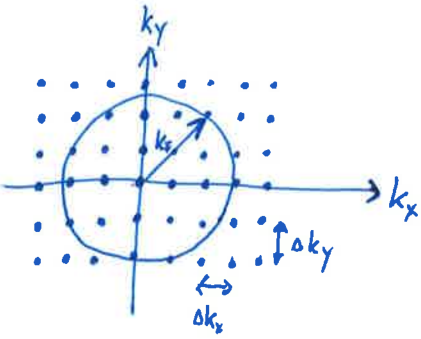
Quantum Transport
Solutions to problem sheet 3

1) Subband energy $\varepsilon_N = \varepsilon_{n_x} + \varepsilon_{n_y} \overset{!}{\lesssim} E_F$

Square-well confinement $\Rightarrow$ equidistant k-states, $\Delta k_i = \frac{\pi}{L_i}$ (periodic boundary conditions)

$$\Rightarrow \frac{\hbar^2 k_{trans}}{2m} = \frac{\hbar^2 (k_x^2 + k_y^2)}{2m} \overset{!}{\lesssim} E_F = \frac{\hbar^2 k_F^2}{2m} \quad \text{or} \quad k_x^2 + k_y^2 \overset{!}{\lesssim} k_F^2$$



If we assume $\Delta k_x \cdot \Delta k_y \ll k_F^2$ we can estimate the number of states inside the Fermi-circle as:

$$\underline{N} \approx \frac{\pi k_F^2}{\Delta k_x \cdot \Delta k_y} = \frac{\pi k_F^2}{\frac{\pi}{L_x} \cdot \frac{\pi}{L_y}} = \frac{k_F^2}{\pi} \underbrace{L_x \cdot L_y}_{=:A} \underset{k_F = \frac{2\pi}{\lambda_F}}{=} \underline{\underline{4\pi \cdot \frac{A}{\lambda_F^2}}}$$

A: cross-section area of lead

2a) conductance G vs. the voltage applied to the split-gate that defines the QPC.

b) The number of transmitted channels is <u>not</u> reduced continuously. One can think of a QPC as a "wire" in which the width is controled by the gate.

c) Ideally, the channels are either fully transmitted or reflected.

d) The conductance value on a plateau is determined by the number of (fully) transmitted channels. Each channel can carry $2e^2/h$ in conductance.

3a) problem in 1D : $-k_F \quad 0 \quad k_F \quad k$

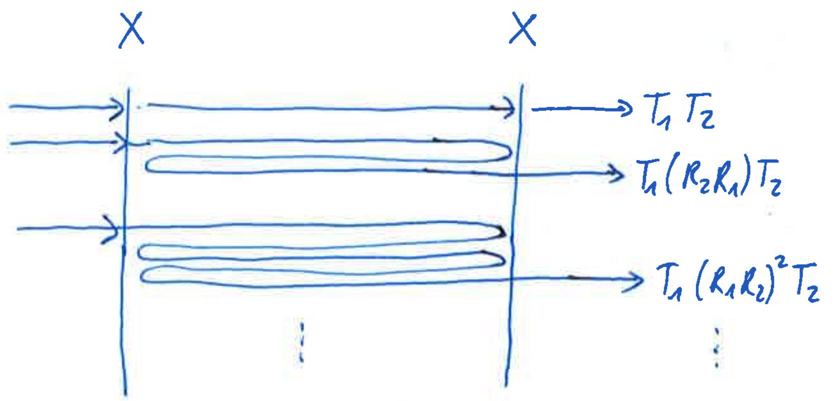
„Fermi-line"

number of transverse modes : $\underline{\underline{M}} \approx \frac{2k_F}{\Delta k} = \frac{2k_F L_x}{\pi} = \underline{\underline{\frac{k_F L_x}{\pi}}}$

see problem 1

b) $\underline{\underline{G}} = \frac{2e^2}{h} \cdot M \cdot T = \frac{2e^2}{h} \cdot \frac{k_F L_x}{\pi} \cdot T = \frac{2e^2}{2\pi\hbar} \cdot \underbrace{\frac{m v_F}{\hbar}}_{k_F} \cdot \frac{L_x}{\pi} T = \underbrace{\frac{m}{\hbar^2 \pi}}_{N} L_x k_F e^2 T/\pi = \underline{\underline{e^2 L_x N \frac{v_F T}{\pi}}}$

c) scatterer 1 scatterer 2



„Feynman paths"

$$\underline{\underline{T_{12}}} = T_1 T_2 \cdot \sum_{n=0}^{\infty} (R_1 R_2)^n = \frac{T_1 T_2}{1 - R_1 R_2}$$

geometric series

$$= \underline{\underline{\frac{T_1 T_2}{1 - (1-T_1)(1-T_2)}}}$$

$R_i = 1 - T_i$

d) $\underline{\underline{\frac{1 - T_{12}}{T_{12}}}} = \frac{1 - \frac{T_1 T_2}{1 - R_1 R_2}}{\frac{T_1 T_2}{1 - R_1 R_2}} = \frac{1 - R_1 R_2 - T_1 T_2}{T_1 T_2} = \frac{1 - (1-T_1)(1-T_2) - T_1 T_2}{T_1 T_2} = \frac{\cancel{1} - \cancel{1} - 2T_1 T_2 + T_1 + T_2}{T_1 T_2}$

$= \left(\frac{1}{T_1} - 1\right) + \left(\frac{1}{T_2} - 1\right) = \underline{\underline{\frac{1 - T_1}{T_1} + \frac{1 - T_2}{T_2}}}$

e) $\frac{1-T}{T}$ is additive (see 3d) $\Rightarrow \frac{1 - T_M}{T_M} = M \cdot \frac{1-T}{T}$

$\Rightarrow \frac{1}{T_M} - 1 = M \cdot \frac{1-T}{T}$

$\frac{1}{T_M} = 1 + M \frac{1-T}{T} = \frac{T + M(1-T)}{T}$ or $\underline{\underline{T_M = \frac{T}{T + M(1-T)}}}$

3f) $T_L = T_M\big|_{M=L\cdot\vartheta} = \dfrac{T}{T + L\vartheta(1-T)}$

$$\frac{1}{T_L} = 1 + \underbrace{L\vartheta\,\frac{1-T}{T}}_{=:\,\frac{1}{L_0}} = 1 + \frac{L}{L_0} = \frac{L+L_0}{L_0} \qquad \text{or} \qquad \underline{\underline{T_L = \frac{L_0}{L+L_0}}}$$

g) $$\underline{\underline{G}} \overset{\text{from b}}{=} e^2 N L_x \frac{v_F T_L}{\pi} = \frac{1}{\pi}\underbrace{e^2 N}_{\sigma/D} \cdot v_F L_x \underbrace{\frac{L_0}{L+L_0}}_{T_L} = \frac{\sigma}{D}\cdot \underbrace{\frac{v_F L_0}{\pi}}_{D} \cdot \frac{L_x}{L+L_0} = \underline{\underline{\sigma \cdot \frac{L_x}{L+L_0}}}$$

$$\underline{\underline{R}} = \frac{1}{G} = \frac{L+L_0}{\sigma\cdot L_x} = \underbrace{\frac{1}{\sigma}\cdot\frac{L}{L_x}}_{\text{Ohm's law}} + \underbrace{\frac{1}{\sigma}\,\frac{L_0}{L_x}}_{\text{contact resistance } \propto \frac{1}{L_x}}$$

Ohm's law:
- $\rho = \frac{1}{\sigma}$ (resistivity)
- $\propto L$
- $\propto \frac{1}{L_x}$