency and time domain, and to interacting particles. This is the subject of current research. A few ideas are contained in the — certainly very incomplete — last chapter, in the introduction to the Luttinger liquid by M. Sassetti in chapter 3 and in the review article by M. Büttiker and T. Christen in chapter 5. It is obvious that future work has to concentrate more than before in this area, last but not least since future electronic devices based on nano–structures will have to operate at very high frequency, and the understanding of the underlying fundamental physics is imperative.

4. What has not been Mentioned

Topics not included in this "crash course" of mesoscopic transport in semiconductor nano–structures are details of the weak localization phenomena, the reproducible fluctuations of the conductance, the non–locality of the quantum transport which is an inherent feature due to the "stiffness" of the quantum states as a consequence of the coherence, stochastic time–dependent effects like "telegraph noise", and all kinds of mesoscopic effects in superconducting systems. Their treatment is beyond the scope of this introduction and also of this book. They are broadly described in [1].

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