

# Quantum Transport FS 2015

lecture given in spring 2015 by:



Christian Schönenberger



Dominik Zumbühl



Andreas Baumgartner



Markus Weiss

lecture is on Tuesdays 16:15 – 18:00

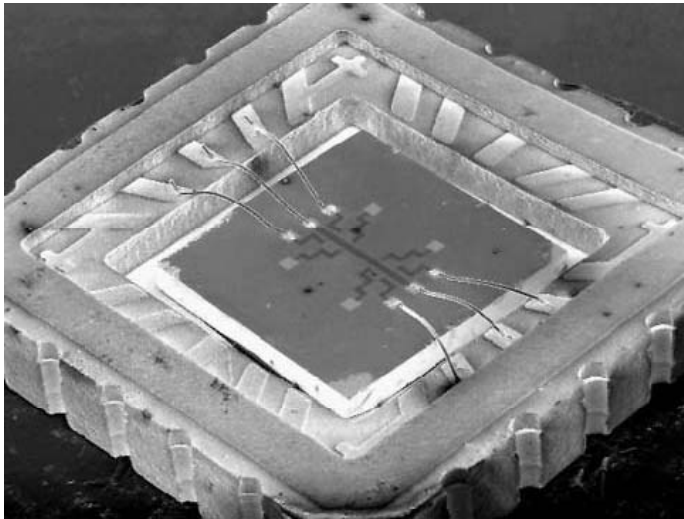
exercises are on Thursdays from 10:15 till 12:00

you must attend the exercises if you want to get the credit points  
in the last exercise we will make a written test

condition for credits: pass the test and have attended 80% of the exercise classes

all in English

# 1. Quantum Transport / Introduction



a chip carrier with a “sample”  
connected by bonding wires

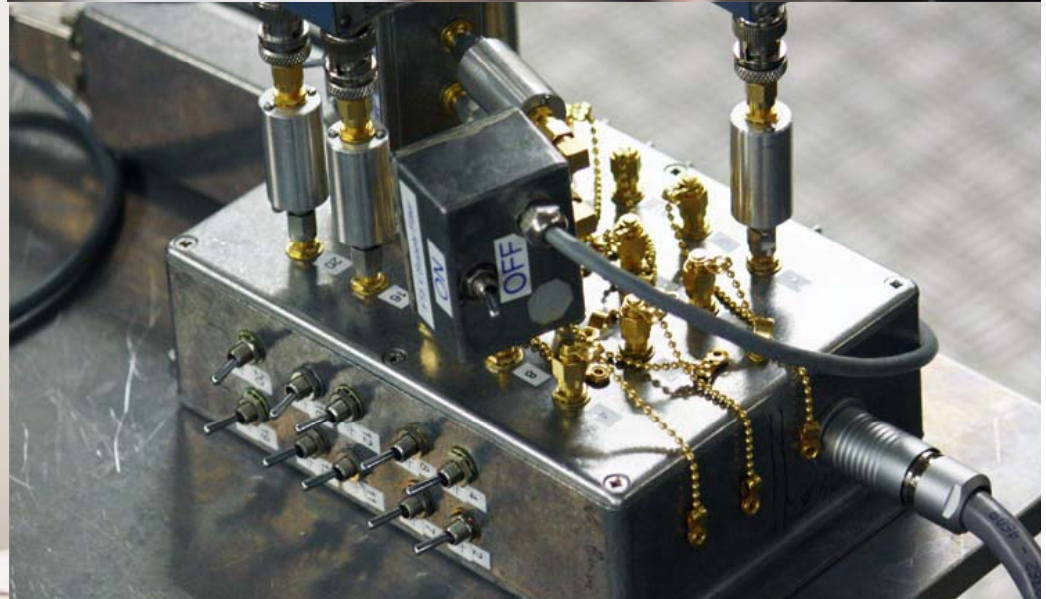
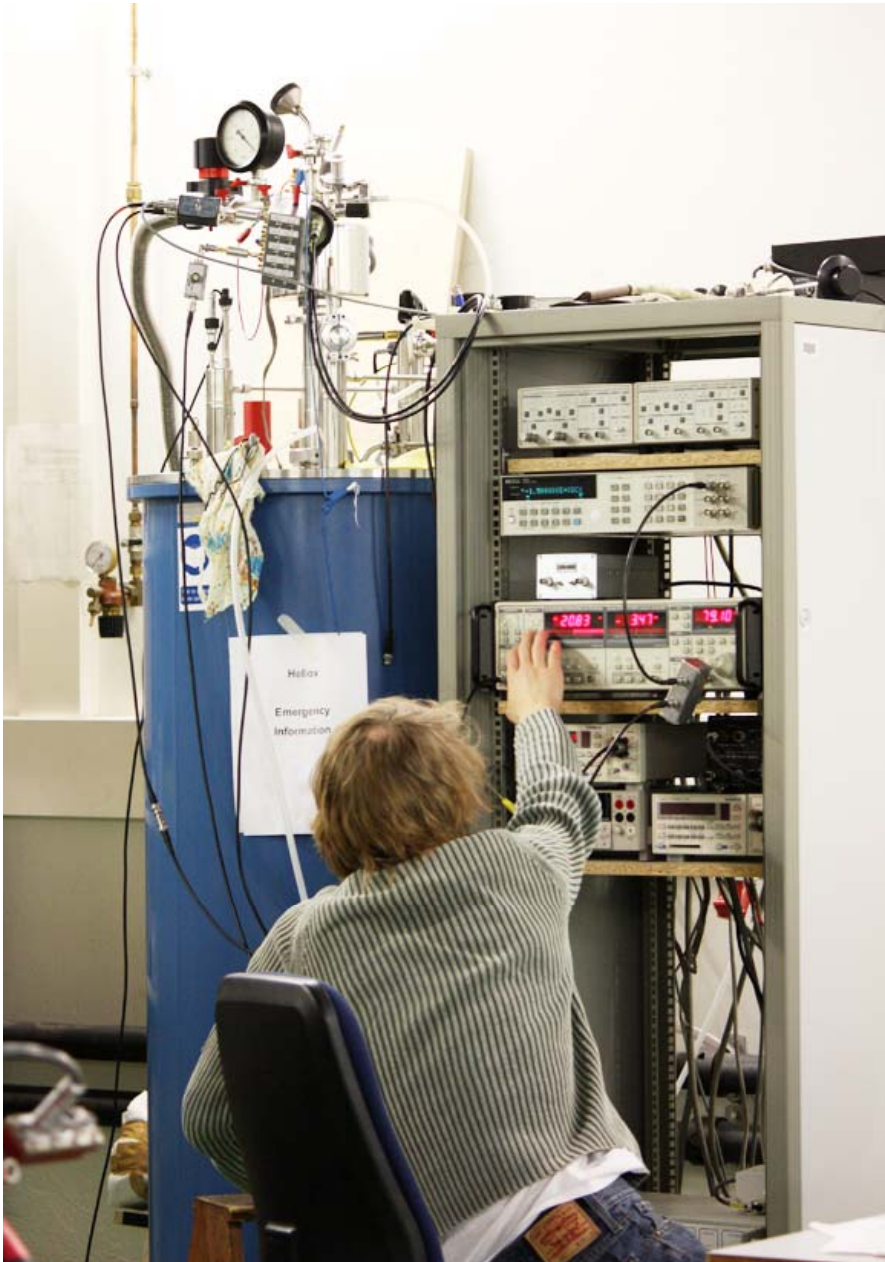
usually fabricated by  
lithography methods

often measured in a cryostat  
measurements mostly electric, but other degrees  
also of interest (e.g. photons)





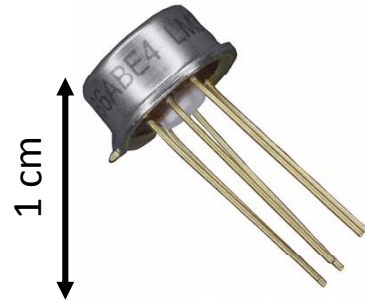
# 1. Intro / Measurement techniques



# 1. Intro / Motivation / history



Si transistor



„Chip“ integrated circuits



GHz electronics



stereo and  
TV for all



bits und bytes  
PC / internet for all

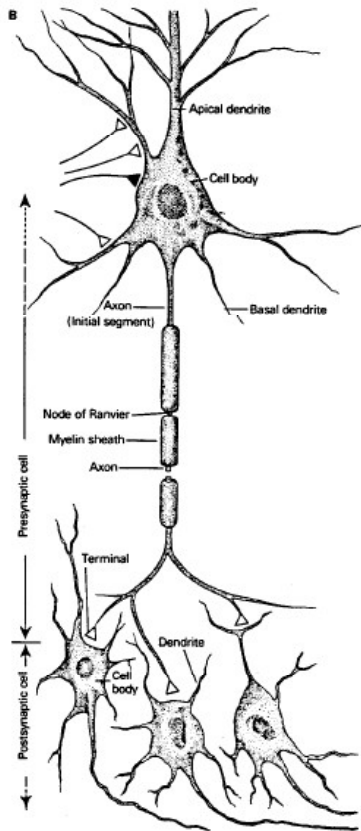


online at  
all times



# 1. Intro / Bioelectronics

Biochip: „brain on a chip“      DNA memory



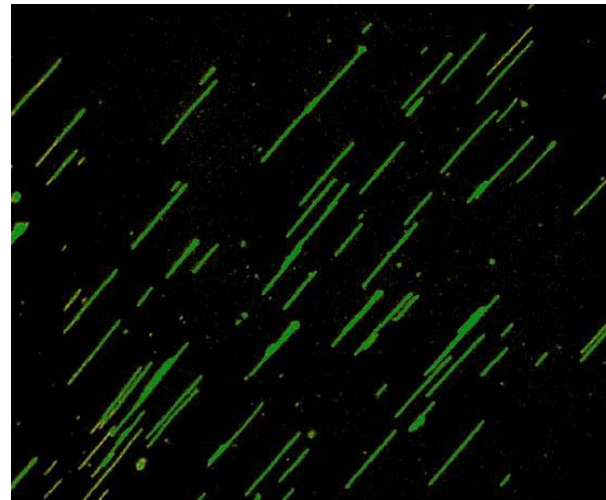
Brain, neural networks, and computation

J. J. Hopfield

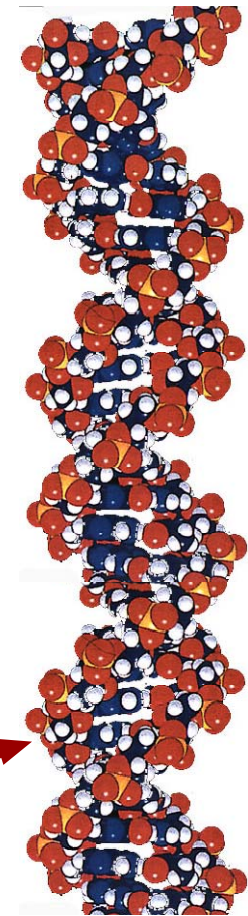


neurons on a chip

image of DNA



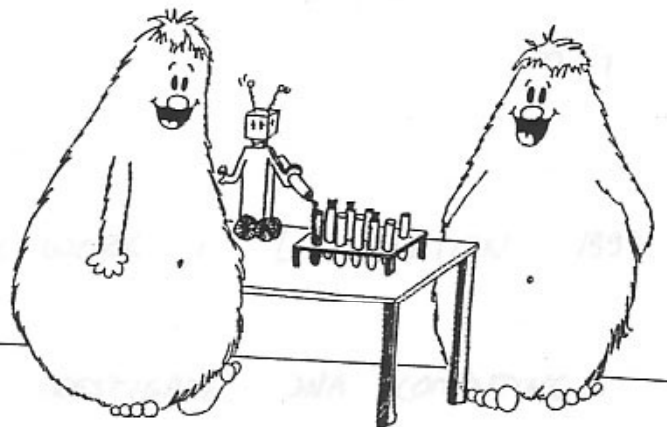
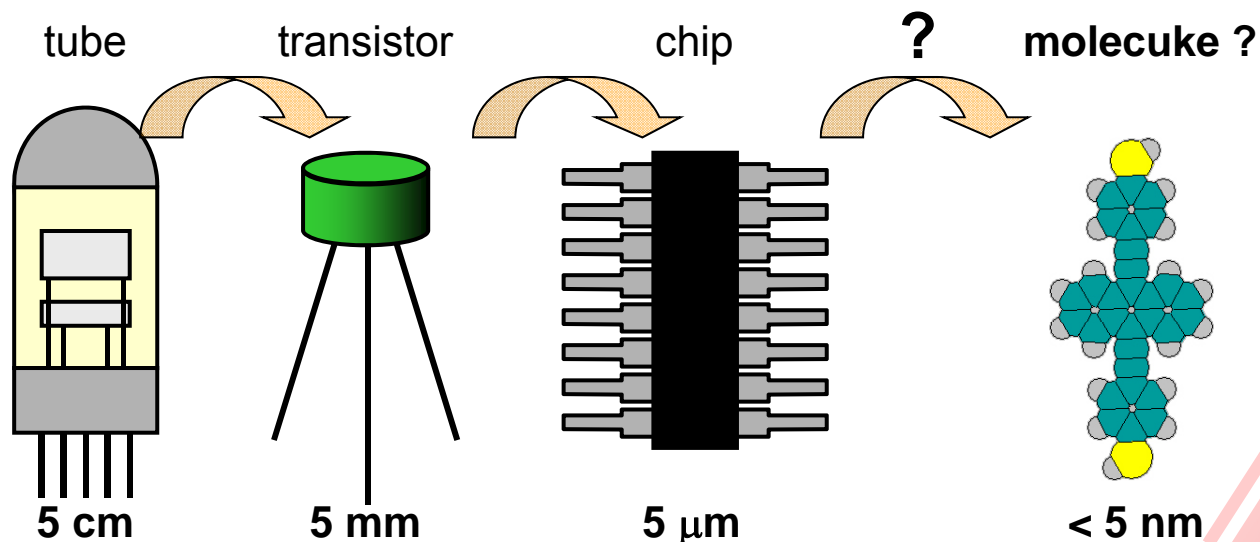
Erbsubstanzmoleküle



# 1. Intro / Molecular Electronics



molecular electronics: the computer in a test tube



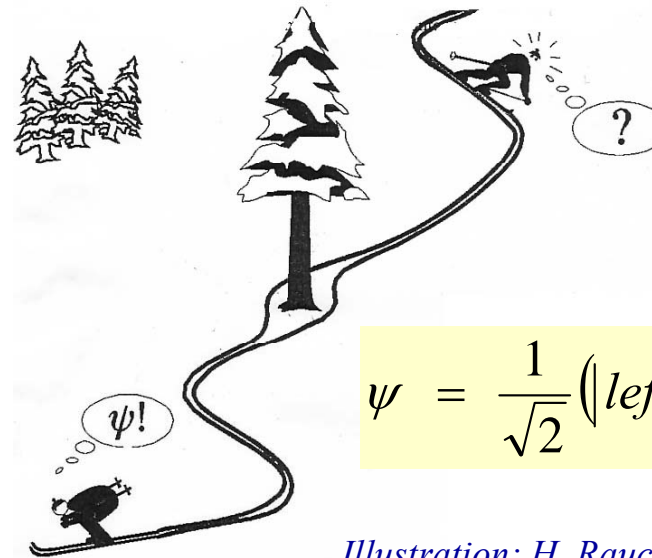
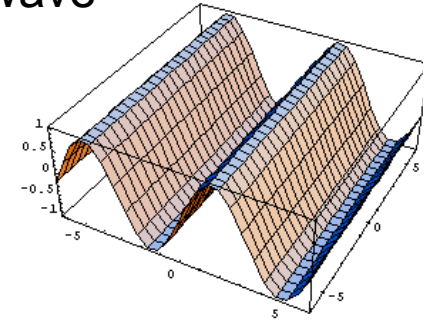


# 1. Intro / Quantum electronics



der Stromfluss **klassischer Elektronen** rauscht wie ein tropfender Wasserhahn

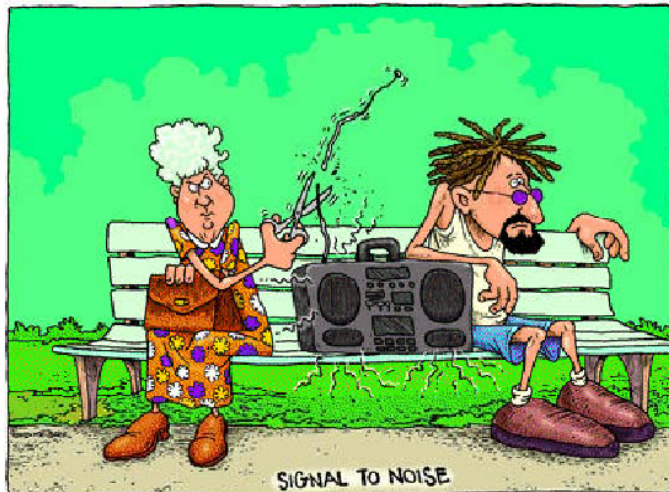
the electron is a **quantum particle**, it can behave as a wave and therefore interfere :



quantum effect

$$\psi = \frac{1}{\sqrt{2}} (|left\rangle + |right\rangle)$$

*Illustration: H. Rauch ATI, Wien*



quantum physics delivers concepts for a new way of computing → **quantum computing**

„The weirdest computer of all“ (The Economist)

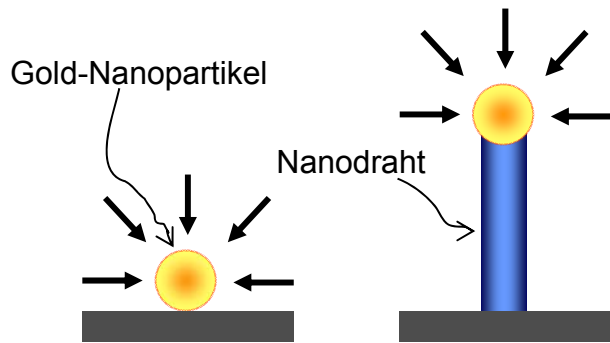
# 1. Technology plays a crucial role (but not only)

- **fabrication technology** (devices need to be made)
- devices get smaller and **smaller**
- signals get also smaller and **faster** and there is much more data
- **imaging** and analytics important
- engineering, physics, chemistry and biology may come together → **language problem**
- **new materials** may also give the field a decisive push

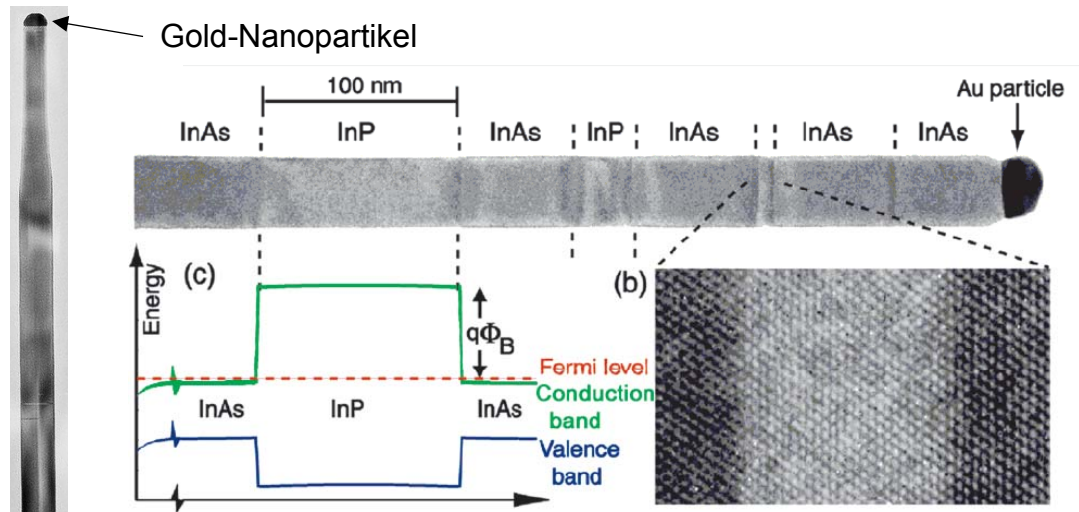
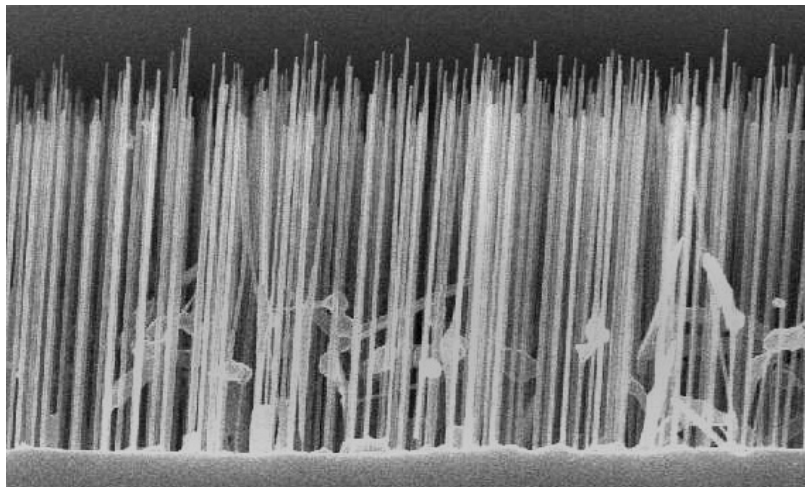


# for example: nanowires

gold nanoparticles are used to catalyze the growth of nanowires



diameter ~ 10-50 nm



aus: *M.T. Björk et al., Nano Lett. 2, 87 (2002)*

composition can be changed during growth



Jesper Nygard (Copenhagen)



Szabolcs Csonka (Budapest)

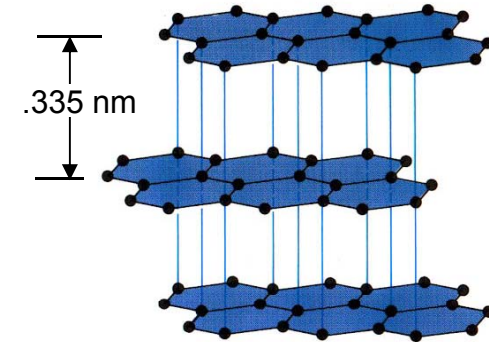
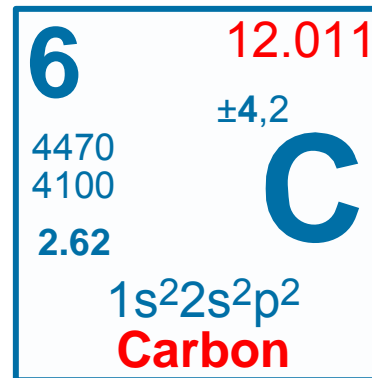
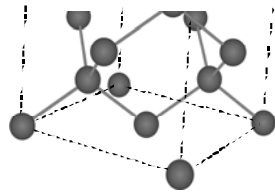


Lukas Hofstetter (Basel)

# Carbon materials

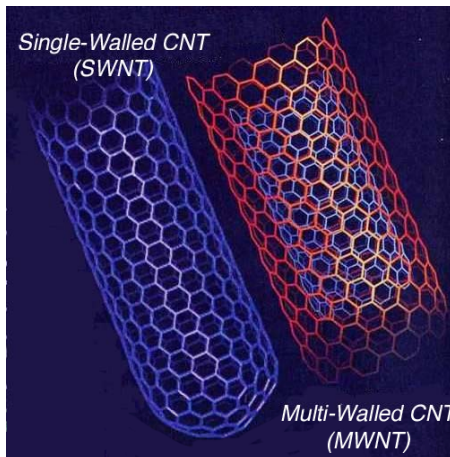
## Diamond ( $sp^3$ carbon):

- hard material
- very good electrical insulator, but still a very good heat conductor (why?)



## Graphite ( $sp^2$ Carbon):

- simple to write with (pencil)
- good electrical conductor



## Nanotubes:

metallic and semiconducting ones:  
diameter 0.5 - 50 nm

## Buckminsterfullerene ( $C_{60}$ ):

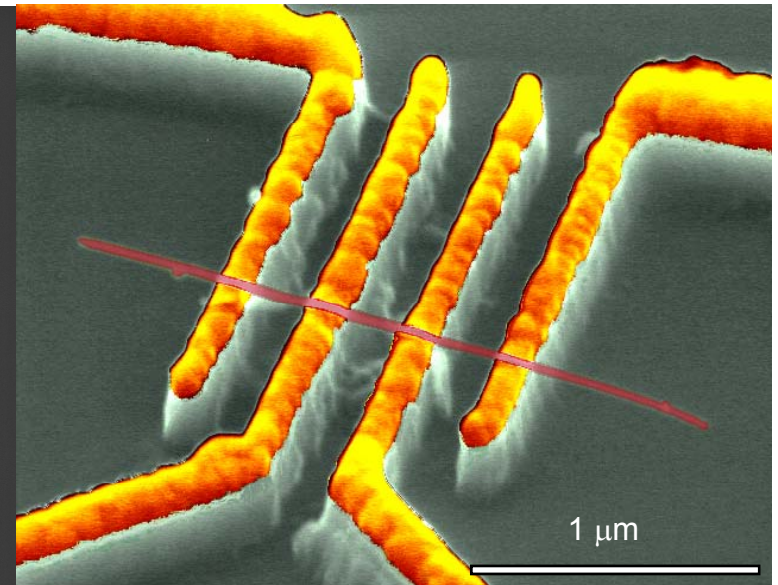
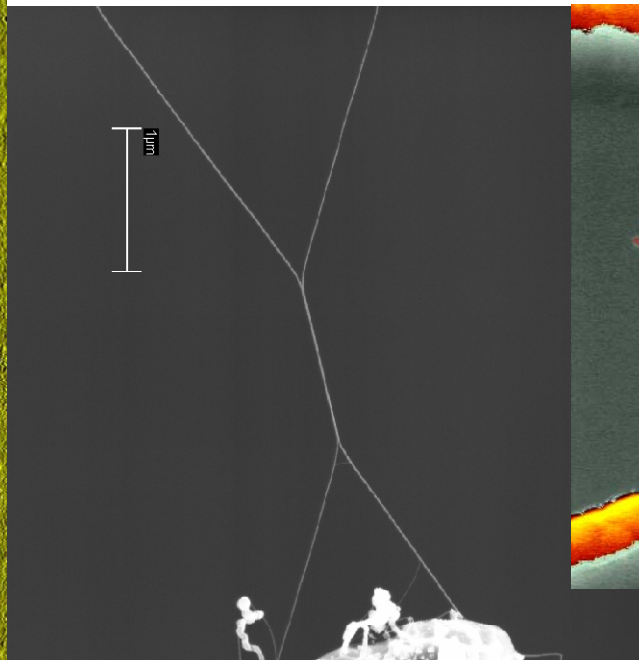
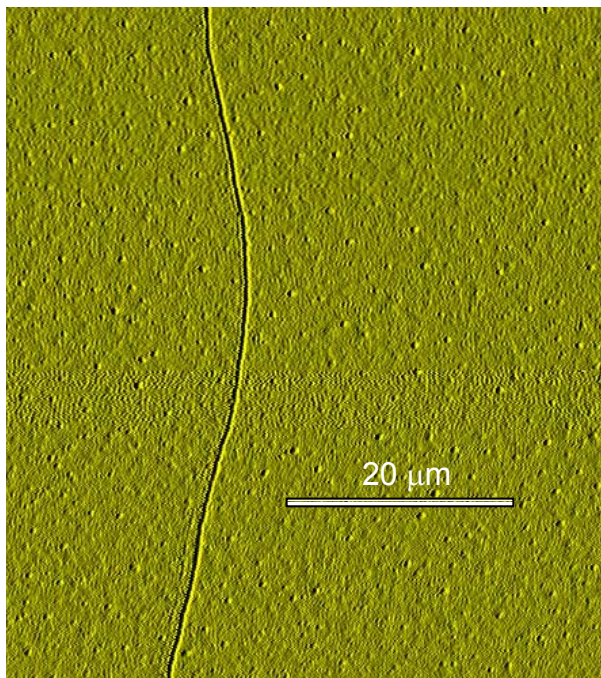
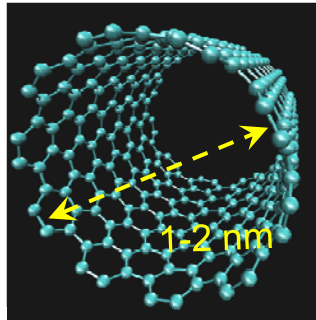
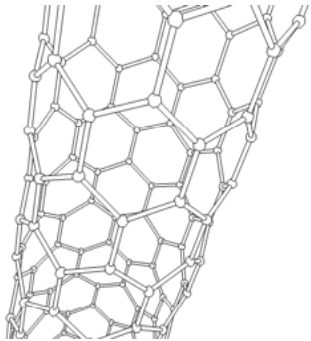
diameter = 0.7 nm



# Carbon Nanotubes

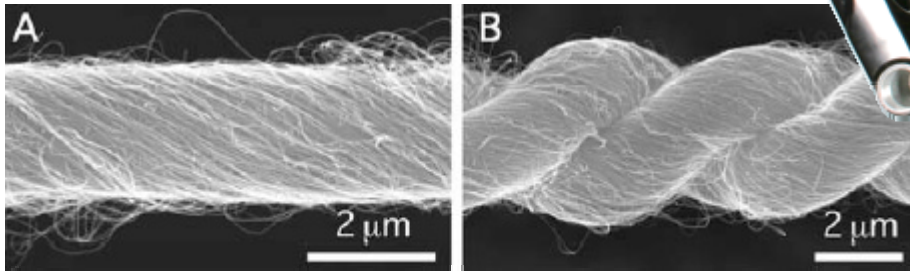


...new forms of graphite





# Carbon Nanotubes



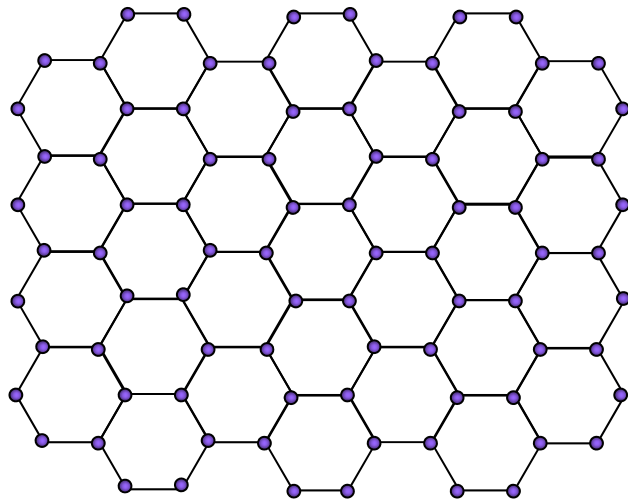
Mit Carbon Nanoröhrchen lassen sich auch ultraleichte und ultrafeste Komposite herstellen, z.B. für Fahrräder, Boeing und Airbus



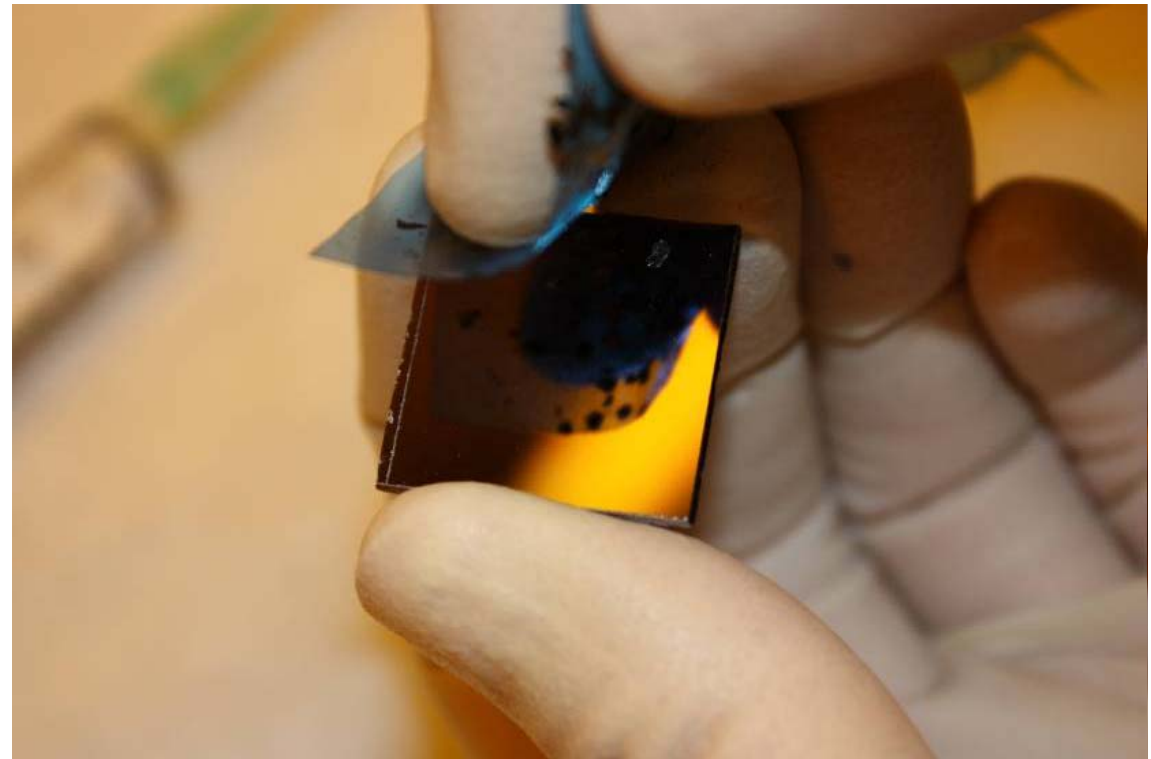
# Graphene



graphite is separated with a Scotch tape into it separate thinner layers



„Graphene“ = Kohlenstoff-Monoschicht



in graphene, electrons behave in a way like relativistic massless particles

# Micro- and Nanofabrication



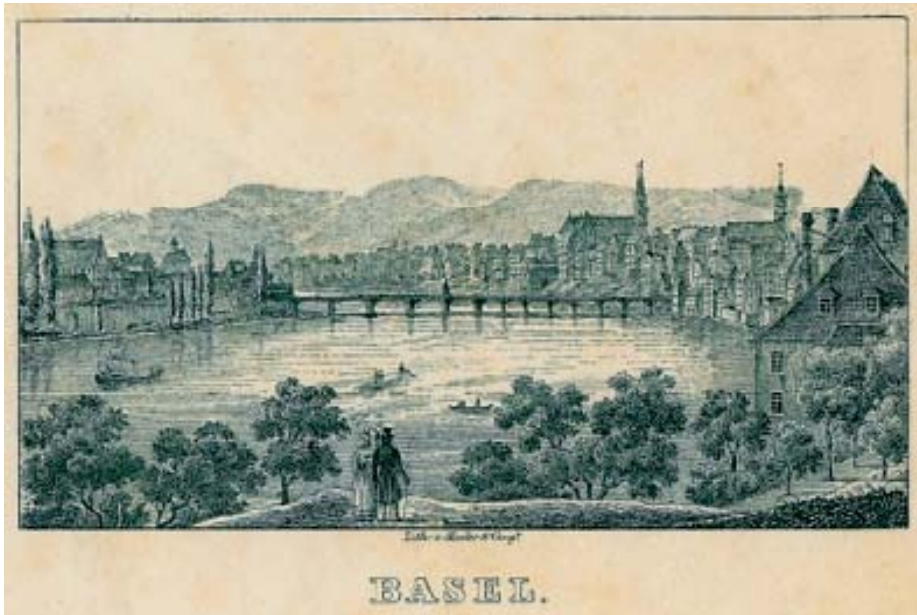
copyright 1997 philg@mit.edu

# 1. Intro / Micro- and Nanofabrication Facility





# How does it work: lithography

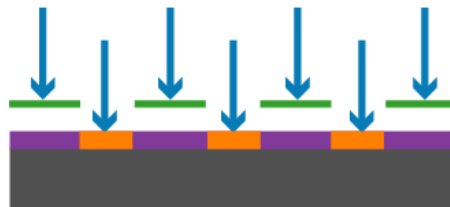


Lithographie von Basel. Druck über eine von Hand vorgezeichnete und danach geätzte Steinplatte

1. **resist on silicon**



2. expose, e.g. through **mask**



3. **develop**



4. **metalization**

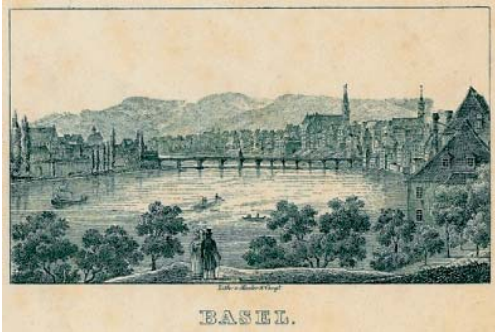
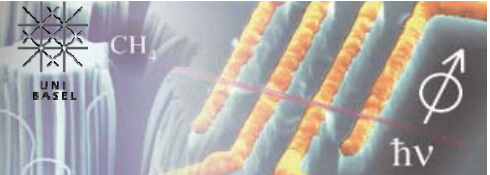


5. **lift-off (resist removal)**



# Lithography for devices

Schönenberger group [www.nanoelectronics.ch](http://www.nanoelectronics.ch)

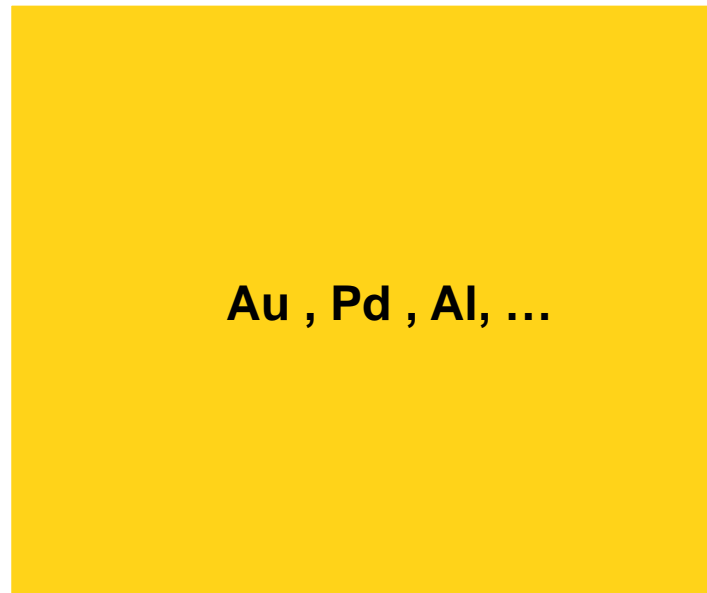


Lithographie von Basel  
Druck über eine von Hand vorgezeichnete  
und danach geätzte Steinplatte

Lithographie mit Licht

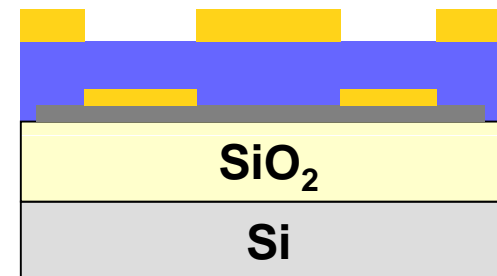


... und mit einem Elektronenstrahl



10  $\mu\text{m}$

fertig !



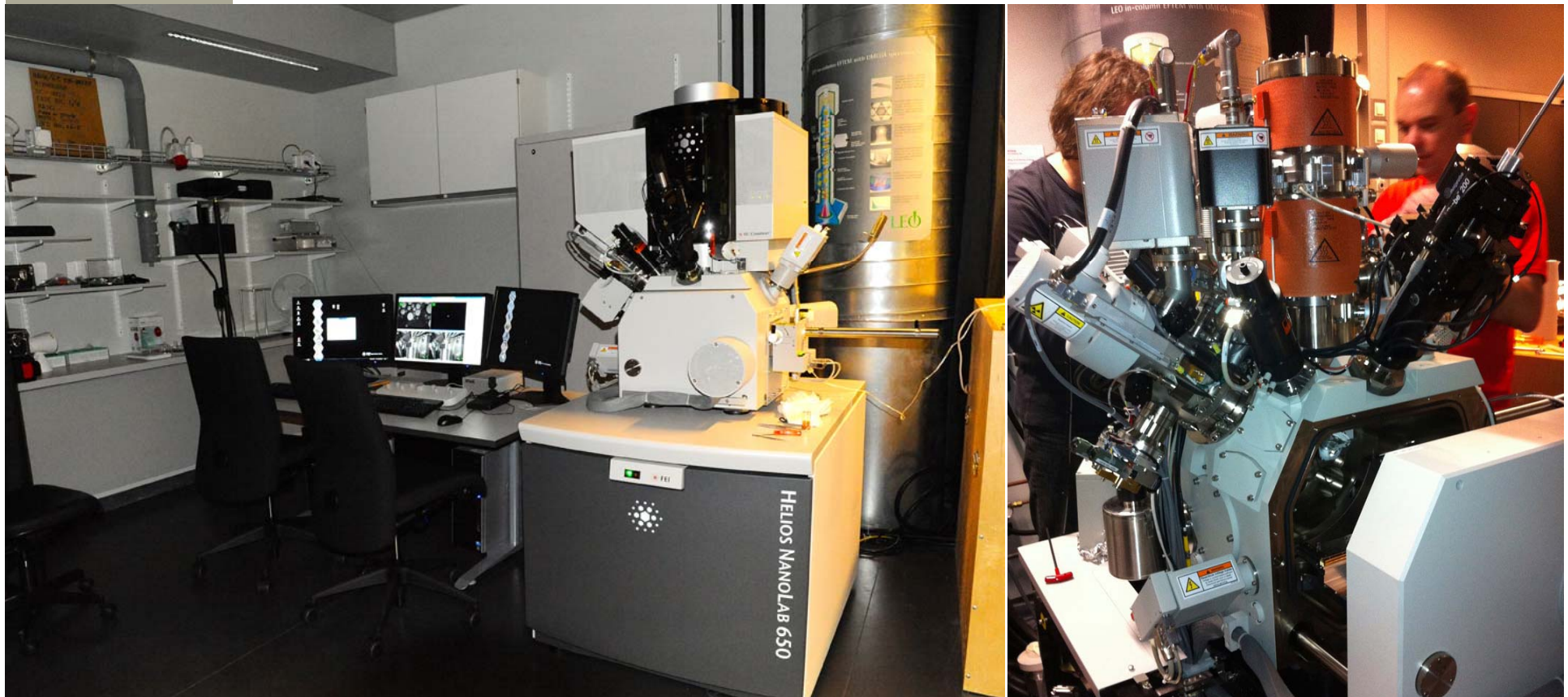
Moderen Lithographie erlaubt uns, Strukturen im Bereich **weniger Nanometer** herzustellen. Hundertfache Wiederholung führt zu einem Pentium Chip.



# Focused Ion Beam (FIB) together with ZMB

operated as a shared facility by ZMB staff and located at the ZMB

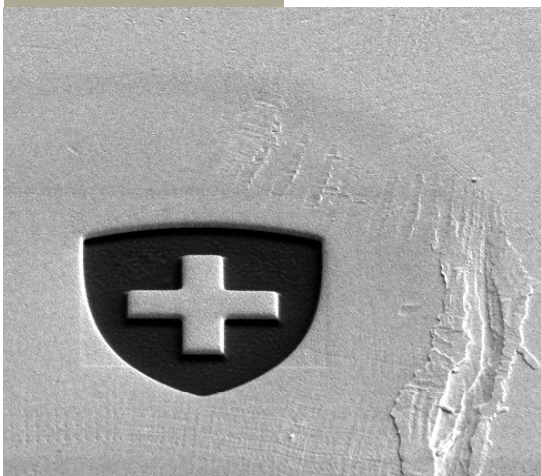
current users are: ZMB, Poggio, Schönenberger, Constable, Gerber, Stahlberg, Hierlemann (D-BSSE), Lim, Maletinsky, Meyer, Richter



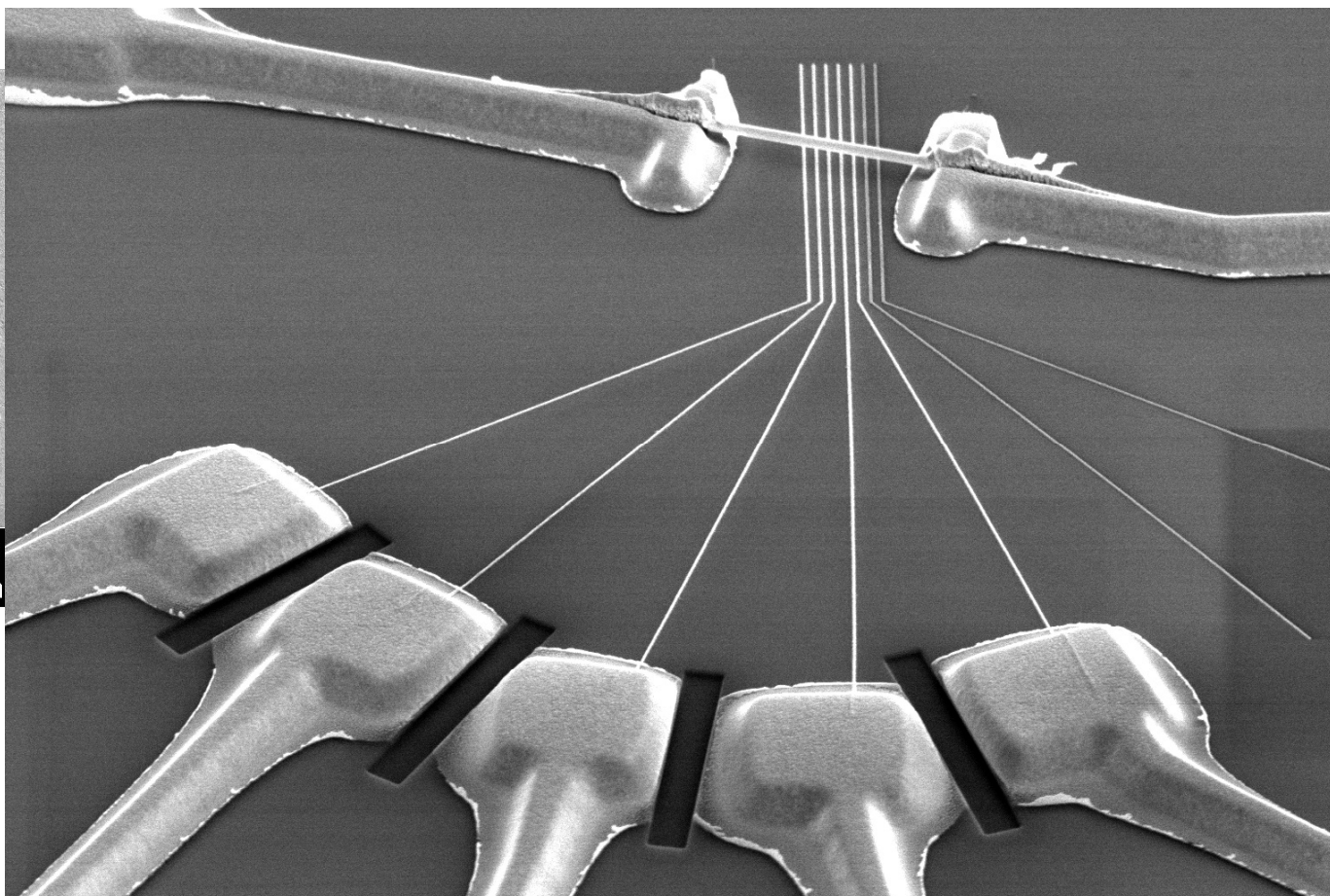


# Focused Ion Beam (FIB) together with ZMB

Poggio lab



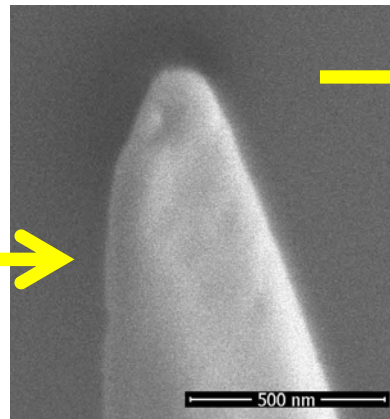
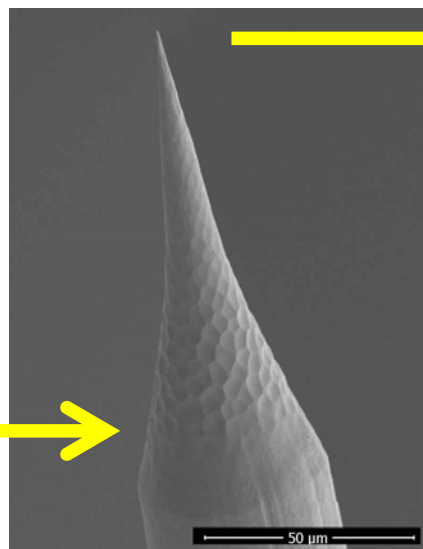
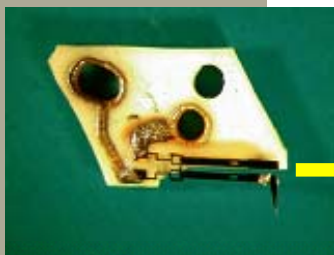
HV	det	tilt	mag	HFV	WD
00 kV	ETD	52 °	3 500 x	59.2 μm	5.6 mm



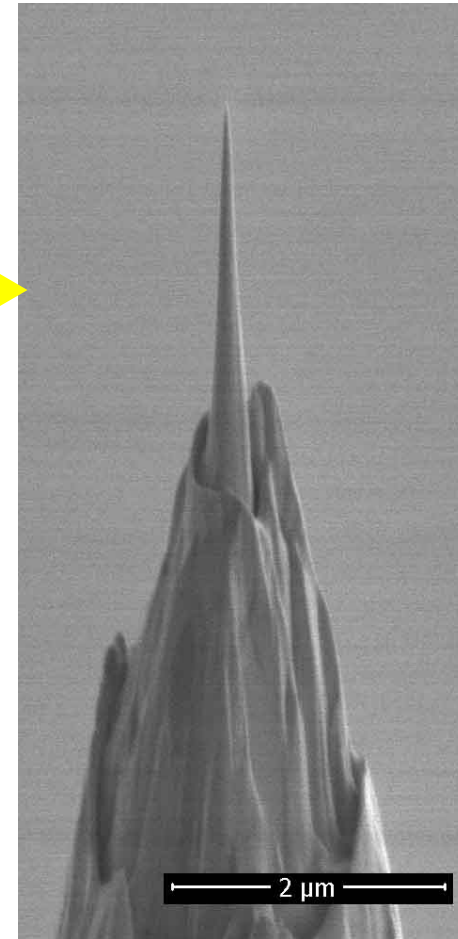
	2/18/2013	HV	curr	dwell	det	mode	tilt	mag	WD	4 μm
	6:56:27 PM	5.00 kV	25 pA	30 μs	ETD	SE	52 °	11 004 x	4.1 mm	

# Focused Ion Beam (FIB) together with ZMB

Ernst Meyer lab



$R_{\text{tip}} \approx 100 \text{ nm}$   
(chemical etching)



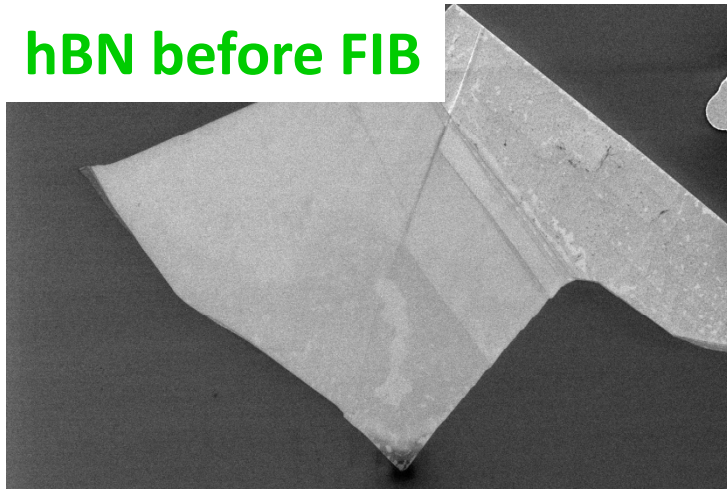
Can be sharpened by FIB milling  
 $R_{\text{tip}} \approx 20 \text{ nm}$

qPlus sensor: F. J. Giessibl, Appl. Phys. Lett. 73, 3956 (1998).

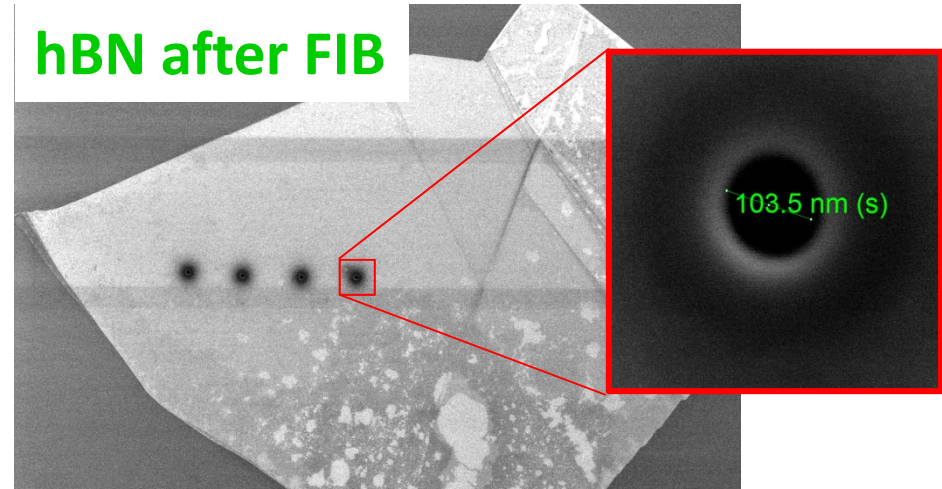


# Inner contact to graphene

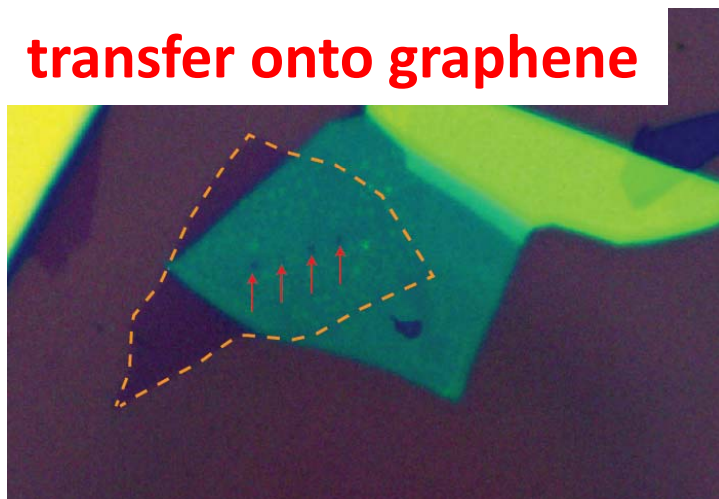
hBN before FIB



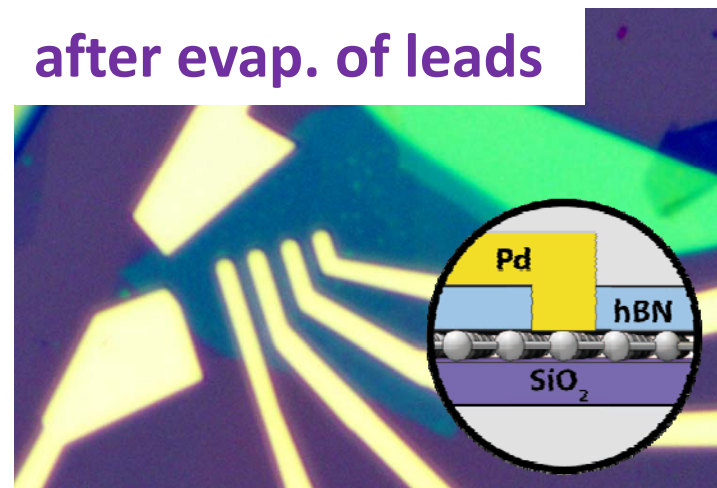
hBN after FIB



transfer onto graphene

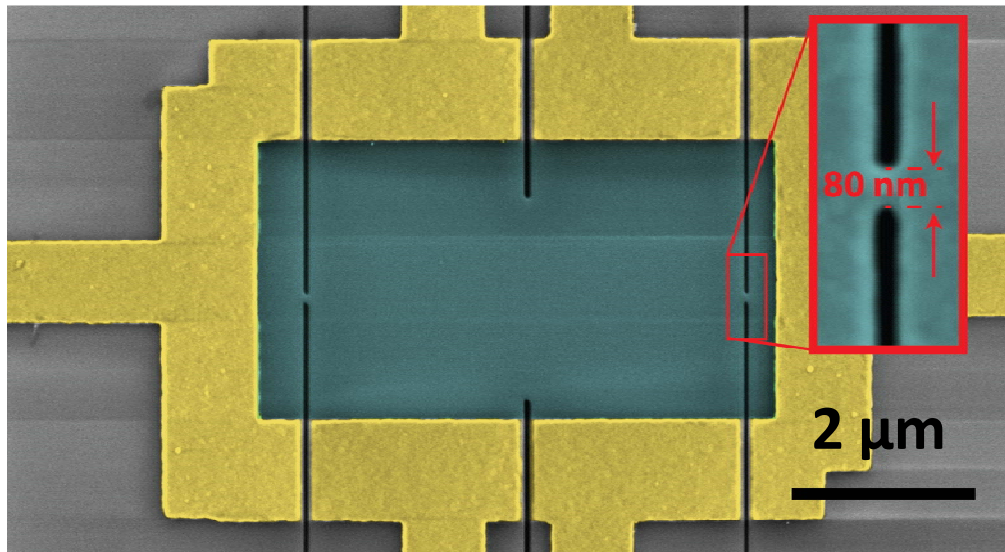


after evap. of leads



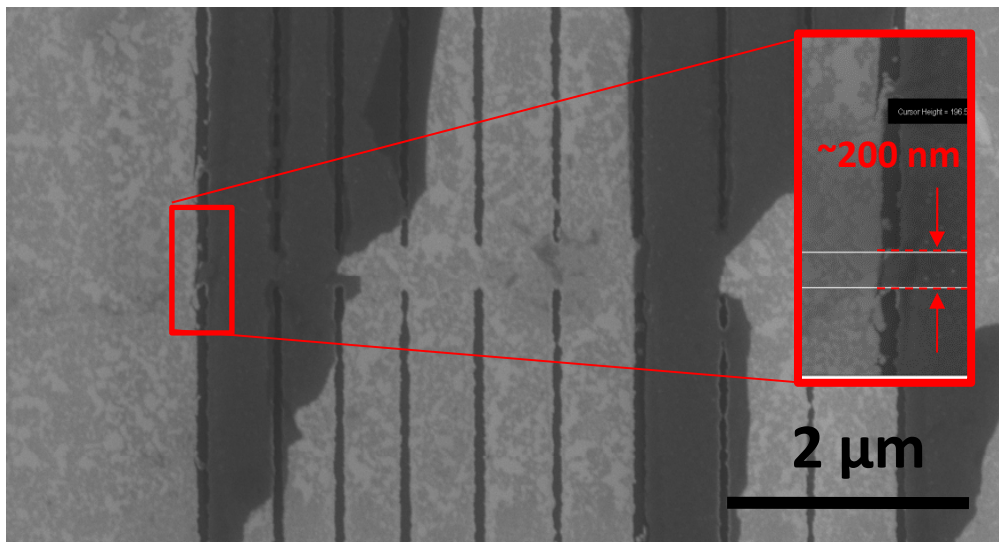


# Cuts for SQUIDs



## Cut with FIB

- $d_{\text{min}} \sim 50\ \text{nm}$  possible
- clean cut

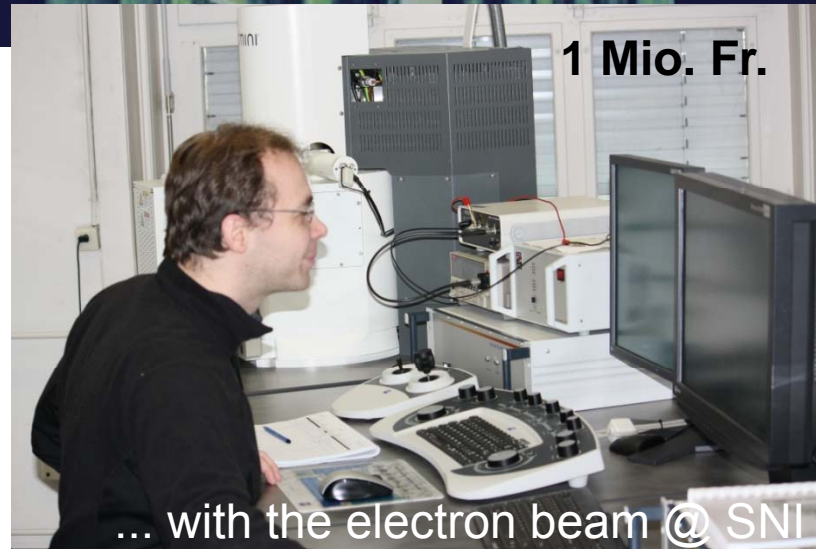


## Etched with SF<sub>6</sub> (mask with E-beam lithography)

- $d_{\text{min}} \sim 150\ \text{nm}$
- etched cuts are not as clean as with FIB

# Key equipment

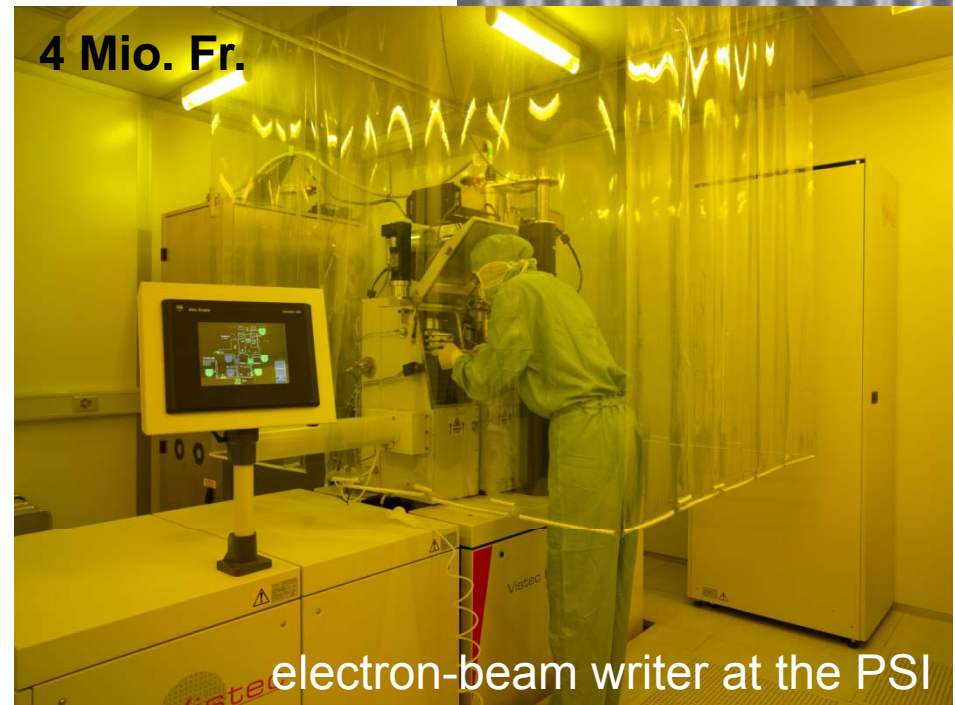
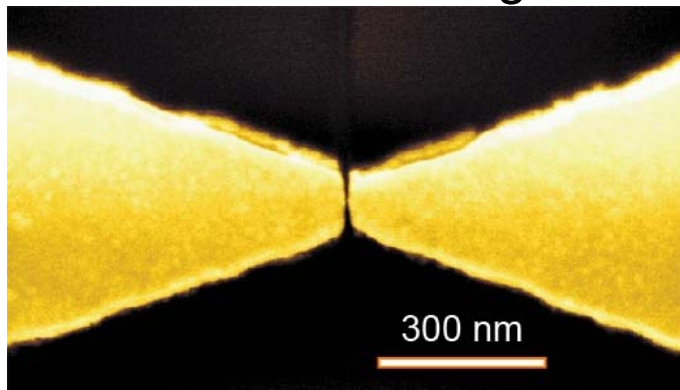
with light (mask aligner)@SNI



Xray-interference  
@ PSI

40nm

- light: 1  $\mu\text{m}$  down to 10 nm
- e-beam: < 10 nm demonstrated
- focused ion beam: < 10 nm
- He-beam: 2 nm range





## SEM-based e-beam writer at SNI/Physics



currently run 2 of those systems (each 1 Mio), but ..

have higher demands:

- a) more groups using it
- b) higher resolution needed (< 10 nm possible)
- c) better overlay accuracy (10nm stitching possible)
- d) large area patterning (large field with 24 bit)

Dedicated Electron Beam Lithography



Home » Products » [EBPG5200](#)

### [EBPG5200](#)

EBPG5000 Plus

VOYAGER

RAITH150 Two

eLINE Plus

PIONEER

ionLINE

ELPHY MultiBeam

ELPHY Plus

ELPHY Quantum

CHIPSCANNER

Produ

Auto

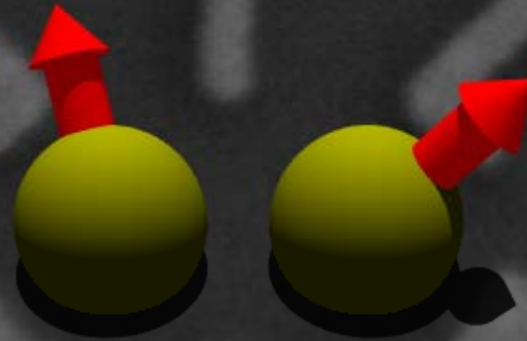
Evolu

The EBPG5200 is a full 200 nm system providing high resolution and field of view solutions for making intricate





# Electron Spins in Quantum Dots



Ti/Au

500 nm

GaAs

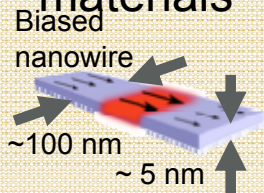
50 nm

Skalierbarer Quantencomputer  
Loss – DiVincenzo (1998)

# Superconducting nanowire single-photon detectors (SNSPD)

Warburton and Schönenberger groups (Basel)

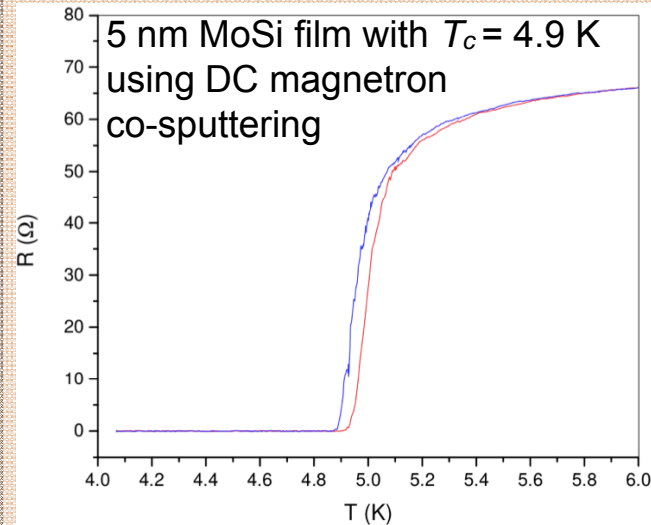
## SNSPDs with amorphous materials



Amorphous materials are promising candidates to yield:

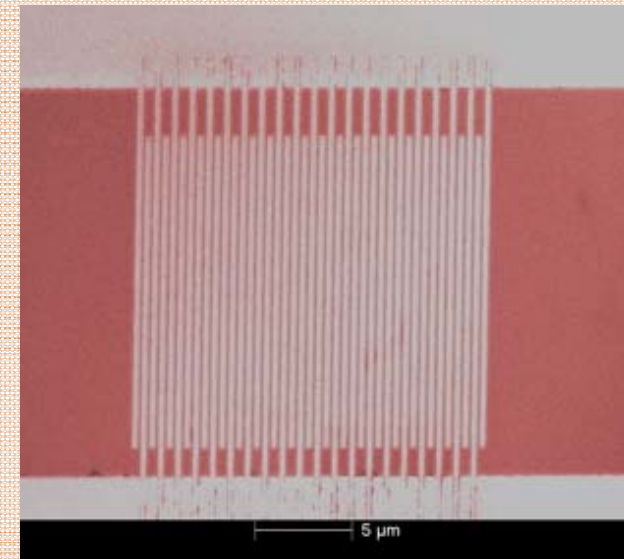
- High-efficiency (90%) from visible to telecom
- Low dark count rates (<1 kHz)
- High count rates (< 20 MHz)
- Low jitter (<100 ps)
- Closed-cycle cryo-operation

## Thin-film deposition



- The large  $T_c$  of MoSi has potential for high-efficiency at 2.5 K
- $T_c$  can be tuned for optimal performance

## Nano-fabrication

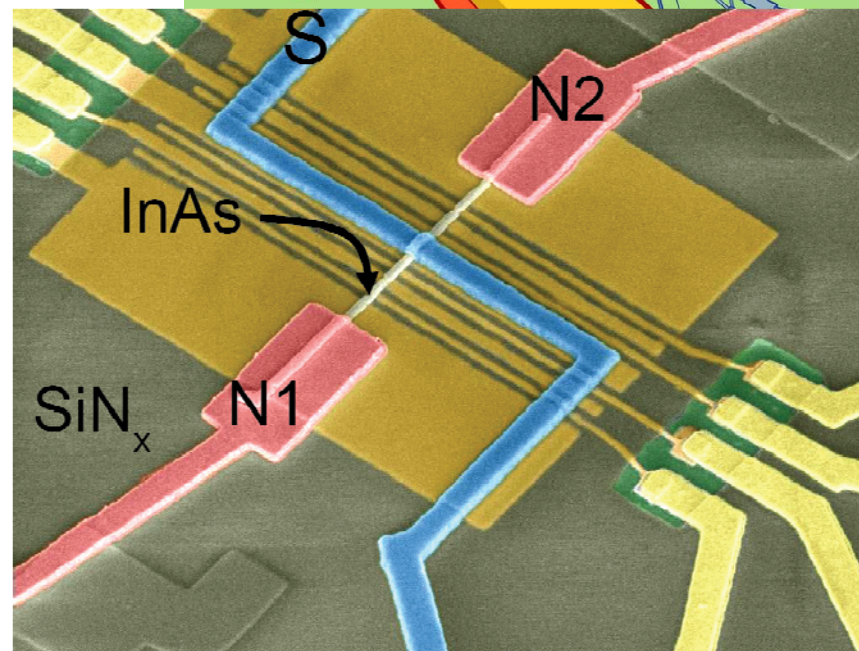
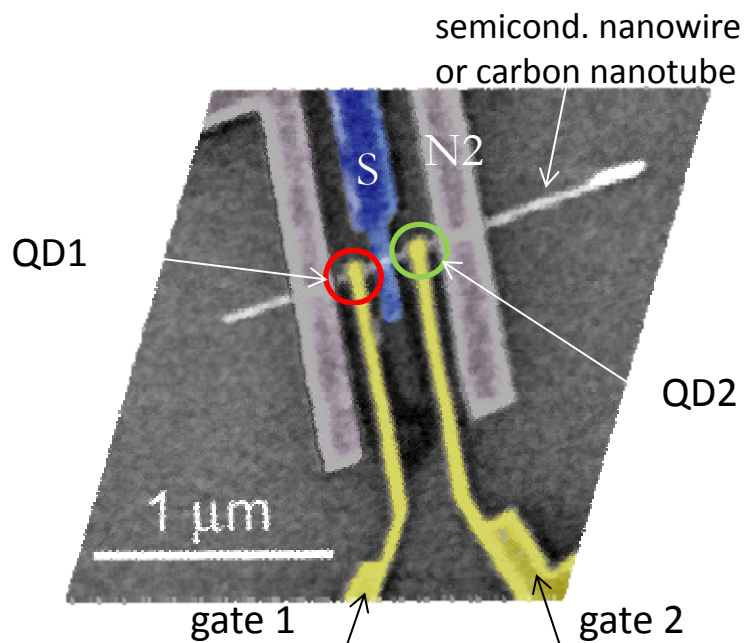
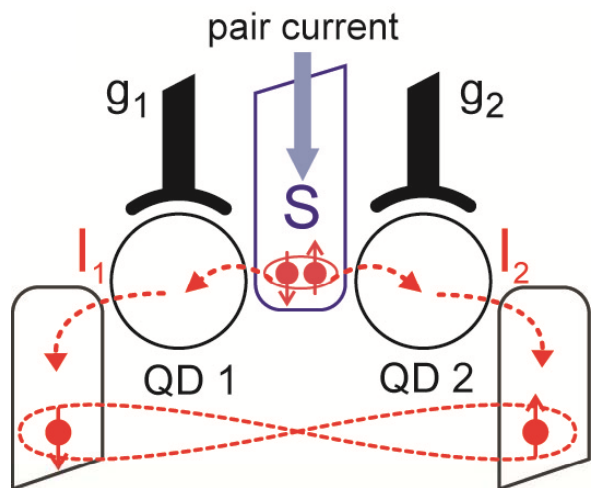


- e-beam lithography for top-down nanowire fabrication



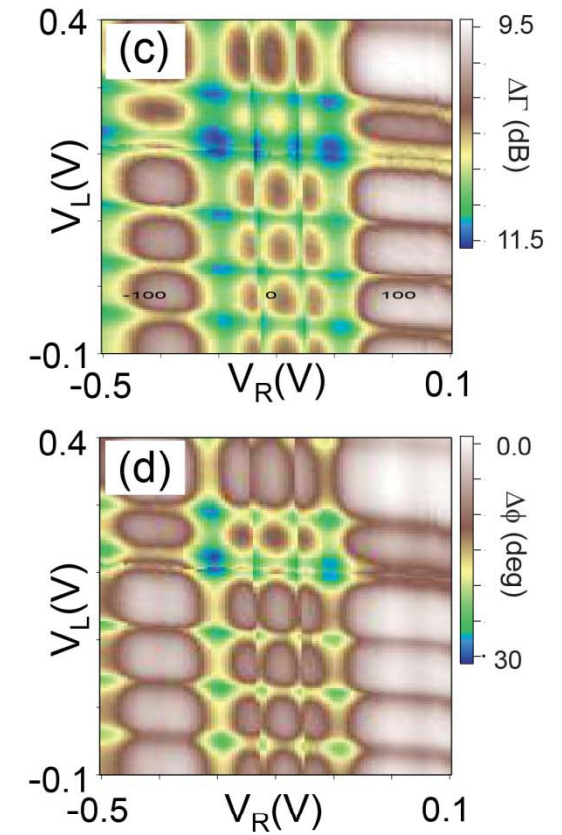
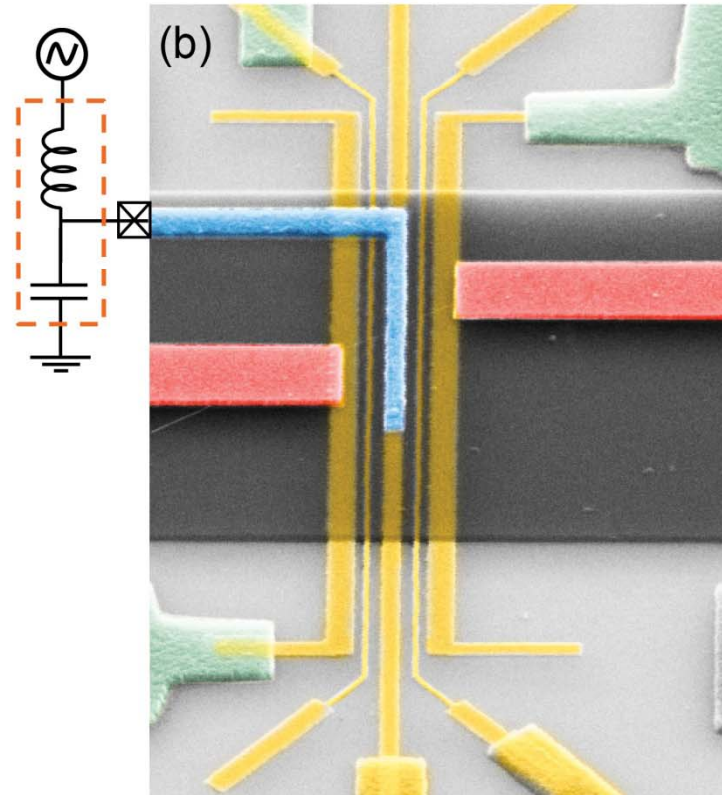
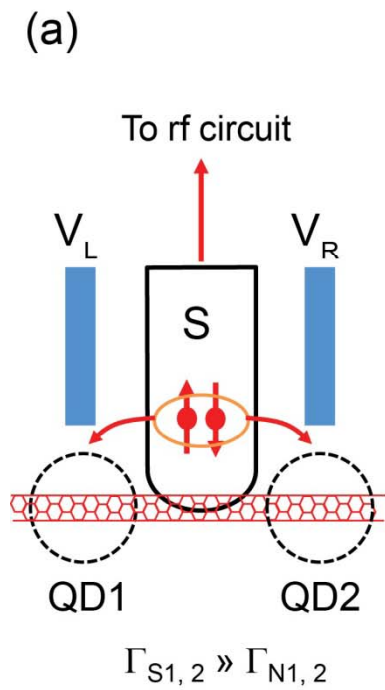
# Cooper-pair splitter

apply quantum dots to split Cooper-pairs;  
*proposal by Recher, Sukhorukov and Loss*



Hofstetter et al. Nature 461, 960-963 (2009)

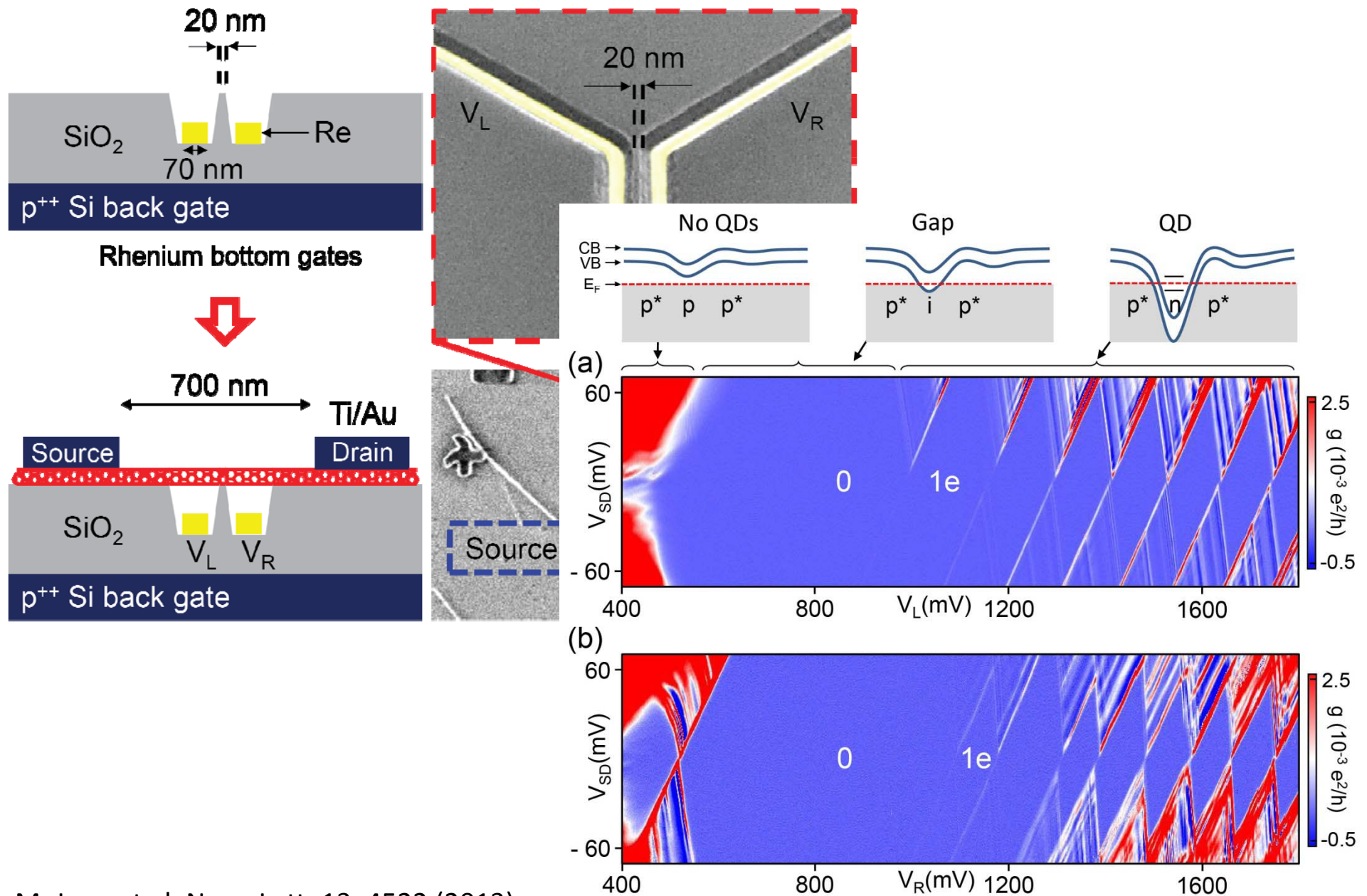
# Cooper-pair splitter



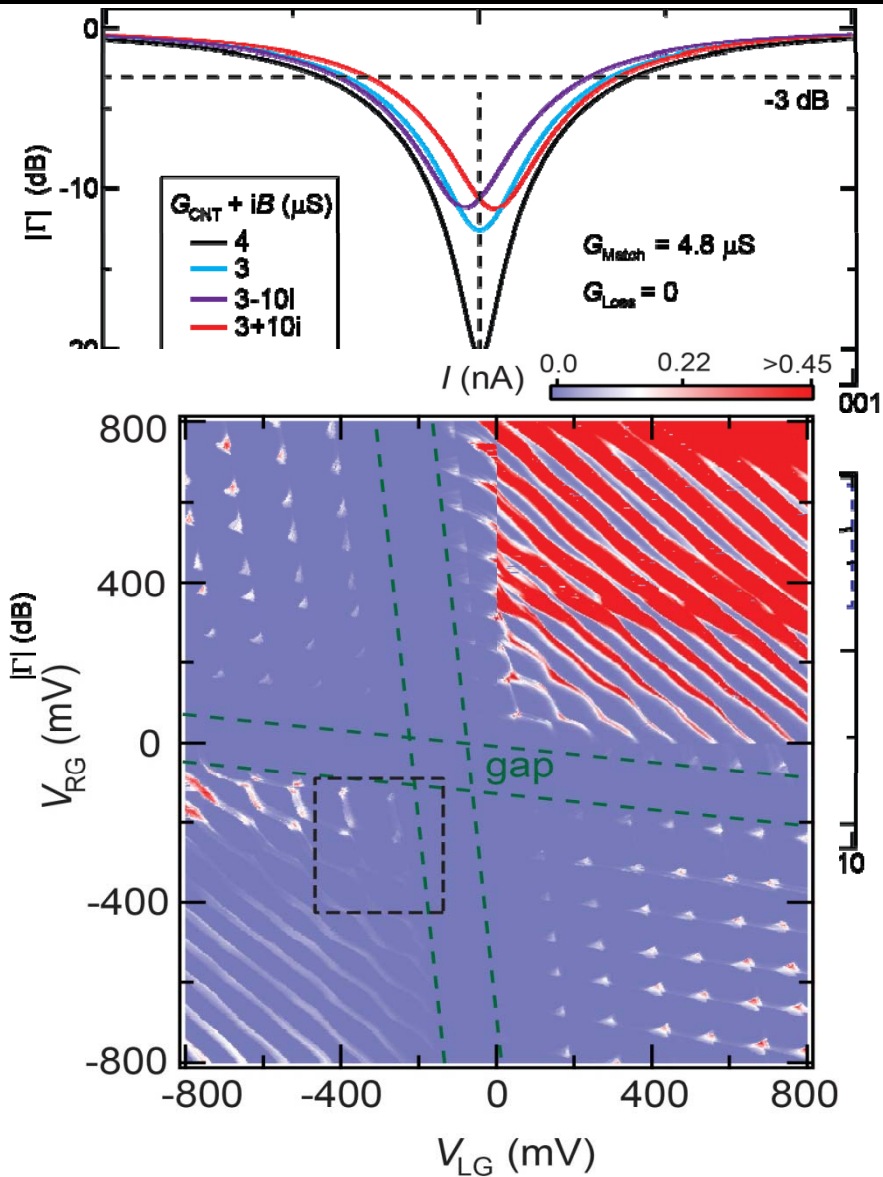
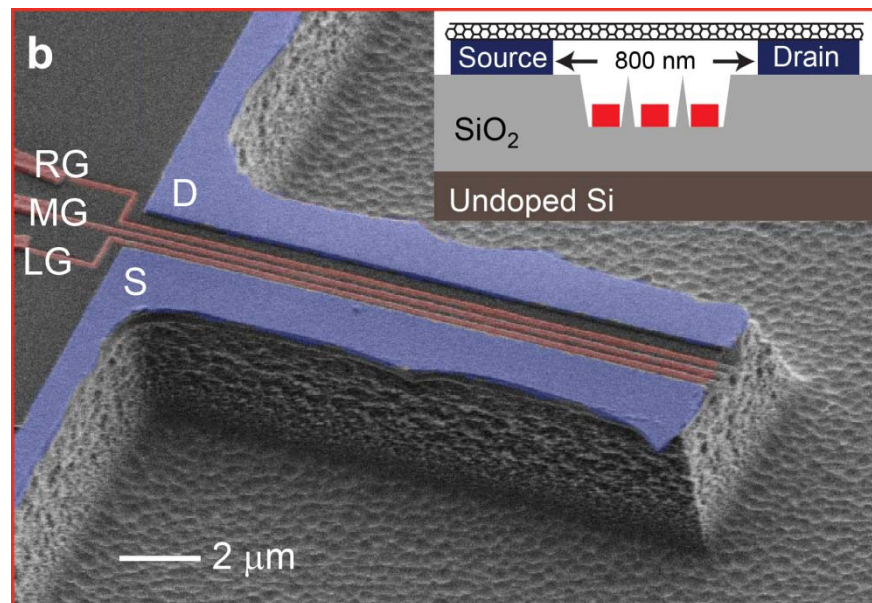
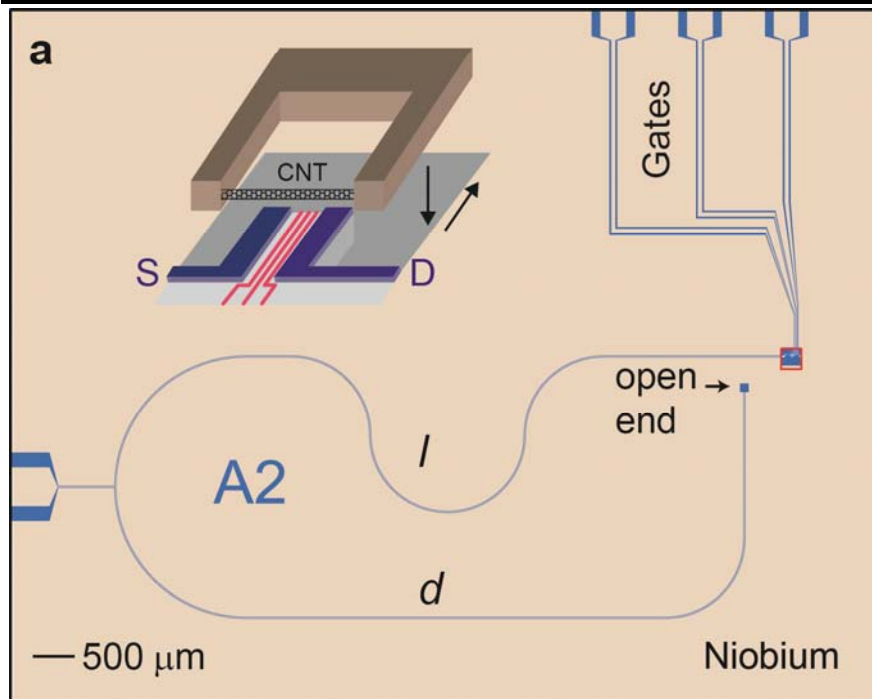
modern one with a single CNT and coupled to an rf circuit  
work in progress



# Carbon Nanotube (CNT) Quantum Dots (qdots)



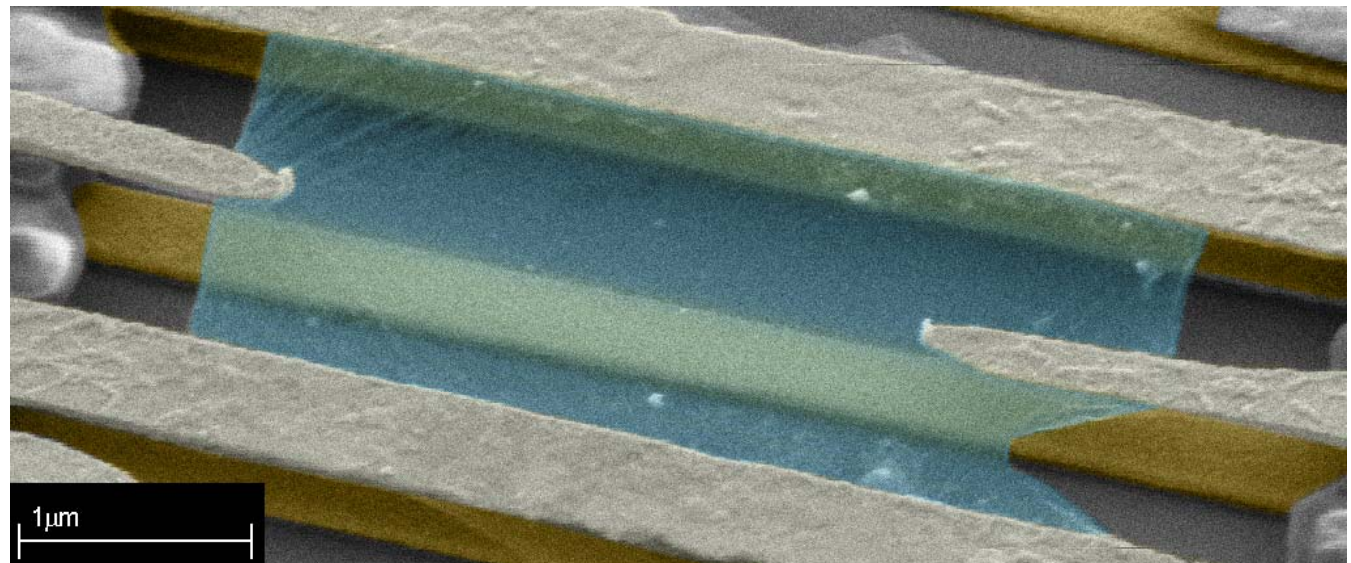
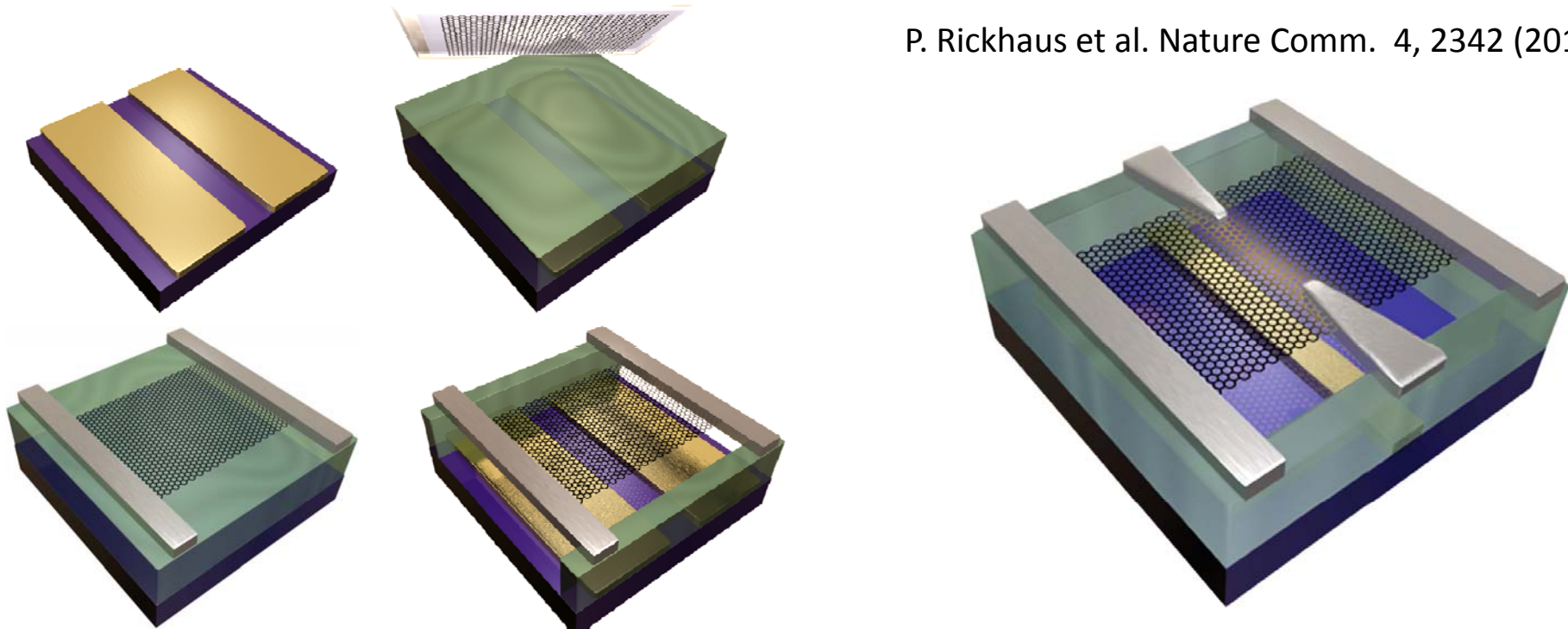
# CNT Qdots coupled to rf cavities





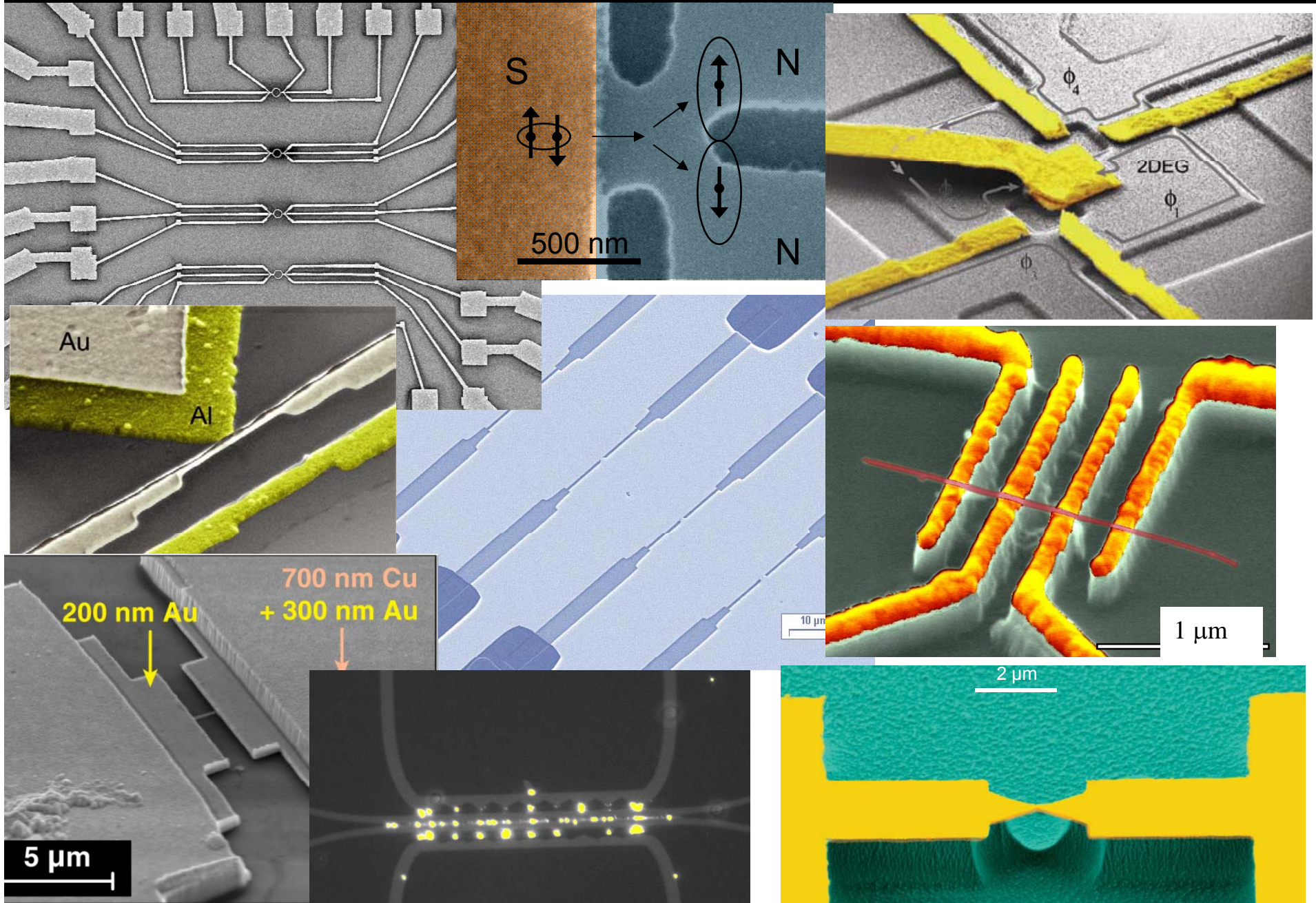
# Graphene devices (suspended)

P. Rickhaus et al. Nature Comm. 4, 2342 (2013)





# some more pics of devices





## 2. Quantum Primer

## a) Wave-Mechanics („like“ optics, acoustics ...)

(energy  $E=const$ )

**Schrödinger**

**Helmholz**

$$\left\{ \Delta + 2m(E - V) / \hbar^2 \right\} \psi = 0 \quad \left\{ \Delta + (E / c\hbar)^2 \right\} \psi = 0$$

$$\left\{ \Delta + k^2 \right\} \psi = 0$$

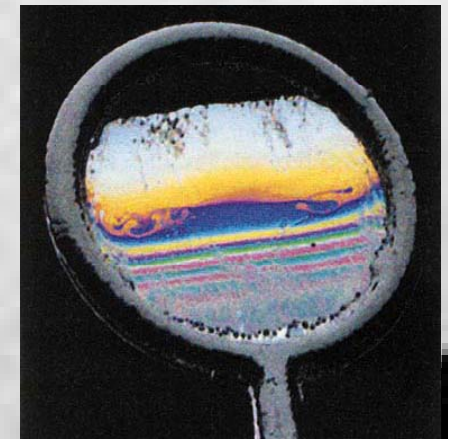
**waves:**

$$\psi = A \exp(iS/\hbar)$$

*A: Amplitude*

$\Theta = S / \hbar$  *Phase* (determined by the action  $S$ )

*(the whole classical physics  
is contained in the phase)*





## 2. Quantum Primer

### b) add some weirdness

uncertainty relations:  $[x, p] = i\hbar$

(not quite weird, formally a consequence of Fourier transformation)

### c) some more weirdness

Copenhagen interpretation of  $\Psi$  „collapse of wavefunction“

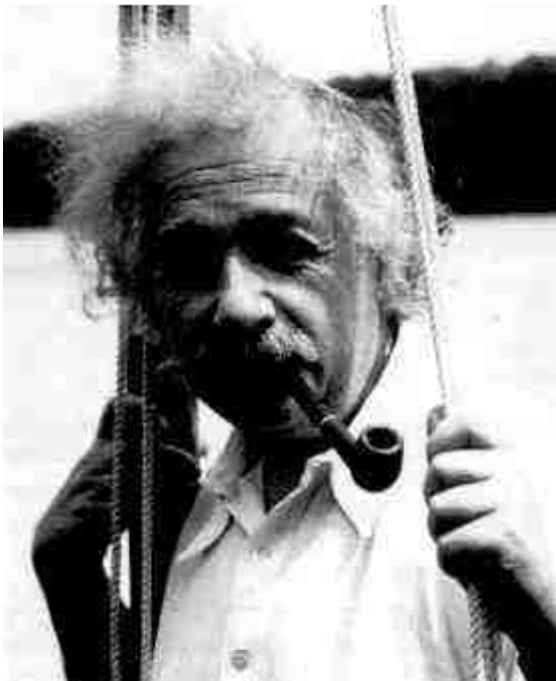
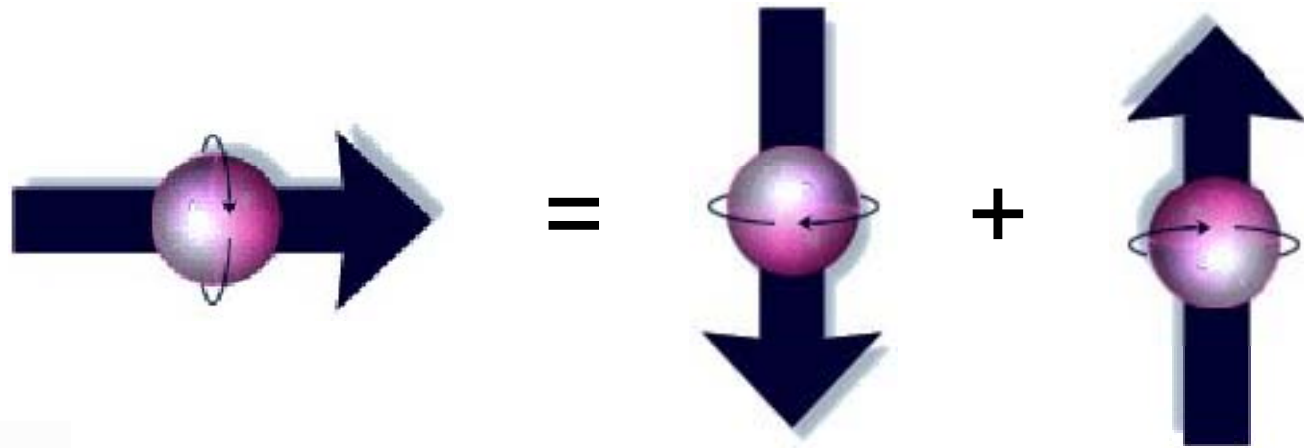
$$\psi \propto a|alive\rangle + b|dead\rangle$$



- linear superposition
- **cat is both dead and alive**
- only if measured is the cat **either dead or alive!**

## 2. Quantum Primer

**superposition**



many measurements:

50% point up  
50% point down

quantum physics is therefore  
probabilistic and not deterministic

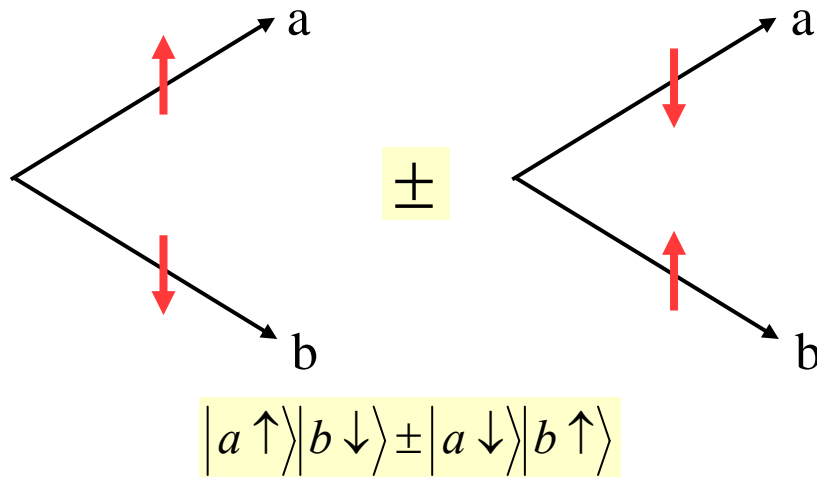
„Gott würfelt nicht!“



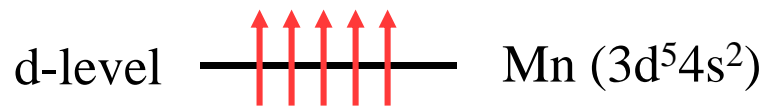
# 2. Quantum Primer

## d) higher level weirdness

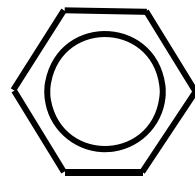
many-particle states, **entangled states!**



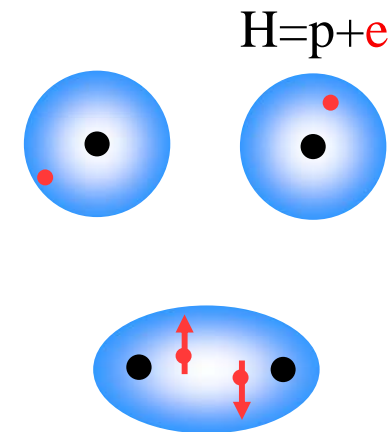
### Magnetism



diamagnetism



### Chemistry



- we all live on quantum mechanics
- we would **not exist without** QM!
- a classical world is unstable!

### 3. Conceptual discussions (blackboard)

- quantities yielding different energies, T, B, f, L ...
- ballistic vs diffusive
- quantum coherence
- energy conserving vs relaxation (damping)
- what is classical, which classical concepts break down
- quantum transport “*phenomenology*”



### 3. Conceptual discussions (blackboard)

<i>Limit</i>	<i>Length</i>	<i>Conserved quantity</i>
Ballistic	$L \ll \ell_{el}, \ell_{e-e}, \ell_{e-ph}$	current for each momentum state
↓	elastic scattering: randomizes momentum direction	
Diffusive, non-equilibrium	$\ell_{el} \ll L \ll \ell_{e-e}, \ell_{e-ph}$	current for each energy
↓	electron-electron scattering: mixes different energies	
Quasi-equilibrium	$\ell_{el}, \ell_{e-e} \ll L \ll \ell_{e-ph}$	charge and energy currents
↓	electron-phonon scattering: energy exchange with environment	
Local equilibrium	$\ell_{el}, \ell_{e-e}, \ell_{e-ph} \ll L$	charge current

**Table 2.1** Different limits for the distribution function, defined by comparing the size  $L$  of the wire to the scattering lengths.

## 4. Phenomena in quantum transport

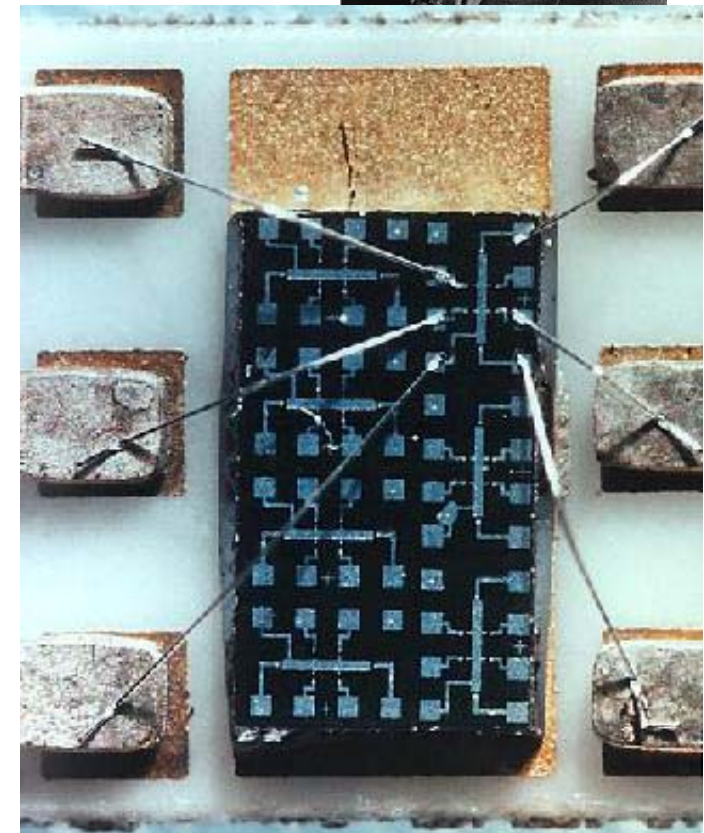
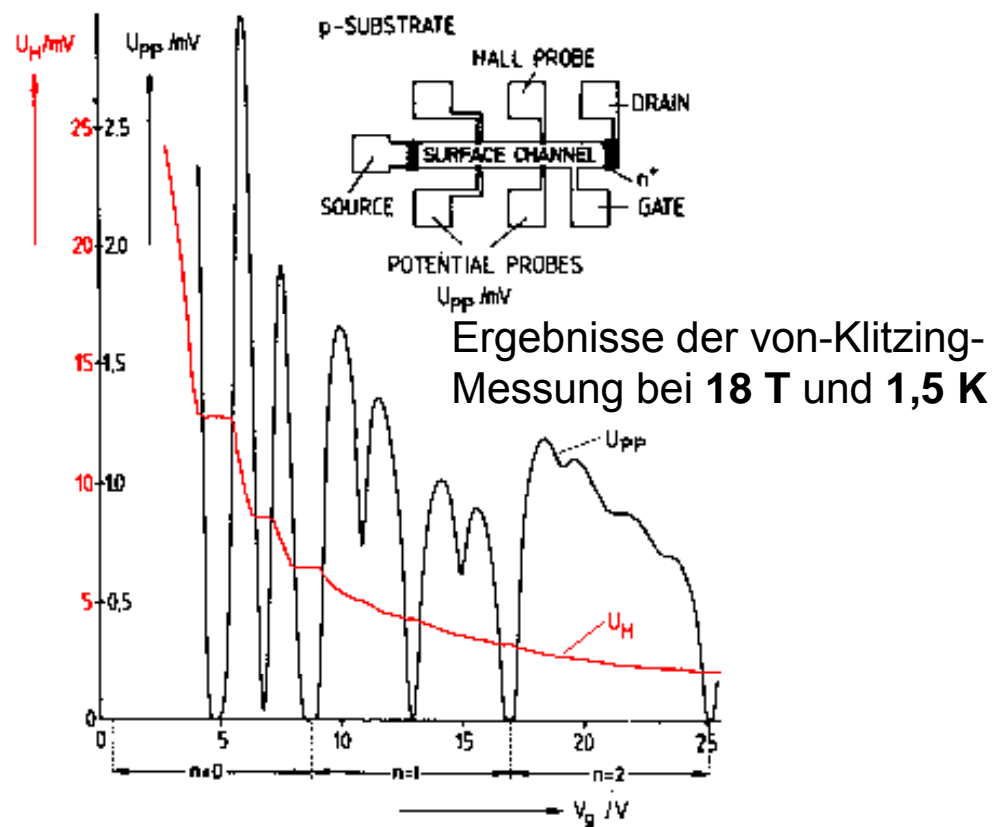


# 4.1 Quantum Hall Effect

1980 discovered by K. von Klaus von Klitzing in Grenoble  
Untersuchung der Hall-Spannung von MOS-FET bei tiefen  
Temperaturen und starken B-Feldern

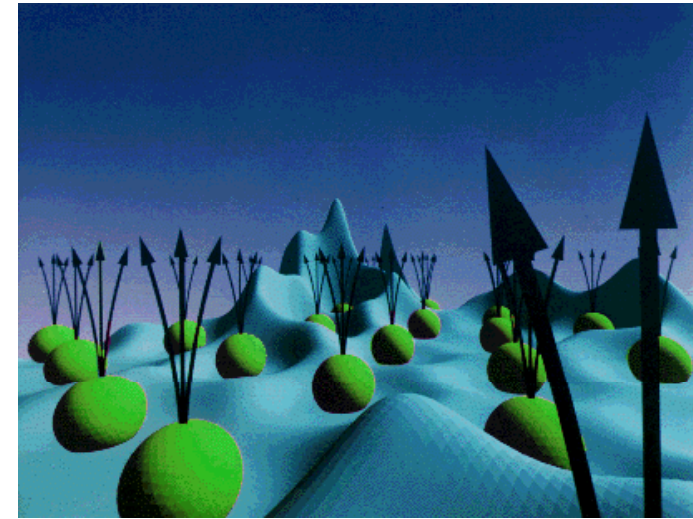
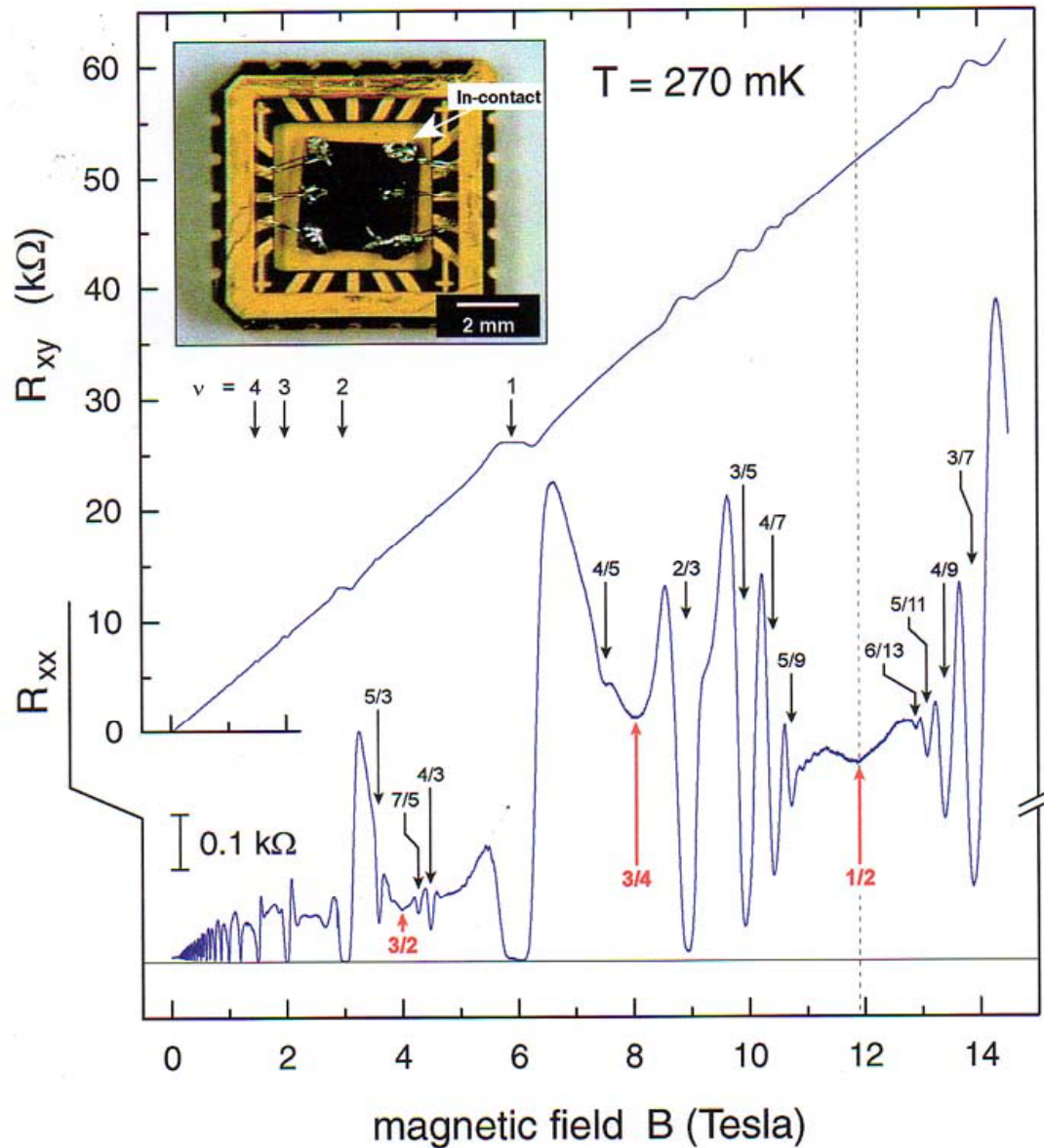
Deutung: **Quantisierung** des Hall-Widerstands

1985 Nobelpreis für Physik

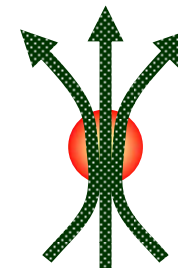


Originalprobe. Deutschen Museum Bonn

# 4.1 Quantum Hall Effect



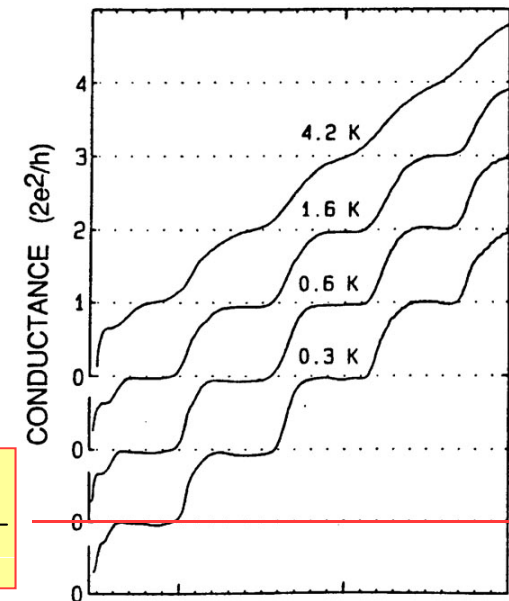
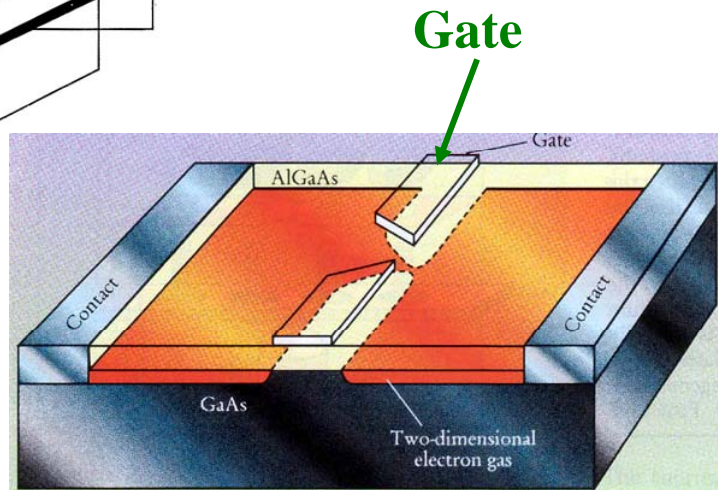
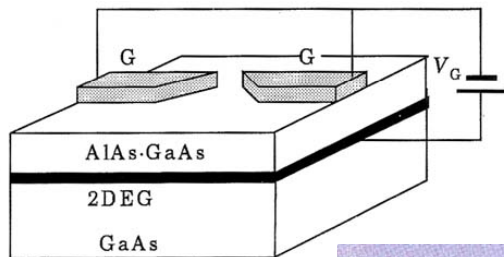
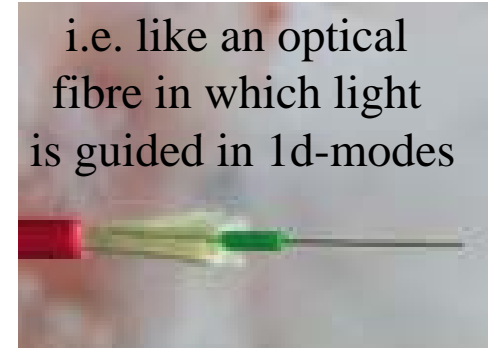
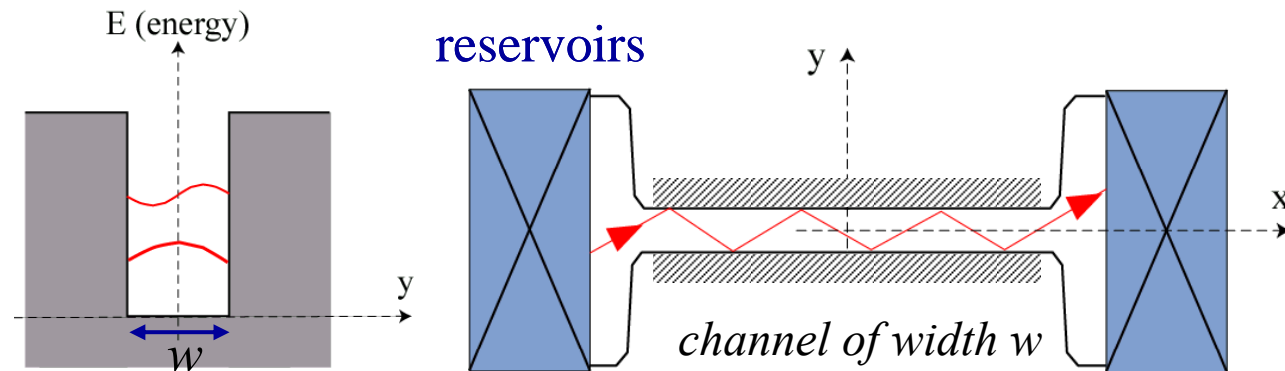
e.g. at filling  $1/3$



composite particle:  
 $1 e + 3 \Phi_0$

excitations (quasiparticles)  
 have **fractional charge**  
 (observed in experiment)

# 4.2 Quantum wire



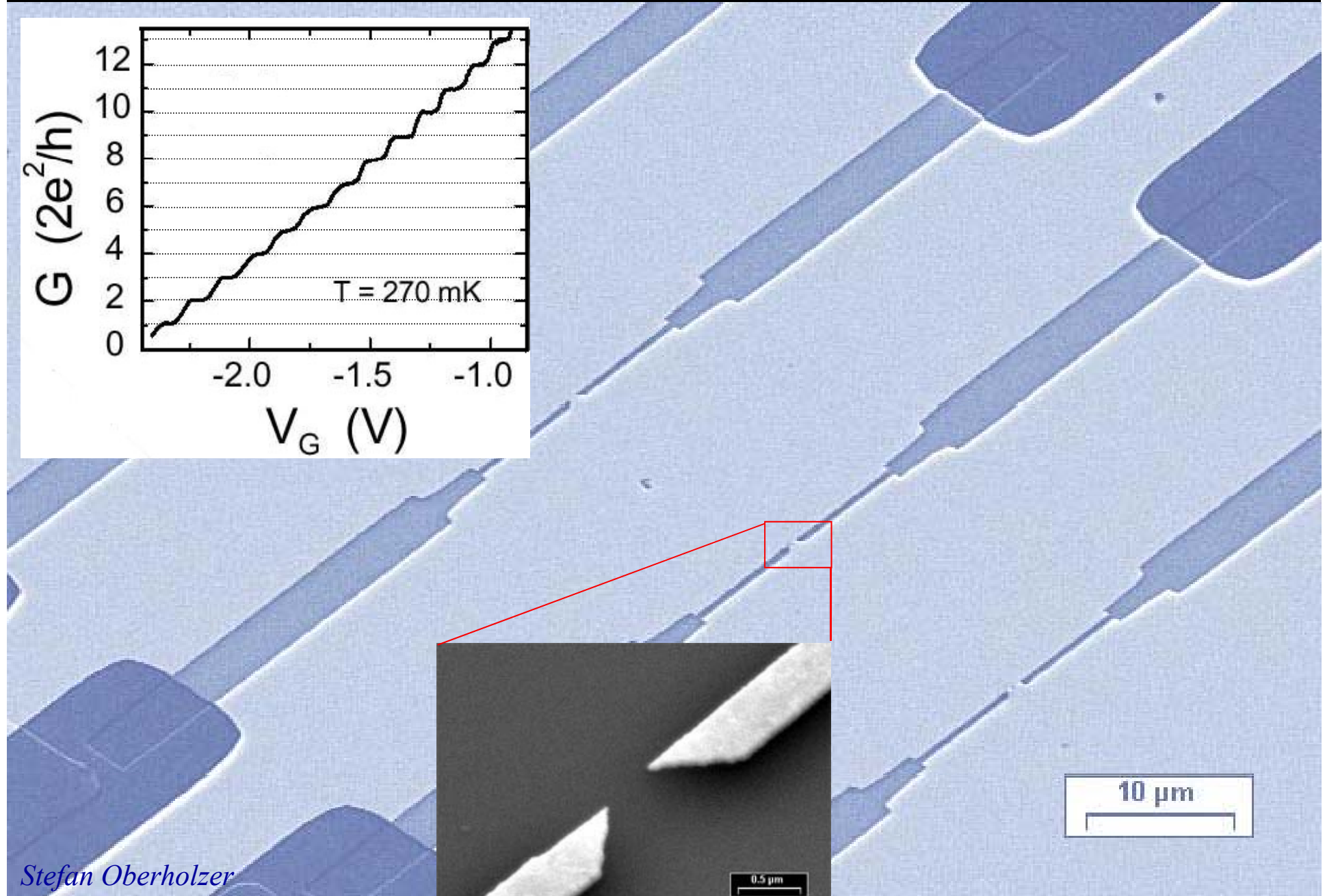
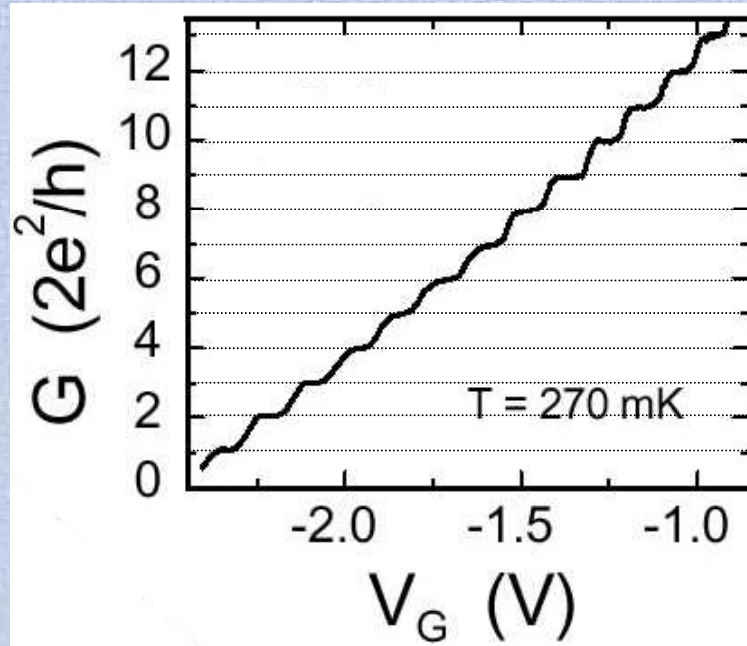
$$G_0 = \frac{2e^2}{h}$$

Gate Voltage  $\propto$  width  $w$

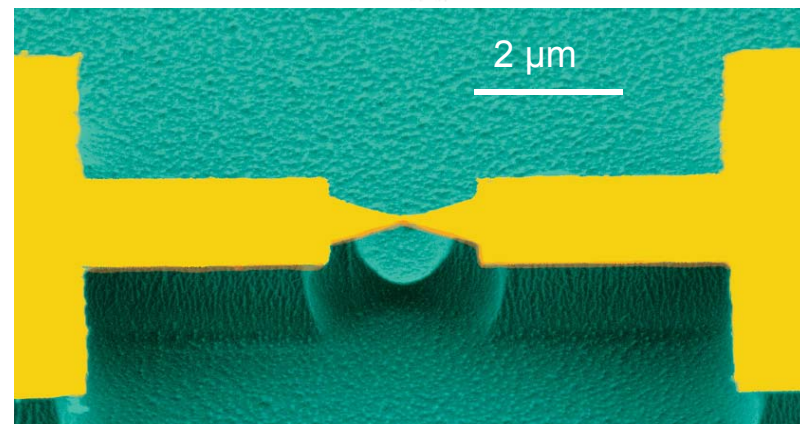
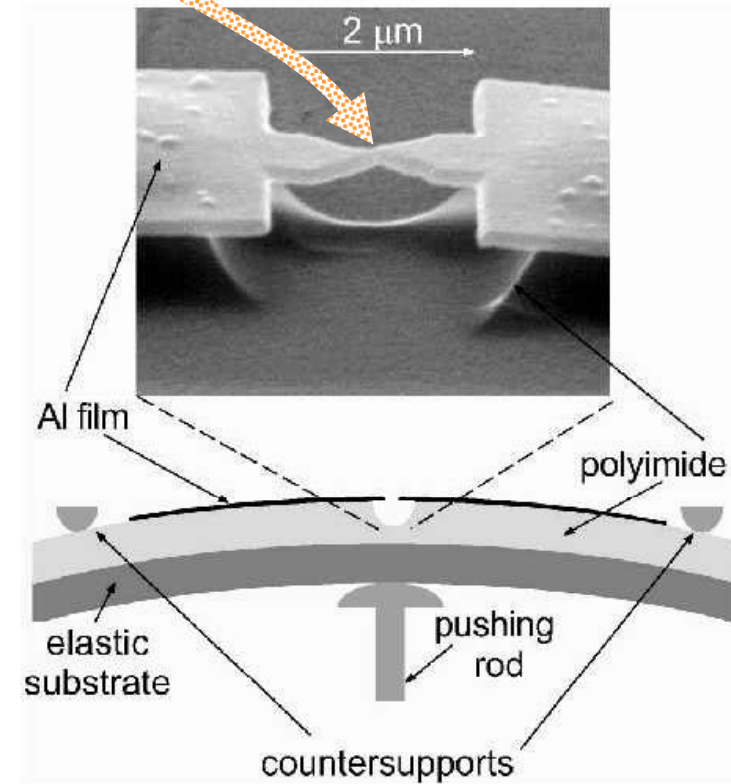
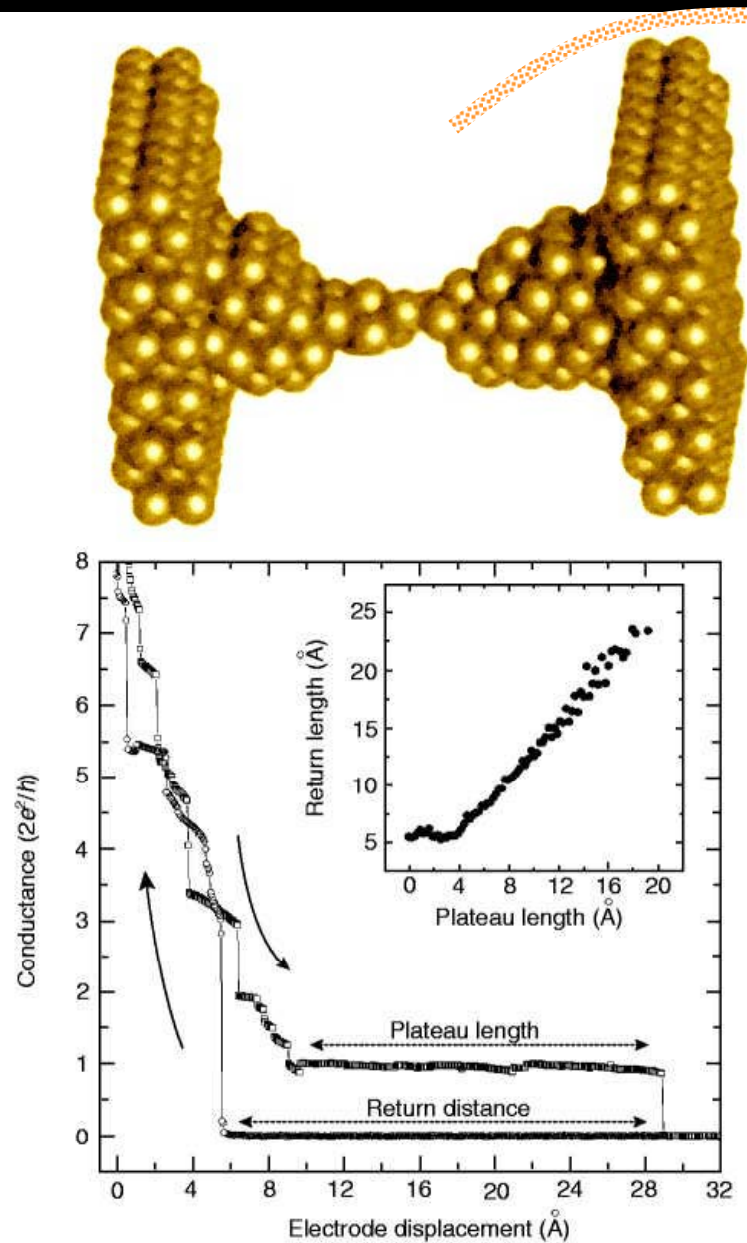
van Wees, van Houten 1988



## 4.2 Quantum (wire) point contact (QPC)

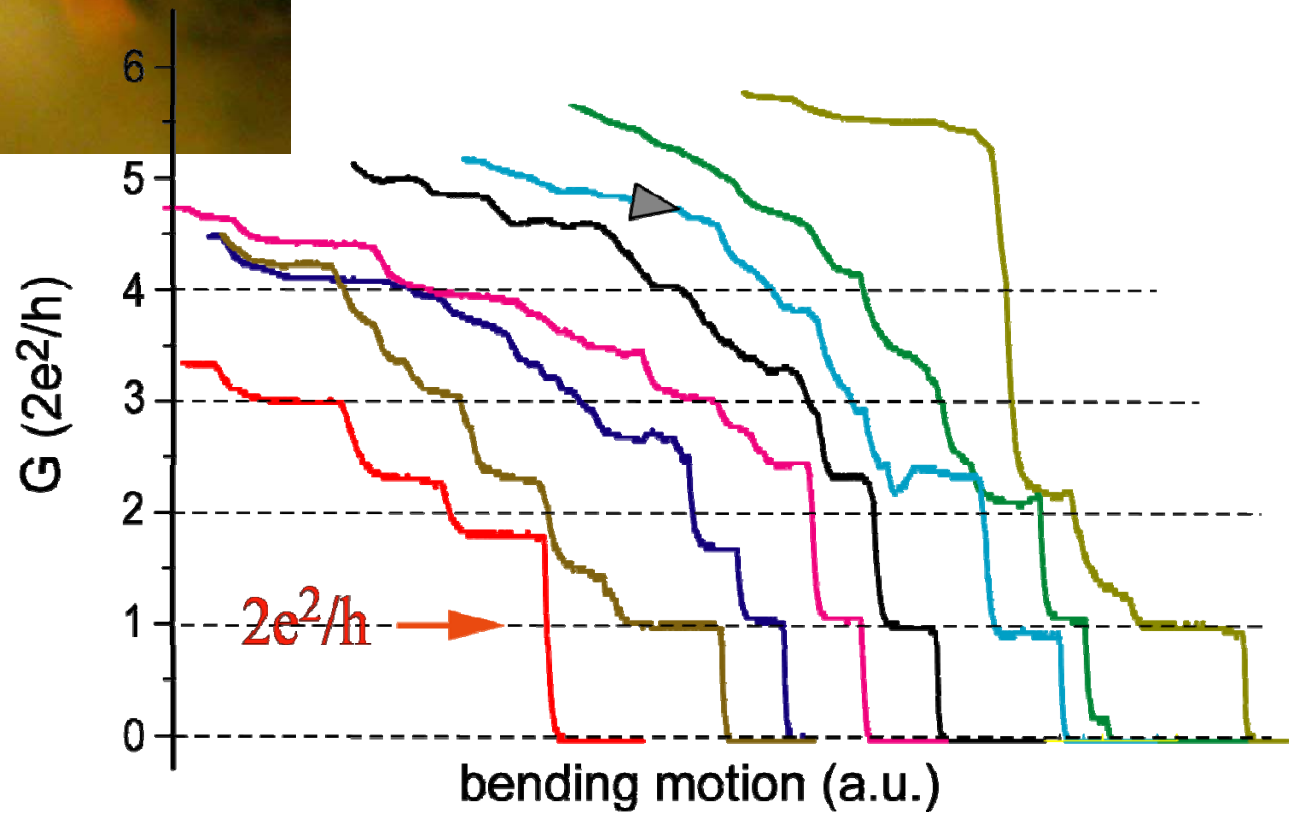
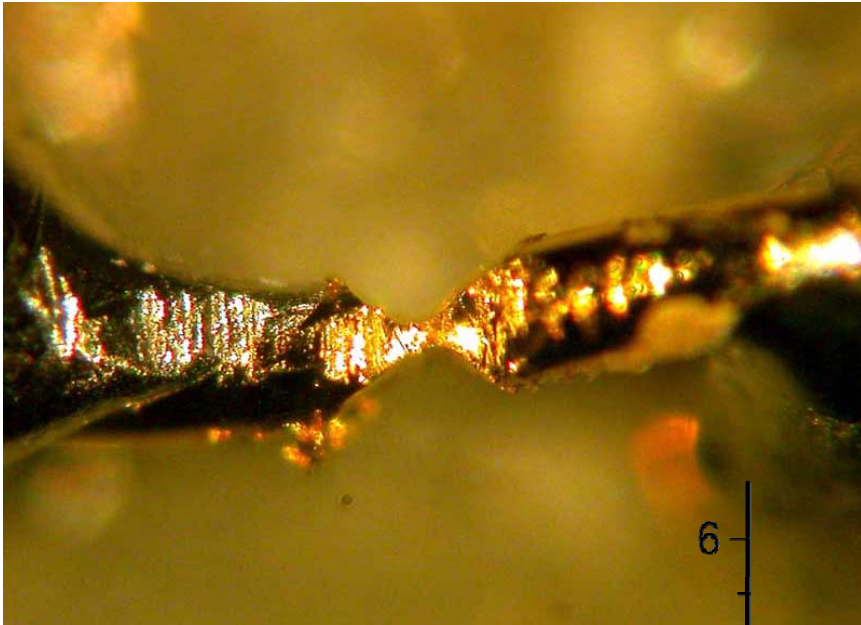


## 4.2 Quantum point contact (QPC)



*pioneered by Jan van Ruitenbeek, C. Urbina et al.*

## 4.2 Quantum point contact (QPC)

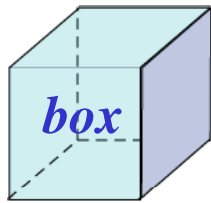




# 4.3 Quantized charge and flux

Millikan 1911

*isolated piece of matter*



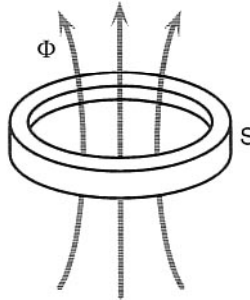
charge  $Q$  is quantized

$$Q = ne$$

(capacitor  $C$ )



*superconducting ring*

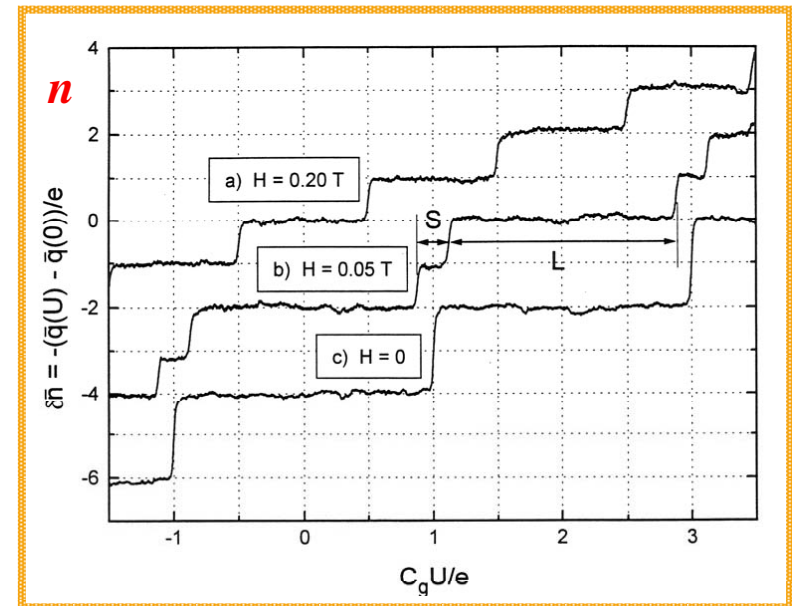


flux  $\Phi$  is quantized

$$\Phi = n(h/2e)$$

(inductor  $L$ )

solid-state version of  
Millikan's oil droplet experiment

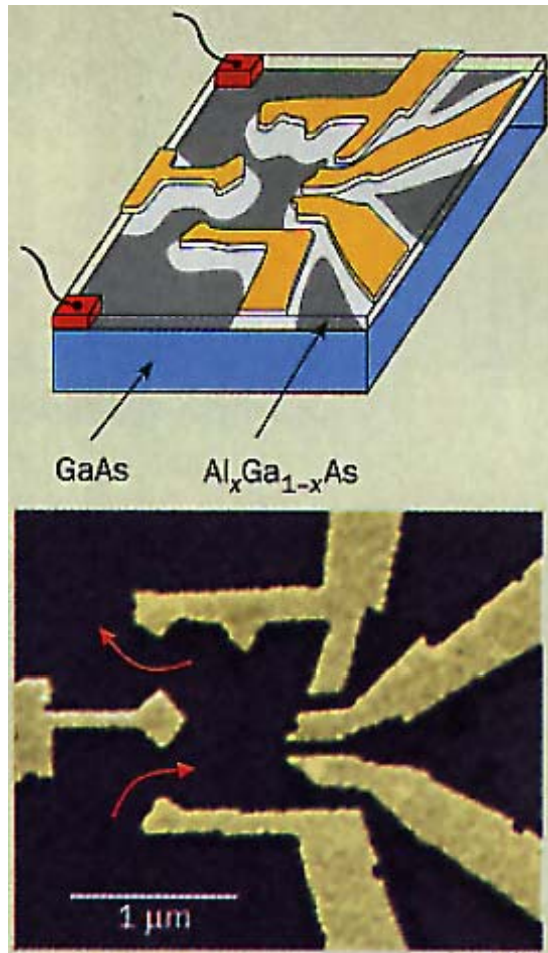


(Lafarge, Pothier, Bouchiat, Esteve, Devoret)

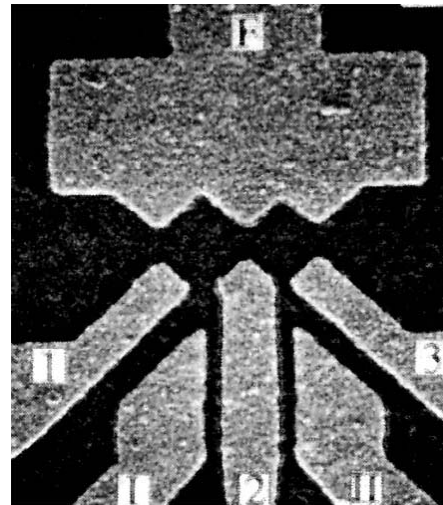
# 4.4 Quantum dots

“small” cavities

planar dot

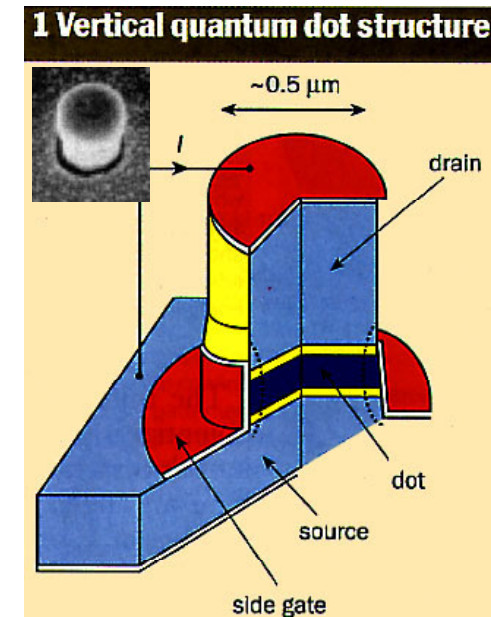


planar double-dot



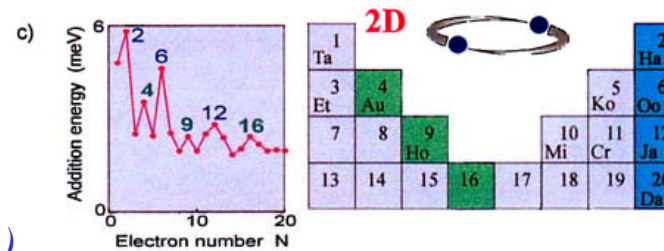
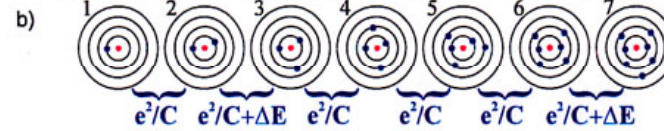
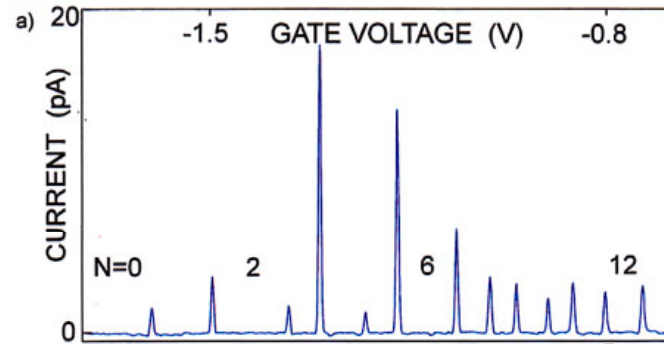
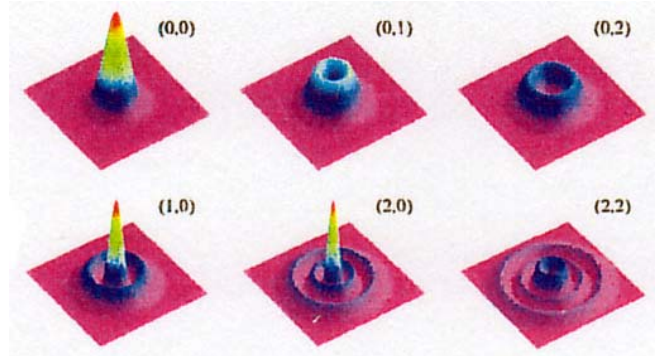
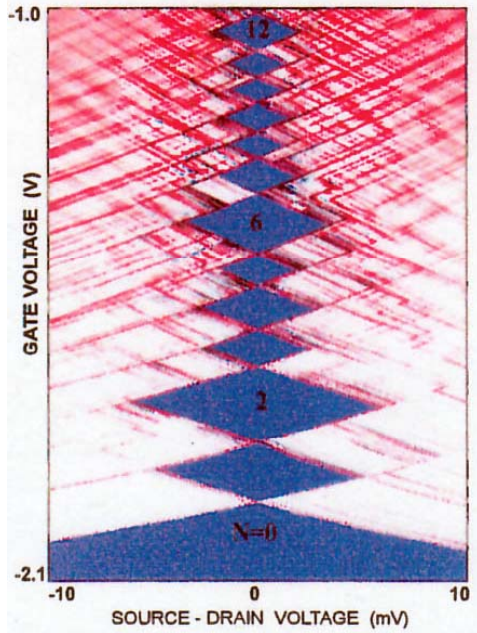
... planar multi-dot

vertical dot

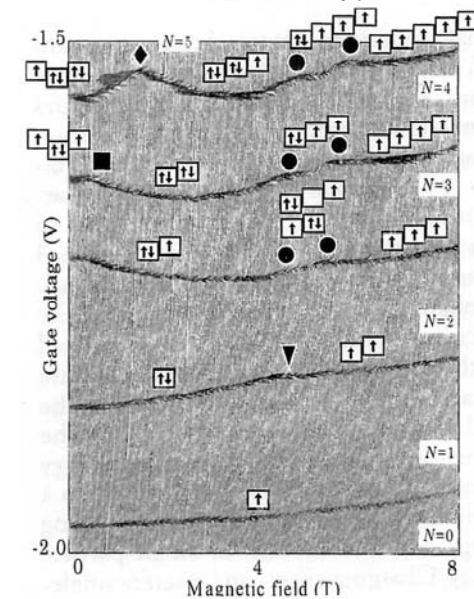
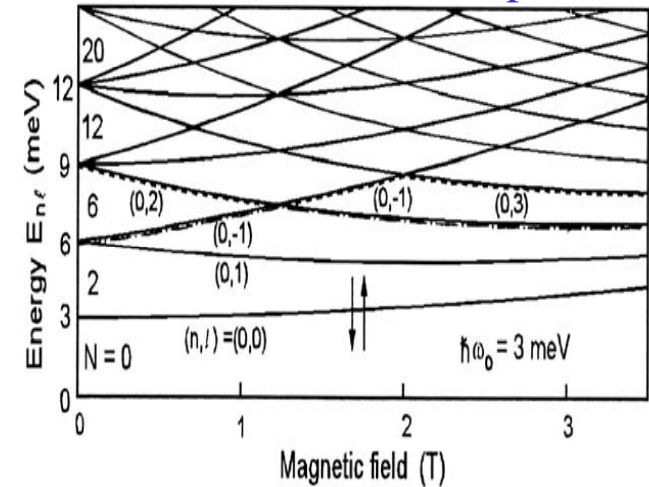


# 4.4 Quantum dots

## few electron quantum dots



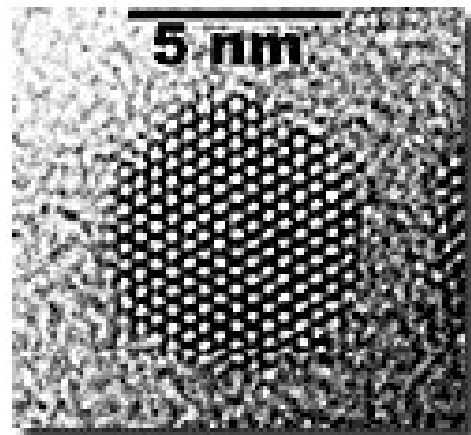
## Fock-Darwin level spectra



(Kouwenhoven, Tarucha et al.)



## 4.4 Quantum dots



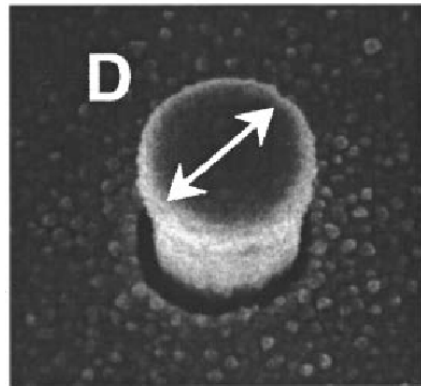
TEM by Andreas Kadavanich. Transmission electron microscopy shows the crystalline arrangement of atoms in a 5 nm CdSe Qdot particle.



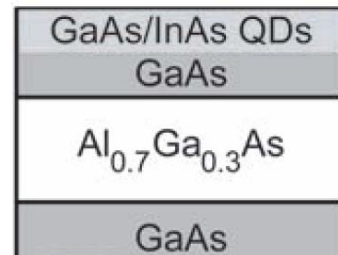
A family of Qdot particles can be made to emit a full spectrum of colors when excited with a single excitation source.



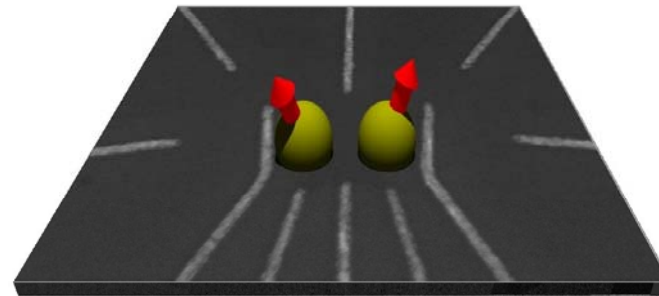
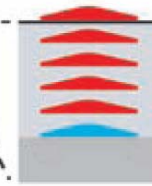
# 4.4 Quantum dots



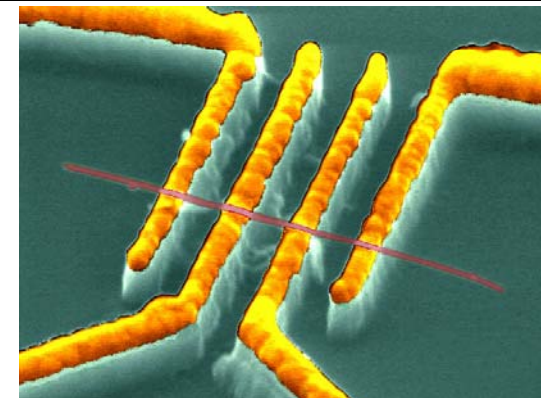
GaAs vertical



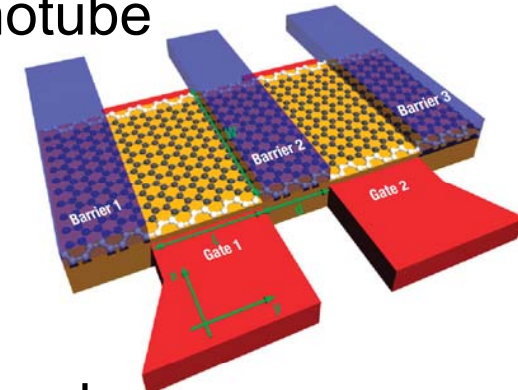
MBE grown



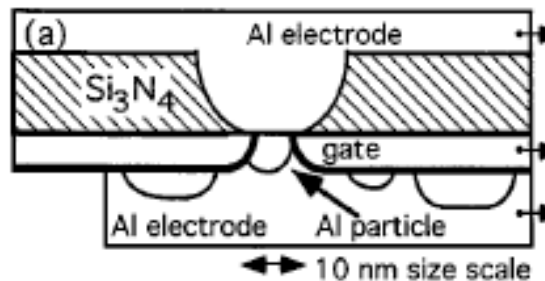
GaAs lateral



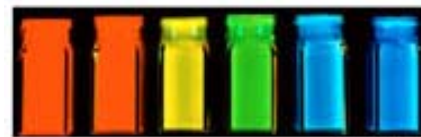
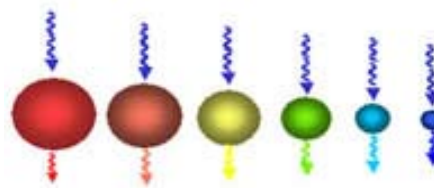
nanotube



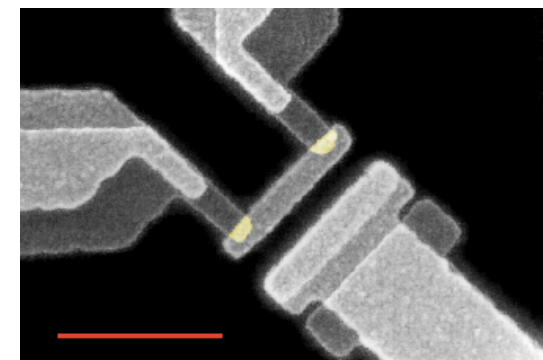
graphene



metal grain

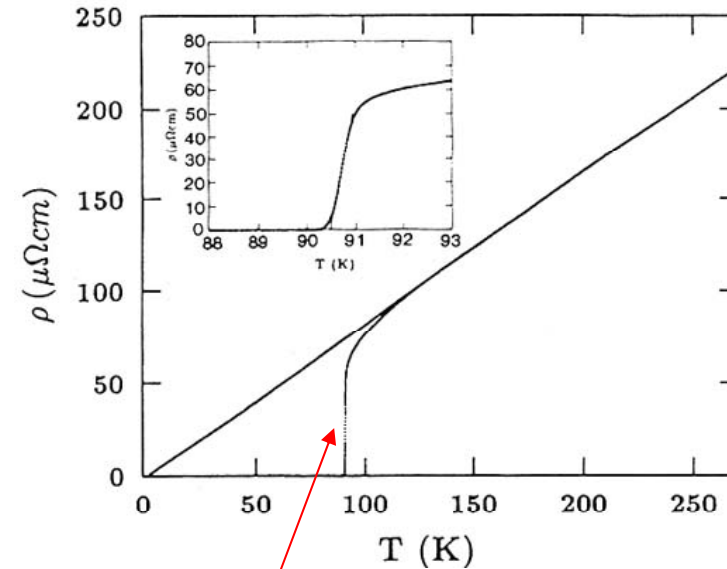
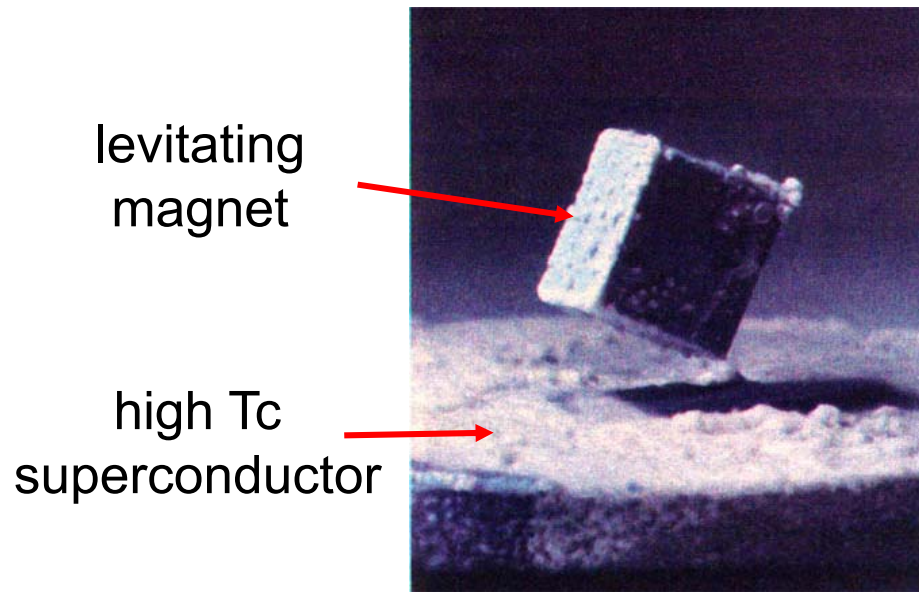


self assembled

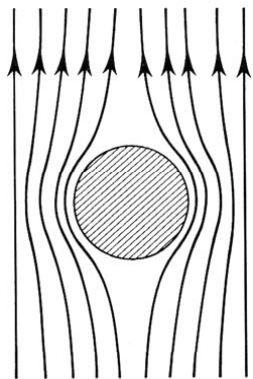


metallic SET

# 4.5 Interference



transition to zero resistance



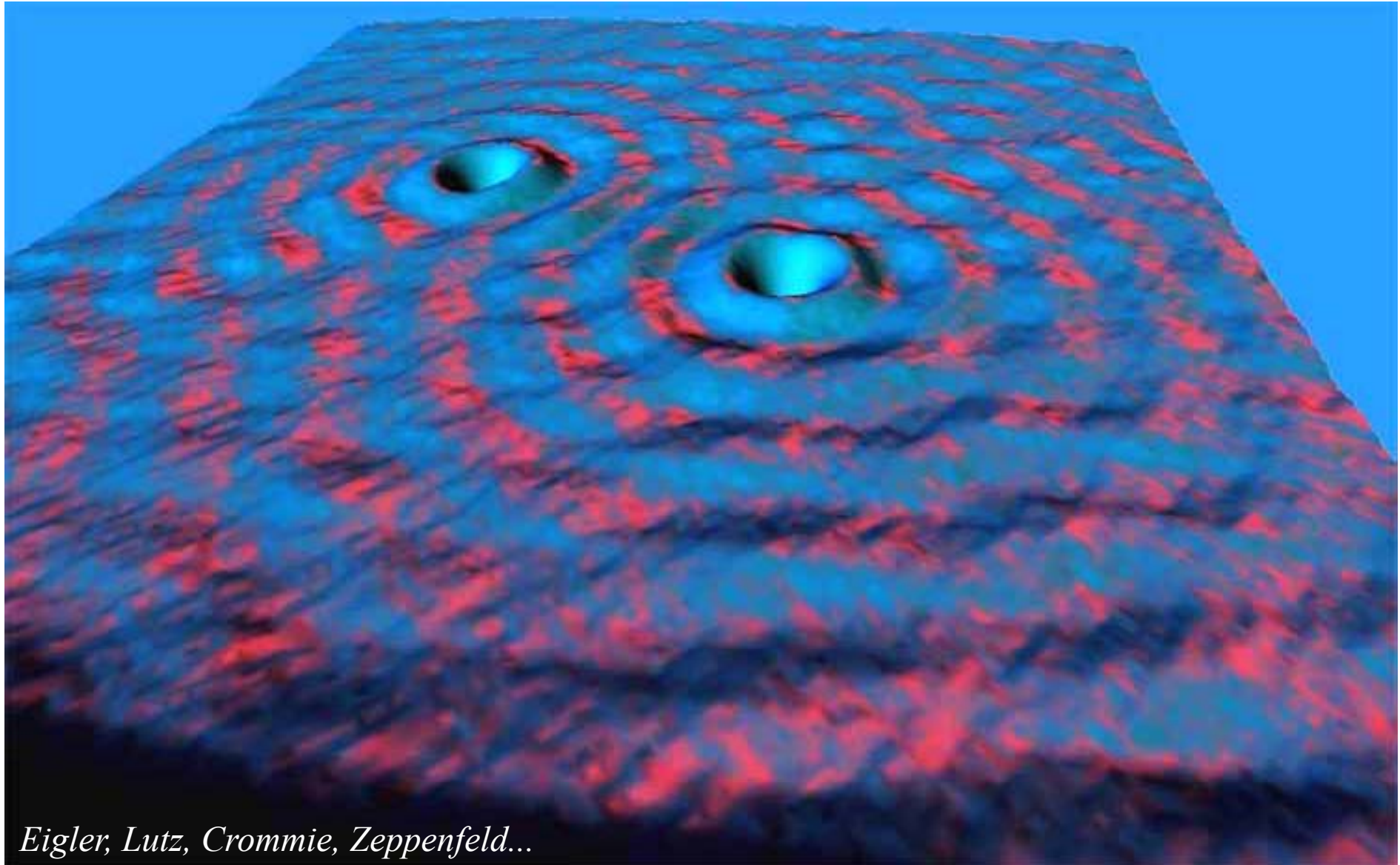
Meissner effect  
(ideal diamagnetism)

$$\psi = A \exp(i\theta)$$

Best known **macroscopic quantum system**: both  $A$  and  $\theta$  are collective „classical“ variables

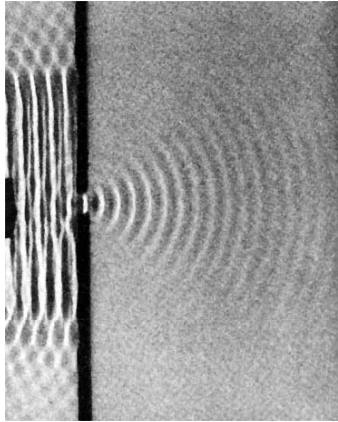


## 4.5 Interference

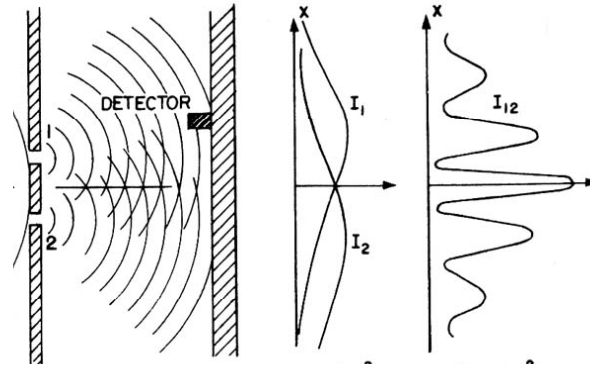


*Eigler, Lutz, Crommie, Zeppenfeld...*

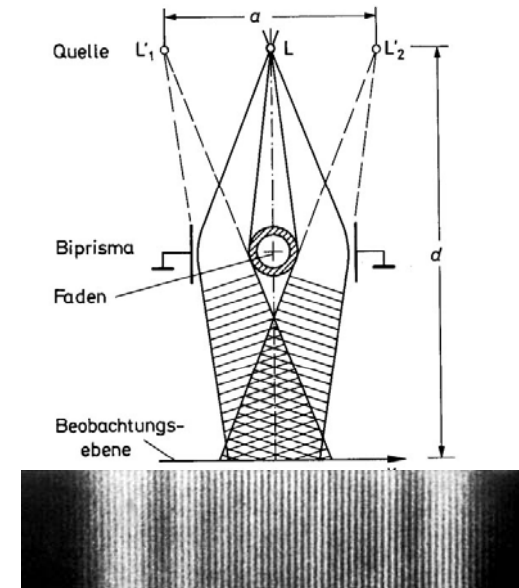
# 4.5 Interference



spherical wave



interference of 2 spherical wave  
Young's double slit experiment



(Moellenstedt, Tonamura)

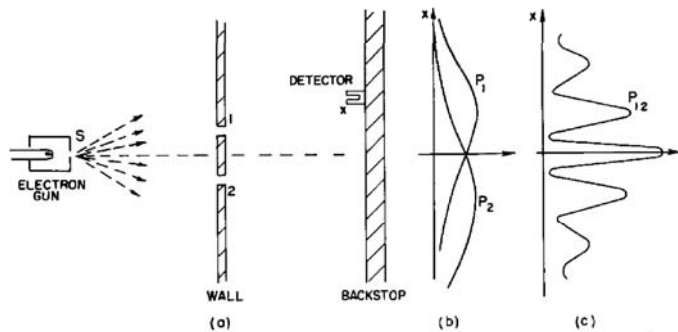
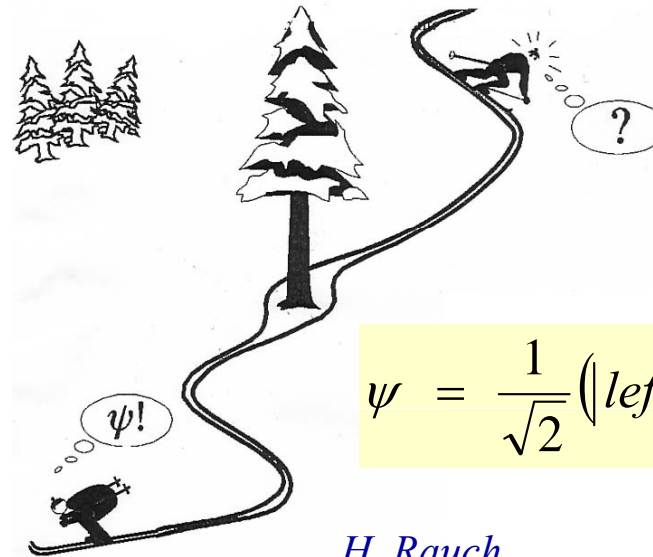


Fig 3-1. Interference experiment with electrons.

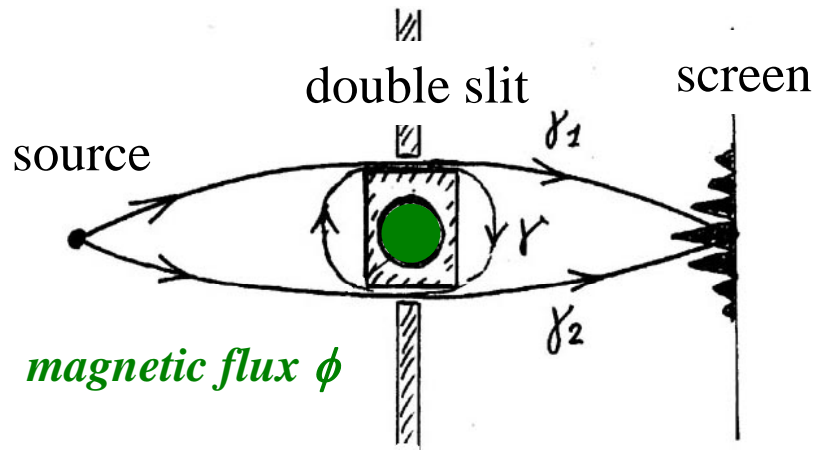
lens-less, low-energy electrons (H.-W. Fink)



$$\psi = \frac{1}{\sqrt{2}} (|left\rangle + |right\rangle)$$

H. Rauch

# 4.5 Interference Aharonov-Bohm effect



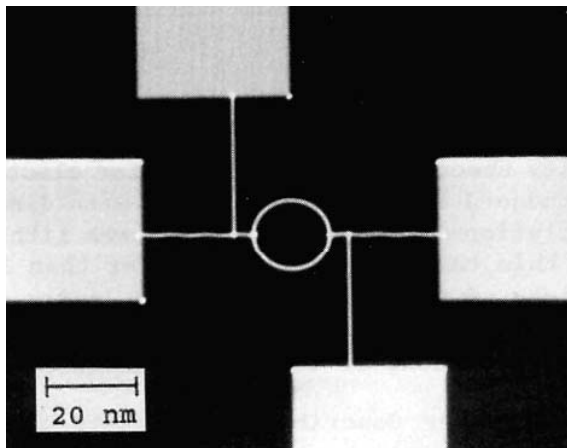
$$\psi = |\psi| \exp(i\theta)$$

$$\theta \rightarrow \theta + \frac{e}{\hbar} \int_{\gamma_i} \vec{A} \cdot d\vec{s} \quad (A = \text{vector potential})$$

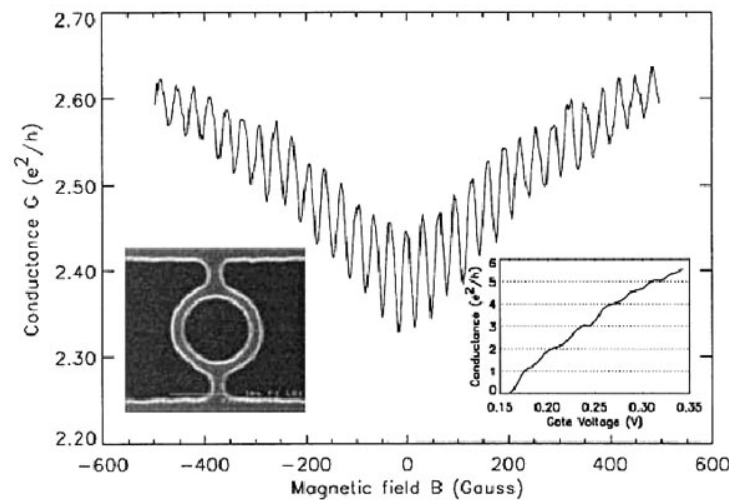
$$\Delta\theta = 2\pi \frac{\phi}{\phi_0}$$

$$\phi_0 = h/e$$

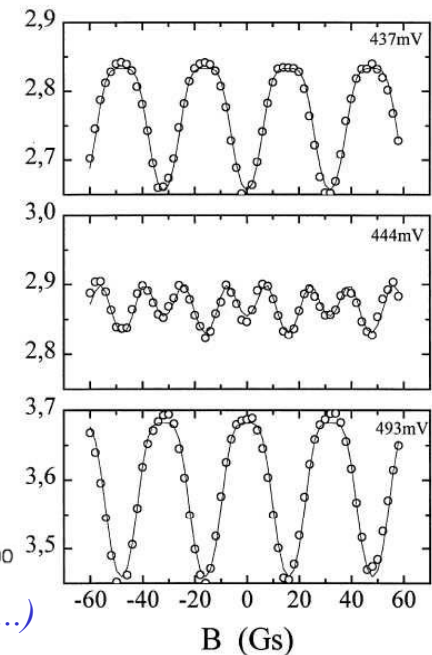
**magnetic flux quantum**



(smallest men-made metallic ring)



(Pederson, Hansen...)





# 4.5 Interference

## which path interferometer

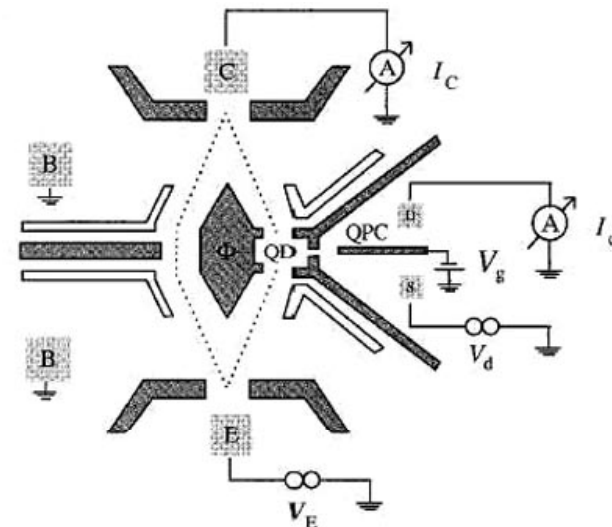
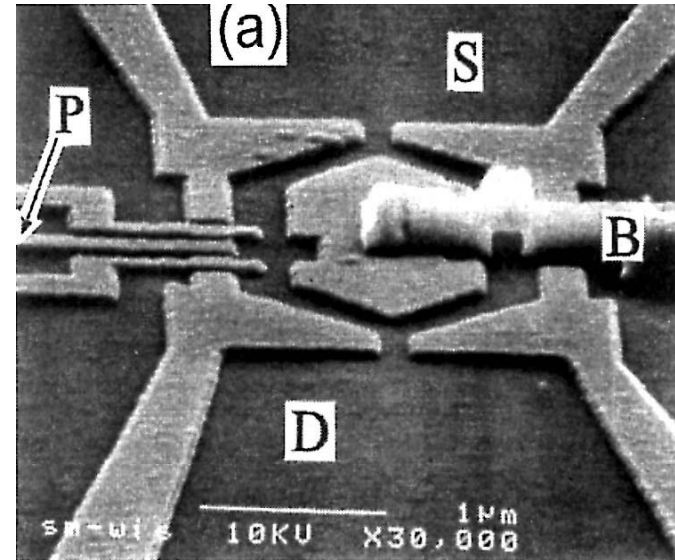
$$\psi = \frac{1}{\sqrt{2}}(|left\rangle + |right\rangle)$$



$$\psi = \frac{1}{\sqrt{2}}(|left\rangle \otimes |O_l\rangle + |right\rangle \otimes |O_r\rangle)$$

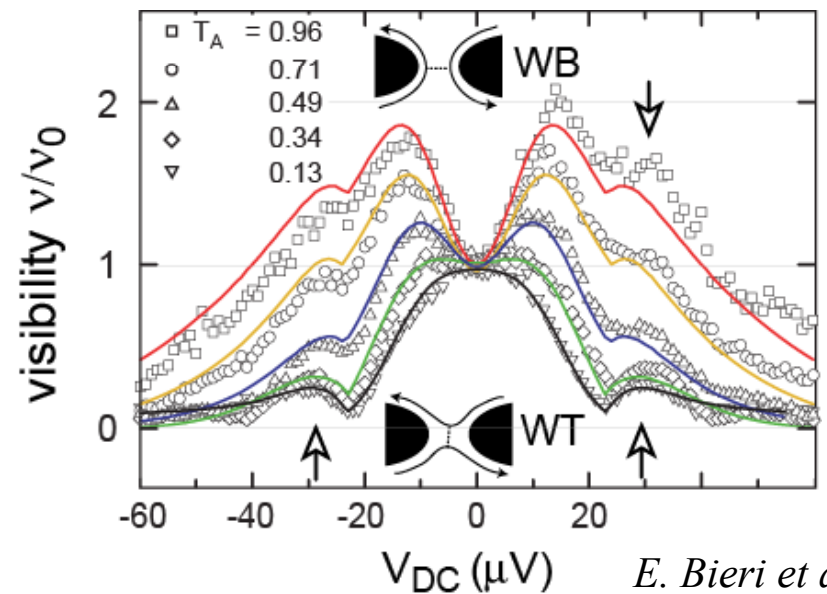
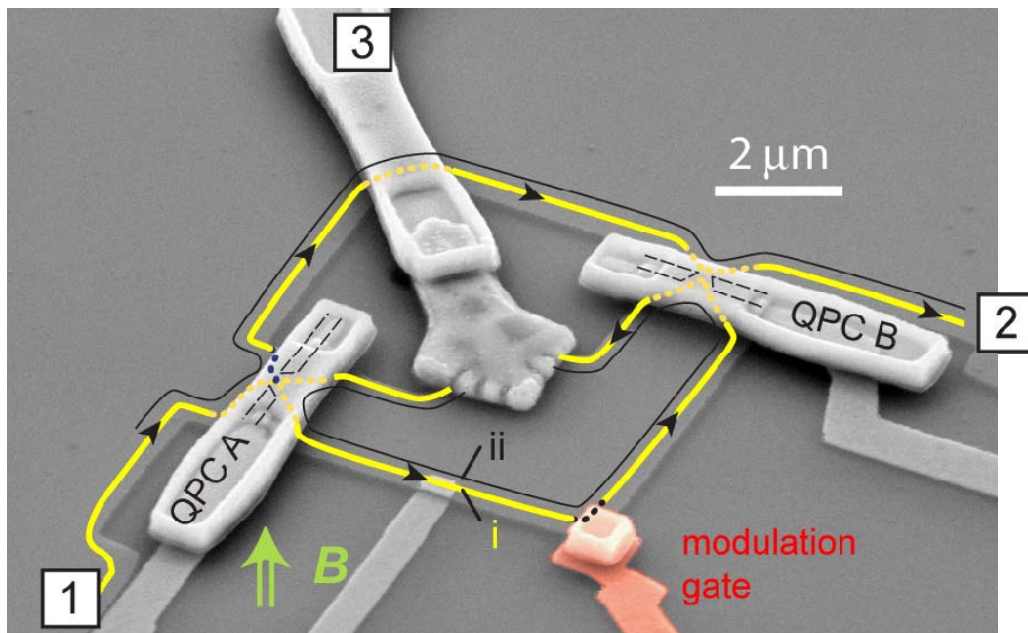
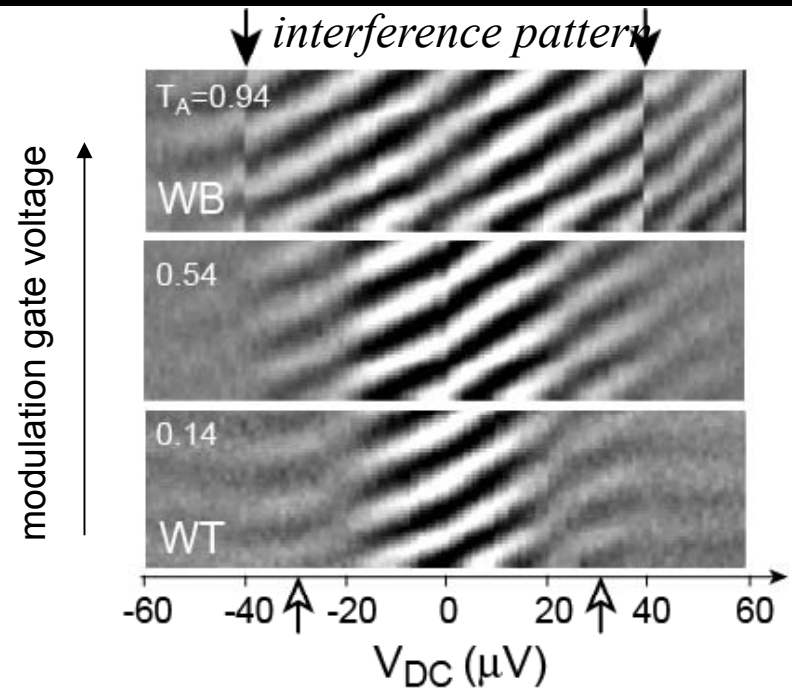
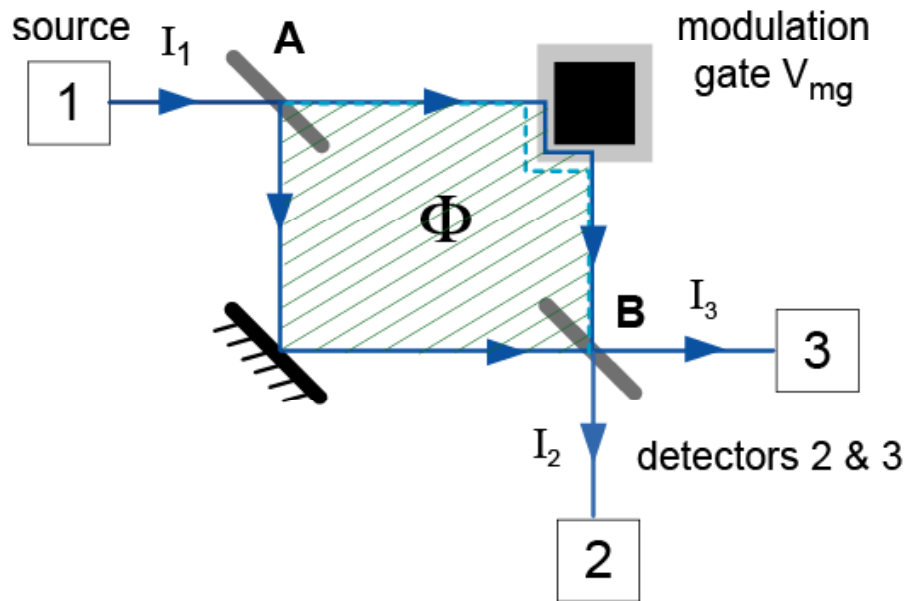
if  $\langle O_r || O_l \rangle = 0 \Rightarrow$  no interference

but we know which path the skier took



(Heiblum, Yacoby, Schuster et al. Weizmann)

# 4.5 Mach-Zehnder Interferometer

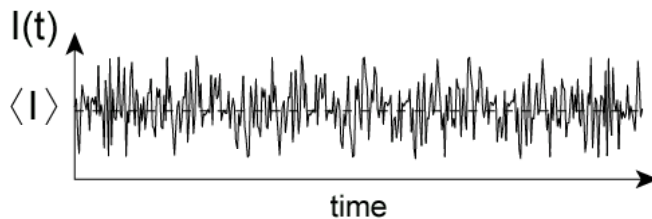
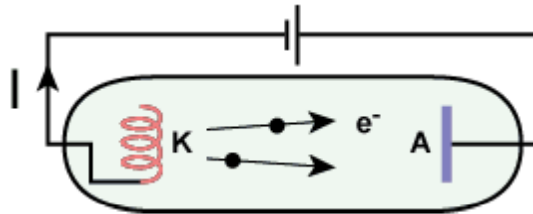


*E. Bieri et al.*

# 4.6 Noise

- A photonmultiplier makes **click** if a photon is absorbed.
- Such detectors are not yet available for charge transport in electronics!
- Still, the **granularity** of the measurement process, caused by the **quantization of charge in units of  $e$** , can still be probed.

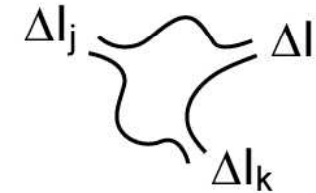
Schottky



$$S = 2e|I|$$

(uncorrelated transfer of „quantas“,  
i.e. Poisson statistics)

$$S_{ij}(\omega) = 2 \int dt e^{i\omega t} \langle \Delta I_i(t) \Delta I_j(0) \rangle \quad [A^2s]$$



Suppression of Shot Noise caused by:

- correlations due to **interactions**
- correlations due to **statistics**
- **fractional** charged quasiparticles



## 4.7 Quantum computing

# 4.7 Quantum computing

classical bit  $b$ :

$$b \in \{0,1\}$$

$b$  is **either 0 or 1**

quantum bit (qubit)  $q$ :

$$q = a|0\rangle + b|1\rangle$$

$q$  is a coherent superposition of 0 **and** 1

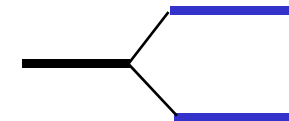
**qubit:**

qubit is a 2-level system

$$H = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1| + \frac{E_c}{2} \{ |0\rangle\langle 1| + |1\rangle\langle 0| \}$$

*coupling  $E_c$*

if  $E_1 = E_2$



E.g. prepare state(s)

$$|0\rangle \Rightarrow a|0\rangle + b|1\rangle$$

through **control of interaction**  
(without loss of coherence..!)

different realizations

qubits =

charge qubit

flux qubit

phase qubit

spin qubit

trapped ions

photon qubit

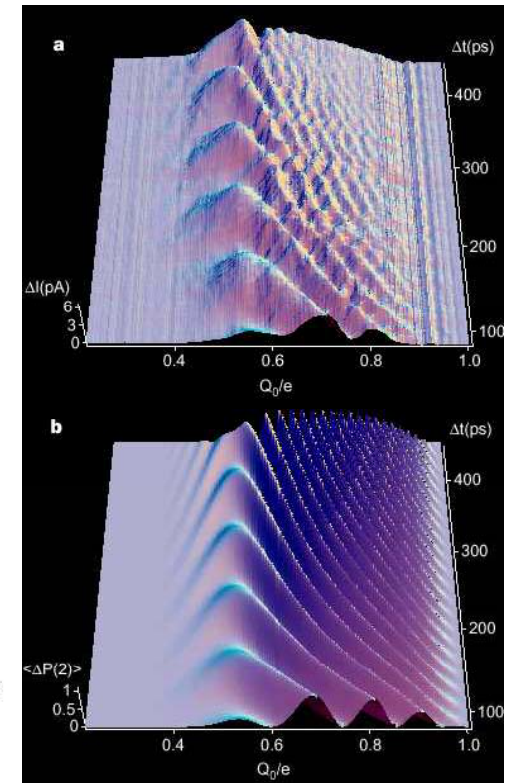
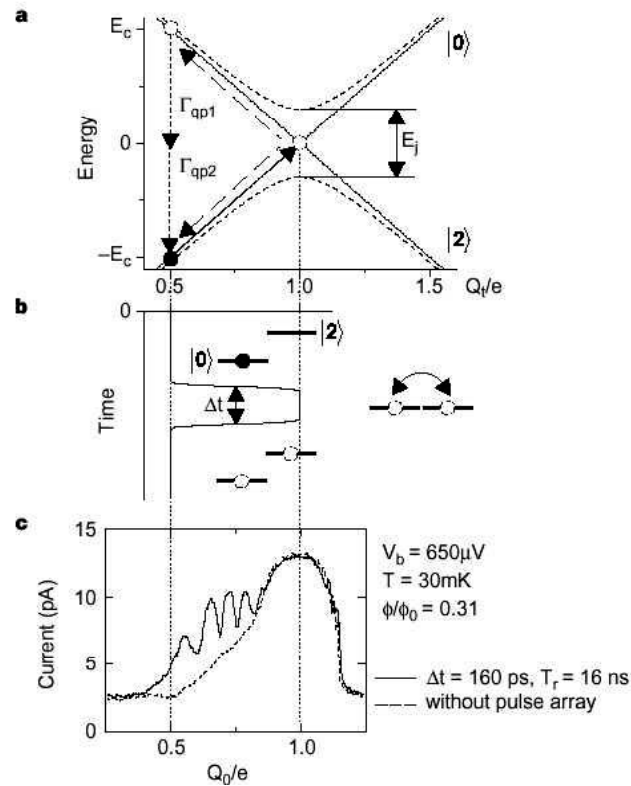
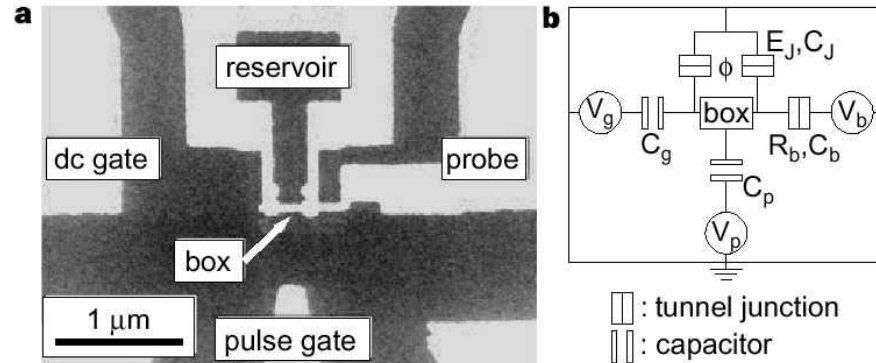
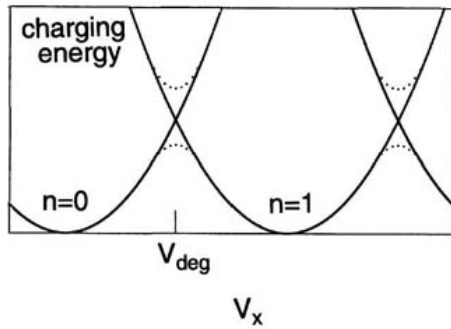
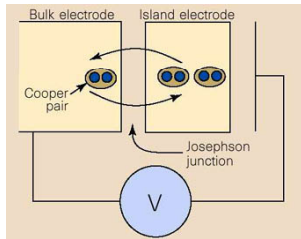
INTERACTION

- charge
- current
- exchange

# 4.7 Quantum computing

## Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura\*, Yu. A. Pashkin† & J. S. Tsai\*



Quantum computation  
**Solid-state qubits under control** ?

D. V. Averin



# Quantum Transport FS 2015 / books

books:

- *Transport in Nanostructures*,  
M. J. Kelly, Clarendon Press, Oxford
- *Mesoscopic Physics*, Leo Kouwenhoven et al. in  
Nato ASI Series E, Vol. 345, p 1-44, Kluwer
- *Mesoscopic Electronics in Solid State Nanostructures*,  
Thomas Heinzel, Wiley-VCH
- *Electronic Transport in Mesoscopic Systems*  
S. Datta, Cambridge University Press
- *Introduction to Mesoscopic Physics*,  
Y. Imry, Oxford University Press
- *The Physics of Low Dimensional Systems*,  
J.H. Davies, Cambridge University Press
- *The Physics of Nanoelectronics*  
Tero T. Heikkilä, Oxford Master Series
- *Quantum Transport: Introduction to Nanoscience*  
Yuli Nazarov and Yaroslav Blanter, Cambridge

# Quantum Transport FS 2015 / books

OXFORD MASTER SERIES IN CONDENSED MATTER PHYSICS

books:

- *Transport in Nanostructures*, M. J. Kelly, Clarendon Press, Oxford
- *Mesoscopic Physics*, Leo Kouwenhover  
Nato ASI Series E, Vol. 345, p 1-44, Kluwer
- *Mesoscopic Electronics in Solid State Nanosystems*  
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- *The Physics of Low Dimensional Systems*  
J.H. Davies, Cambridge University Press
- *The Physics of Nanoelectronics*  
Tero T. Heikkilä, Oxford Master Series
- *Quantum Transport: Introduction to Nanoscale Transport*  
Yuli Nazarov and Yaroslav Blanter, Cambridge University Press

## The Physics of Nanoelectronics

*Transport and Fluctuation Phenomena  
at Low Temperatures*

Tero T. Heikkilä

OXFORD



# Basics and 1. exercises

all below you need to know. Repetition on Thursday, but you must prepare yourselves by studying the notes provided, i.e. *1.background\_knowledge.pdf*

- Ohm's law, Kirchhof's laws
- Fermi gas, Fermi parameters like  $E_F$ ,  $\lambda_F$ ,  $k_F$ ,  $T_F$ ...
- band structure (metal and semiconductor, band gap)
- effective mass approximation
- density of states, DOS,  $\rho(E_F)$ ,  $N(E_F)$ , chemical potential, electrochemical potential
- Schrödinger equation with confining potential in different dimensions
- density of states of Fermi gas in different dimensions
- capacitance spectroscopy (R. Ashoori 1993)
- thermodynamics, Fermi-Dirac and Bose-Einstein distribution
- generalized electron distribution function → week 2

homework 2: read “Quantum Transport in Nano-Structured Semiconductors” by B. Kramer *1.Quantum\_Transport\_Intro\_B.Kramer.pdf*

- how to calculate a current (I do in the class right now !)