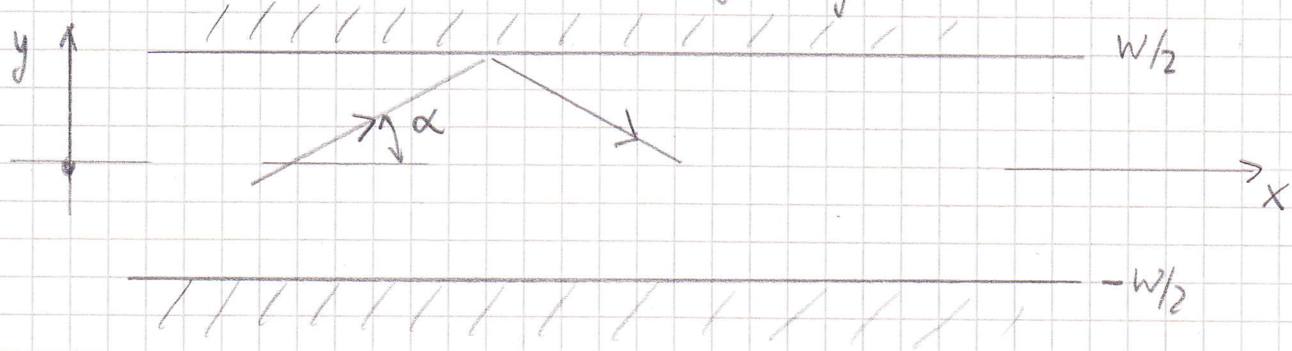
Exercise week 2 Semiclassical transport

2.1. Transport in a channel of width w realized in a 2DEG with elastic scattering only



$\vec{v} = v_F\,(\cos(\alpha), \sin(\alpha))$ for an electron with Fermi velocity

We assume $\dfrac{df}{dt} = \vec{v}\cdot\dfrac{\partial f}{\partial \vec{x}} + \dot{\vec{v}}\cdot\dfrac{\partial f}{\partial \vec{v}} = [I_{coll}]$

with $[I_{coll}] = -\dfrac{f}{\tau} + \dfrac{\langle f\rangle}{\tau}$ und $f = f(\vec{x}, \vec{v})$

Let us now assume that a magnetic field $\vec{B} = B\vec{e}_z$ is applied.

$\Longrightarrow$ a) show that $\dot{\vec{v}}\cdot\dfrac{\partial}{\partial \vec{v}} = \omega_c \dfrac{\partial}{\partial \alpha}$ with $\omega_c = \dfrac{eB}{m}$

We only consider electrons at the Fermi surface. Hence, the magnitude of $\vec{k}$ or $\vec{v}$ is fixed, only the direction matters.
We write now $\underline{F(\vec{x}, \alpha) = f(\vec{x}, \vec{v})}$ $\Rightarrow$

$$(*)\quad \frac{dF}{dt} = \vec{v}\cdot\frac{\partial F}{\partial \vec{x}} + \omega_c\frac{\partial F}{\partial \alpha} = -\frac{F}{\tau} + \frac{1}{2\pi\tau}\int_0^{2\pi} d\beta\; F(\vec{x}, \beta)$$

$\Longrightarrow$ b) Show that
$F(\vec{x}, \alpha) = -c\,(x - \omega_c \tau y) + c\cdot l\cdot\cos(\alpha)$ with $c = \text{const}$
$l = v_F\tau$ is a solution of $(*)$ satisfying

boundary conditions for specular reflection, i.e.:

$$F(\vec{x}, \alpha) = F(\vec{x}, -\alpha) \quad \text{if } y = \pm W/2$$

$\Rightarrow$ c) calculate $n(x,y) = \int_{-\pi}^{\pi} F(\vec{x}, \alpha)\, d\alpha = \ldots$

$\Rightarrow$ d) calculate the particle current I:

$$I = \int_{-W/2}^{W/2} dy \int_{-\pi}^{\pi} d\alpha \; F(\vec{x}, \alpha)\, v_F \cos(\alpha)$$

$\Rightarrow$ e) show that for the current density j, the following equation holds

$$j = -D \frac{\partial n}{\partial x}$$

$\Rightarrow$ f) what is D?

2.2 Relaxationsapproximation

Dirac distribution

assume that $f(\vec{x},\vec{k}) = f_0 + \delta f$ with

$$\delta f = -\tau\left(\vec{v}\cdot\frac{\partial f_0}{\partial \vec{x}} - \frac{e\vec{\mathcal{E}}}{\hbar}\frac{\partial f_0}{\partial \vec{k}}\right) \quad ; \quad \vec{\mathcal{E}} = \text{electric field (only)}$$

show that δf can be written as follows:

$$\delta f = \left(\frac{\partial f_0}{\partial E}\right)\tau\,\vec{v}\cdot\left\{\vec{\nabla}\mu + e\vec{\mathcal{E}} + \left(\frac{E-\mu}{T}\right)\vec{\nabla}T\right\}$$

here $f_0 = \dfrac{1}{e^{(E(\vec{k})-\mu)/k_B T}+1}$ and $\mu = \mu(\vec{x})$, $T = T(\vec{x})$

2.3 Currents

charge current density $\vec{j}_e = \dfrac{-e}{4\pi^3}\displaystyle\int d^3k\ \delta f\ \vec{v}$

energy current density $\vec{j}_E = \dfrac{1}{4\pi^3}\displaystyle\int d^3k\ (E-\mu)\ \delta f\ \vec{v}$

$$\boxed{\begin{aligned}\vec{j}_e &= (\sigma/e)\,\vec{\nabla}\mu_E + \mathcal{L}_{12}\,\vec{\nabla}T \\ \vec{j}_E &= \mathcal{L}_{21}\,\vec{\nabla}\mu_E \underbrace{- \mathcal{K}\,\vec{\nabla}T}_{\text{heat current}}\end{aligned}}$$

with $\vec{\nabla}\mu_E = \vec{\nabla}\mu + e\vec{\mathcal{E}}$

calculate $\mathcal{K} = \cdots$

conductivity

one obtains $\mathcal{K} = \dfrac{\pi^2 k_B^2}{3e^2}\,\sigma\,T$

2.4 Read paper Phys. Rev. Lett. 79, 3490 (97) and ask questions

Find an interesting paper that cites the above one!