Quantum transport

Andreas Baumgartner

Problem sheet 3

1) Estimate the number of transverse quantum channels in an infinitely long lead of width L_x and thickness L_y for a given Fermi energy and infinite square well potentials as confinement.

2) Find, download and read the paper Phys. Rev. Lett. 60, 848 (1988).
   a) What is plotted in Fig. 2?
   b) In few words: why are there steps in the conductance of the quantum point contact (QPC), though the gate voltage is tuned continuously?
   c) What happens to the many quantum channels outside of the QPC region?
   d) What determines the conductance values on the plateaus?

3) Derivation of Ohm's law (in one dimension) in the scattering picture.
   a) Determine the number of transverse modes in an infinitely long lead of width L_x.
   b) Determine the conductance G of this "wire" for a given transmission T. Express G using the density of states $N = \frac{m}{\pi\hbar^2}$ of a two-dimensional electron gas (2DEG) and the Fermi velocity $v_F = \frac{\hbar k_F}{m}$.

3c) Assume that there are <u>two</u> scatterers along the wire with transmissions T_1 and T_2 and that transport is coherent. What is the transmission through both scatterers, T_{12}?
Hint: it is <u>not</u> $T_{12} = T_1 \cdot T_2$ due to multiple reflections and transmissions!

d) Show that the quantity $\frac{1-T}{T}$ is additive when placing 2 scatterers in series, i.e. show that $\frac{1-T_{12}}{T_{12}} = \frac{1-T_1}{T_1} + \frac{1-T_2}{T_2}$.

e) Use the result of 3d to obtain the transmission through M scatterers.

f) Use a given density of scatterers, ν, to estimate the number of scatterers in a wire of length L and express the transmission T_L of this wire using the characteristic length $L_0 := \frac{T}{\nu(1-T)}$, which is related to the scattering length. Assume that all scatterers have the same transmission T.

g) Use the results of 3b and 3f to obtain an expression for G. Assuming that L_0 is related to the scattering length, we can then define the diffusion coefficient $D = v_F \cdot L_0/\pi$ and obtain the <u>conductivity</u> $\sigma = e^2 N D$ (Einstein relation), which gives G as a function of σ, L, L_x and L_0. Then write down $R = G^{-1}$ and separate a term that scales with L and the remaining contact resistance.