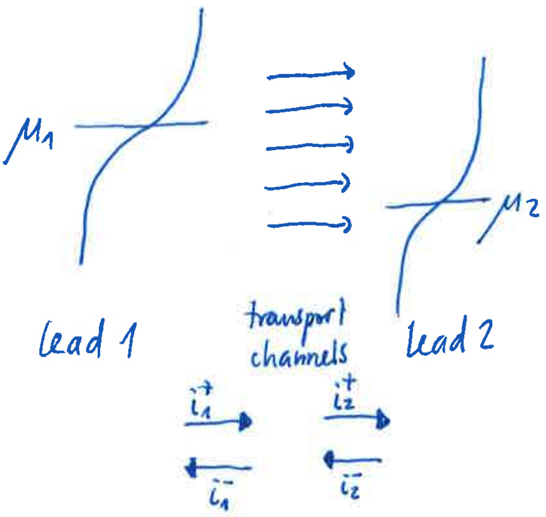
Quantum Transport

Week 4: more on the scattering approach and quantum Hall effect (QHE)

One result from last week: conductance from transmission for a 2-terminal device:

$$G = \frac{2e^2}{h}\cdot T \quad \text{for temperature } T=0.$$

Finite temperature:



intuitively (see schematic)

lead 1: $i_1^+(E) = \frac{2e}{h}\cdot M_1(E)\cdot f_1(E)$

lead 2: $i_2^-(E) = \frac{2e}{h} M_2(E)\cdot f_2(E)$

from this $i_1^-(E) = \overbrace{[1-T(E)]}^{R(E)}\cdot i_1^+(E) + \tilde{T}(E)\cdot i_2^-(E)$

$i_2^+(E) = T(E)\cdot i_1^+(E) + \underbrace{[1-\tilde{T}(E)]}_{\tilde{R}(E)}\cdot i_2^-(E)$

$$\Rightarrow i(E) = i_1^+(E) - i_1^-(E) \left(= i_2^+(E) - i_2^-(E)\right)$$
$$= \cancel{i_1^+(E)} - \cancel{i_1^+(E)} + T(E)\, i_1^+(E) - \tilde{T}(E)\cdot i_2^-(E)$$
$$= \frac{2e}{h}\left[M_1 T(E) f_1(E) - M_2(E)\tilde{T}(E)\cdot f_2(E)\right]$$
$$= \frac{2e}{h}\left[\bar{T}(E)\cdot f_1(E) - \bar{\tilde{T}}(E)\cdot f_2(E)\right]$$

assume $\tilde{T} = T$

$$\Rightarrow \underline{\underline{I}} = \int i(E)\, dE \overset{\downarrow}{=} \underline{\underline{\frac{2e}{h}\int \bar{T}(E)\cdot[f_1(E) - f_2(E)]\, dE}}$$

Some small-print:
- magnetic field might lead to non-orthogonal states in the leads. One can show that one can choose a separate gauge in each lead with orthogonal states [Phys. Rev. B 40, 8169 (1989)]
- should we not insert a factor $[1-f_i]$ to account for empty states where the electrons can tunnel to?
  ↳ NO! Scattering state (one total wave function) in equilibrium with origin-reservoir → ok as long as NO inelastic scattering.

Linear response:

– „sanity check": $\mu_1 = \mu_2 \Rightarrow I = 0$

small bias $V = \frac{\mu_1 - \mu_2}{e} =: \frac{\Delta\mu}{e}$

$$\delta I = \frac{2e}{h} \int \Big[\overline{T}_{eq}(E) \cdot \underbrace{\delta(f_1 - f_2)}_{\left.\frac{\partial f}{\partial \mu}\right|_{eq} \cdot (\mu_1 - \mu_2) = -\frac{\partial f}{\partial E} \cdot \Delta\mu} + \delta \overline{T}_{eq}(E) \cdot \underbrace{[f_1 - f_2]_{eq}}_{=0} \Big] \cdot dE$$

$$f = \frac{1}{1 + e^{\frac{E-\mu}{kT}}}$$

$$\Rightarrow \delta I = -\frac{2e}{h} \int \overline{T}_{eq}(E) \cdot \frac{\partial f}{\partial E} \cdot \Delta\mu \cdot dE \quad \Rightarrow \quad G = \frac{\delta I}{\Delta V} = -\frac{2e^2}{h} \int \overline{T}_{eq}(E) \frac{\partial f}{\partial E} \cdot dE$$

– check: $T \to 0 \Rightarrow -\frac{\partial f}{\partial E} \to \delta(E - E_F) \Rightarrow G = \frac{2e^2}{h} \cdot \overline{T}_{eq}(E_F)$

↳ When is the system response linear?

a) clearly for $kT \gg \Delta\mu$ (Fermi function wider than transport window → all features washed out)

↳ very restrictive: no linear transport for $T \to 0$!

b) if $T(E) = \text{const}$ between μ_1 and μ_2

↳ $I = \frac{2e}{h} \overline{T}(E_F) \int (f_1 - f_2) dE = \frac{2e}{h} \cdot \overline{T}(E_F) \cdot \Delta\mu$ for all T!

↳ „correlation energy" $\mathcal{E}_c$: $\overline{T}(E)$ independent of E for intervall $[E, E + \mathcal{E}_c]$

⇒ linear response for $\boxed{\Delta\mu \ll kT + \mathcal{E}_c}$

implicit assumption: $\overline{T}(E)$ <u>independent</u> of bias!

(e.g. tunnel barrier does <u>not</u> change with bias, ...)

Landauer - Büttiker Formalism

↳ more than 2 terminals
↳ all probes/terminals on equal footing
(i.e. no distinction between voltage probe and current contact)

- 2-terminals : $I = \frac{2e}{h} \cdot T \cdot [\mu_1 - \mu_2]$
- N-terminals : $\underset{\substack{\uparrow \\ \text{terminal } p}}{\underline{I_p}} = \frac{2e}{h} \sum_{q=1}^{N} [T_{q\leftarrow p} \cdot \mu_p - T_{p\leftarrow q} \cdot \mu_q] \overset{V_i = \mu_i/e}{=} \underline{\underline{\sum_q [G_{qp} V_p - G_{pq} V_q]}}$

with $G_{pq} = \frac{2e^2}{h} \cdot T_{pq}$

„conductance matrix"

<u>properties of G_{pq}:</u>

- all terminals on same potential $V \Rightarrow$ all $I_p = 0 = V \cdot \sum_q [G_{qp} - G_{pq}]$
  ↳ $\sum_q G_{qp} = \sum_q G_{pq}$

  $\Rightarrow \underline{\underline{I_p = \sum_q G_{pq} \cdot [V_p - V_q]}}$

- experimentally (and exact for coherent transport, see below):

  $\underline{\underline{G_{pq}|_{-B} = G_{qp}|_{+B}}}$ (swapp contacts and invert magnetic field → same conductance)
  ↳ related : Onsager relations in thermodynamics.

- a voltage probe: $I_M = 0$ (by definition!)
  ↳ $V_M = \frac{\sum G_{Mq} V_q}{\sum_{q \neq M} G_{Mq}}$ ⇔ average over potentials of other terminals, with the weight G_{Mq}.

- $B = 0 \Rightarrow G_{pq} = G_{qp}$

More properties of G_{pq} from scattering matrix:

↳ coherent transport!

↳ if $\vec{A}$ and U (and bandstructure) are known, one can (in principle) calculate the full scattering matrix S, which relates all out-going states to all in-coming states.

- transmission probabilities: $\underbrace{T_{m\leftarrow n}}_{\text{modes}} = |S_{m\leftarrow n}|^2$

  from one lead to another: $\underbrace{\bar{T}_{p\leftarrow q}}_{\text{leads}} = \sum_{\substack{m\in q\\ \text{modes in } q}} \sum_{\substack{n\in p \\ \text{modes in } p}} T_{nm}$

- Matrix notation. $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}$ in-coming states (amplitudes)

  $\underline{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_M \end{pmatrix}$ out-going states

current conservation: $\sum_m |a_m|^2 = \sum_m |b_m|^2$ or $\underline{a}^\dagger \underline{a} = \underline{b}^\dagger \underline{b} = (\underline{\underline{S}}\underline{a})^\dagger (\underline{\underline{S}}\underline{a}) = \underline{a}^\dagger \underline{\underline{S}}^\dagger \underline{\underline{S}} \underline{a}$

$\Rightarrow \boxed{\underline{\underline{S}}^\dagger \underline{\underline{S}} = \mathbb{1}, \; S \text{ is unitary!}}$

explicitly: $\sum_{\substack{m\\ \text{modes}}} |S_{nm}|^2 = 1$ (no obvious interpretation?)

$\sum_m |S_{mn}|^2 = 1$ (every electron has to go somewhere!)

$$\hookrightarrow \sum_{\substack{q\\ \text{leads}}} \bar{T}_{qp} = \sum_{\substack{n\in p\\ \text{modes in lead } p}}^{M_p} \left(\sum_{m=1}^{M_q} T_{mn} \right) \underset{\text{from just above}}{=} \sum_{n\in p}^{M_p} 1 = M_p$$

$$\sum_q \bar{T}_{pq} = \sum_{n\in p} \left(\sum_{m=1}^{M_q} T_{nm} \right) \overset{!}{=} \sum_{n\in p} 1 = M_p$$

sum of $\underline{\underline{T}}$ over a row or a column $= M_p$, the number of modes in the corresponding lead

$\Rightarrow$ sum rule for conductance matrix: $\boxed{\sum_q G_{pq} = \sum_q G_{qp} = \frac{2e^2}{h} \cdot M_p(E_F) \quad (\text{low } T)}$

- Two-terminal device: $\underline{\underline{T}} = \begin{pmatrix} M_1 - x & x \\ x & M_2 - x \end{pmatrix} \Rightarrow T_{12} = T_{21}$ independent of B!

Reciprocity for coherent transport:

Schrödinger equation: $\left[E_s + \frac{(i\hbar\underline{\nabla} + e\underline{A})^2}{2m^*} + U(\underline{r})\right]\psi_B(\underline{r}) = E\psi_B(\underline{r})$

E_s: bottom of band

complex conjugate: $\left[E_s + \frac{(-i\hbar\underline{\nabla} + e\underline{A})^2}{2m^*} + \underset{\text{real}}{U(\underline{r})}\right]\psi_B^* = \underset{\text{real}}{E}\,\psi_B^*$

reverse $\underline{B}$ (and thus $\underline{A}$): $\left[E_s + \frac{(i\hbar\underline{\nabla} + e\underline{A})^2}{2m^*} + U(\underline{r})\right]\psi_{-B}^* = E\psi_{-B}^*$ or $\underline{\underline{H\psi_{-B}^* = E\psi_{-B}^*}}$

complex conjugate in leads: incoming $(e^{ikz}) \longrightarrow$ out-going (e^{-ikz})

$\hookrightarrow \psi_{-B}^* = \psi_{+B}$

$\Longrightarrow$ $\underline{b} = \underline{\underline{S}}|_{+B} \cdot \underline{a}$ $\quad\Rightarrow\quad$ $\underline{b}^* = \underline{\underline{S}}^*|_{+B} \cdot \underline{a}^*$

but also $\quad \underline{a}^* = \underline{\underline{S}}|_{-B} \cdot \underline{b}^*$ $\quad$ or $\quad \underline{b}^* = (\underline{\underline{S}}|_{-B})^{-1}\,\underline{a}^*$

$\hookrightarrow$ $\underline{\underline{S}}^*|_{+B} = (\underline{\underline{S}}|_{-B})^{-1} \underset{S\text{ unitary}}{=} \underline{\underline{S}}^{\dagger}|_{-B}$ $\quad$ or $\quad \boxed{\underline{\underline{S}}|_{+B} = \underline{\underline{S}}^{t}|_{-B}}$ $\quad$ (t: transpose without complex conj.)

$\hookrightarrow$ $\boxed{S_{mn}|_{+B} = S_{nm}|_{-B}}$ (Element-wise)

$\Rightarrow$ sum over all modes in lead p and q:

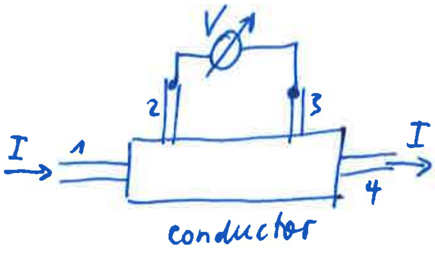
$$\sum_{m\in p}\sum_{n\in q} |S_{mn}|_{+B}^2 = \sum_{m\in p}\sum_{n\in q} |S_{nm}|_{-B}^2$$

$\hookrightarrow$ $\boxed{T_{pq}|_{+B} = T_{qp}|_{-B}}$

and thus $\underline{\underline{G_{pq}|_{+B} = G_{qp}|_{-B}}}$

Example: A <u>4-terminal device</u>:

Without any microscopic knowledge (scattering matrix) we can describe ~~a~~ the following device with 4 contacts:



<u>Boundary conditions</u>: from experiment!

$I_2 = I_3 = 0$ (voltage measurement)

$I_1 = I$, $I_4 = -I$

$$\Rightarrow I_p = \sum_q G_{pq}[V_p - V_q] = \sum_q G_{pq} V_p - \sum_q G_{pq} V_q$$

$$\Rightarrow \begin{pmatrix} I \\ 0 \\ 0 \\ -I \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13}+G_{14} & -G_{12} & -G_{13} & -G_{14} \\ -G_{21} & G_{21}+G_{23}+G_{24} & -G_{23} & -G_{24} \\ -G_{31} & -G_{32} & G_{31}+G_{32}+G_{34} & -G_{34} \\ -G_{41} & -G_{42} & -G_{43} & G_{41}+G_{42}+G_{43} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

This system of equation is not independent (current conservation, Kirchhoff's rules).

↳ e.g. $I_4 = -(I_1 + I_2 + I_3) = -I_1 = -I$ $\Rightarrow$ remove line 4

↳ set $V_4 \equiv 0$ (only voltage differences matter) $\Rightarrow$ remove ~~row~~ column 4

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13}+G_{14} & -G_{12} & -G_{13} \\ -G_{21} & G_{21}+G_{23}+G_{24} & -G_{23} \\ -G_{31} & -G_{32} & G_{31}+G_{32}+G_{34} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \quad \text{or} \quad \underline{\tilde{I}} = \underline{\underline{\tilde{G}}} \cdot \underline{\tilde{V}}$$

$\underline{\underline{\tilde{G}}}$ can be inverted! $\Rightarrow \underline{\tilde{V}} = \underline{\underline{\tilde{G}}}^{-1} \cdot \underline{\tilde{I}} \equiv \underline{\underline{\tilde{R}}} \cdot \underline{\tilde{I}}$

↑ given!

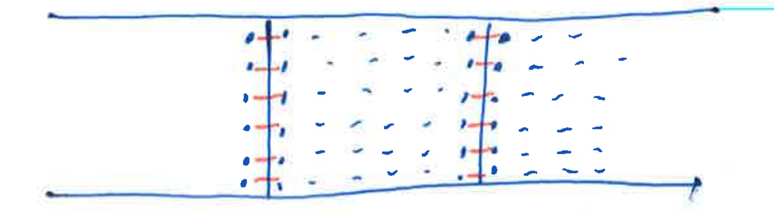
Example: $\underline{\tilde{I}} = \begin{pmatrix} I \\ 0 \\ 0 \end{pmatrix}$

The measured 4-terminal voltage (V in schematic) is given by

$$R_{4t} = \frac{V}{I} = \frac{V_2 - V_3}{I} = \tilde{R}_{21} - \tilde{R}_{31}$$

Comment: since in terminals 2 and 3 the current is 0, R_{4t} does not contain any contact resistance, in contrast to a 2-terminal experiment, where the current flows through the voltage terminals.

Schematically: calculation of the scattering matrix:



infinite lead 1 → but simple → analytical green's function

finite scattering region → but complex → numerics (finite differences)

infinite lead 2 → but simple

$\Rightarrow$ Finite-size matrix Green's function with additional „self-energy" Σ^{ret}

$$G^{ret} = \left[E \cdot \mathbb{1} - \underset{\substack{\uparrow \\ \text{finite scattering region}}}{H_c} - \Sigma^{ret}\right]^{-1}$$ → can be inverted!

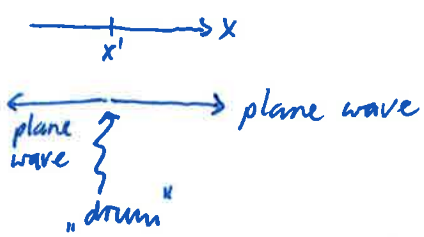
Connection to scattering matrix: Fisher-Lee relation

$$S_{qp} = -\delta_{qp} + i\hbar\sqrt{v_p v_q} \cdot \underset{\substack{\uparrow \\ \text{leads}}}{G^{ret}_{qp}}$$

(Green's function: system response to δ-excitation)

↳ $(E-H)G = \delta(x-x')$

e.g. 1D wire:



retarded: out-going („wave after bang")

advanced: in-coming („wave join for bang")

Quantum Hall effect (QHE)

2DEG in (strong) <u>magnetic field</u> perpendicular to 2DEG, „bar"-shape.

Schrödinger equation: $\left[E_S + \frac{(i\hbar\nabla + eA)^2}{2m^*} + U(y)\right]\Psi(x,y) = E\Psi(x,y)$

$\uparrow$ confinement only along y

- <u>No confinement</u> $(U \equiv 0)$:

Gauge choice: $\underline{B} = \begin{pmatrix}0\\0\\B\end{pmatrix} \Rightarrow \underline{A} = \begin{pmatrix}By\\0\\0\end{pmatrix}$, so that $\underline{\nabla} \wedge \underline{A} = \begin{pmatrix}0\\0\\B\end{pmatrix}$

$$\Rightarrow \left[E_S + \frac{(i\hbar\partial_x + eBy)^2}{2m^*} + \frac{(i\hbar\partial_y)^2}{2m^*}\right]\Psi = E\Psi$$

Ansatz: $\Psi(x,y) = \frac{1}{\sqrt{L}} \cdot e^{ikx} \cdot \chi(y)$ $\Rightarrow$ plane waves along x

$\hookrightarrow \left[E_S + \frac{p_y^2}{2m^*} + \frac{(eBy + \hbar k)^2}{2m^*}\right]\chi = E\chi$

or with $\underline{\underline{y_k = \frac{\hbar k}{eB}}}$ and $\underline{\underline{\omega_c = \frac{eB}{m^*}}}$ (cyclotron frequency)

$$: \left[E_S + \frac{p_y^2}{2m^*} + \frac{1}{2}m\omega_c^2(y+y_k)^2\right]\chi = E\chi$$

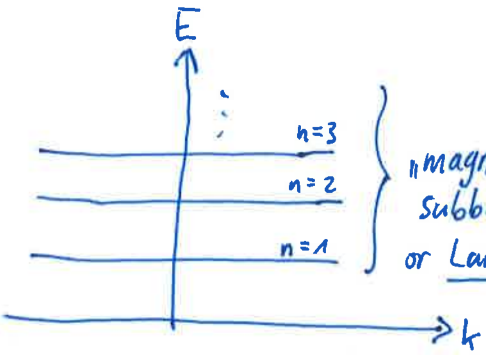
$\hookrightarrow$ harmonic oscillator!

<u>Solutions:</u> $\chi_{n,k} = e^{-\frac{|eB|}{2\hbar}\cdot(y-y_k)^2} \cdot H_n\left(\frac{eB}{\hbar}[y-y_k]\right)$

$\uparrow$ Gaussian envelope

$\boxed{\text{width: } \ell_B = \sqrt{\frac{\hbar}{eB}} \text{ „magnetic length"}}$

$\uparrow$ Hermite polynomials



„magnetic subbands" or <u>Landau levels</u> (LL)

$$\underline{\underline{E_n = E_S + (n + 1/2)\cdot\hbar\omega_c}}, \quad n \in \mathbb{N}$$

For $U \equiv 0$: $v_{n,k} = \frac{1}{\hbar}\frac{\partial E_n}{\partial k} \equiv 0$ $\leftrightarrow$ classically: circular orbits!

Degeneracy of subband n: spacing in k: $\Delta k = \frac{2\pi}{L}$

$\hookrightarrow$ spacing in y: $\Delta y = \frac{\hbar\Delta k}{eB} = \frac{2\pi\hbar}{eBL} = \frac{h}{eBL}$

$\Rightarrow$ number of states in sample of width W: $\underline{\underline{N}} = \underset{\uparrow\,\text{spin}}{2}\frac{W}{\Delta y} = \frac{2e\cdot BLW}{h} = \underline{\underline{2\cdot\frac{\phi}{\phi_0}}}$; $\phi_0 = \frac{h}{e}$

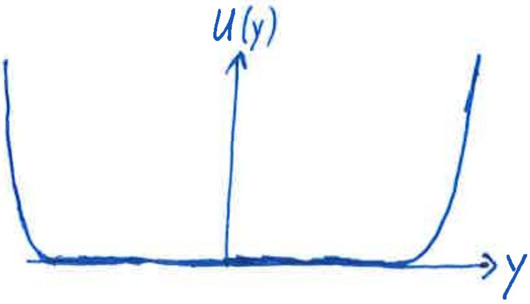
$\hookrightarrow \Delta E = E_n - E_{n-1} = \hbar\omega_c \propto B$ <u>and</u> $N_n = \frac{2\phi}{\phi_0} \propto B$

(Bulk-) filling factor: how many Landau levels (LL) are occupied?

$$\underline{\underline{\nu}} = \frac{N_{tot}}{N_{LL}} \underset{\substack{\uparrow \\ \text{count spin} \\ \text{separately}}}{=} \frac{n\cdot WL}{\frac{eBWL}{h}} = \underline{\underline{\frac{h\cdot n}{eB}}}$$

$n \leftarrow$ electron density

Now we add a confinement potential along the y-direction:



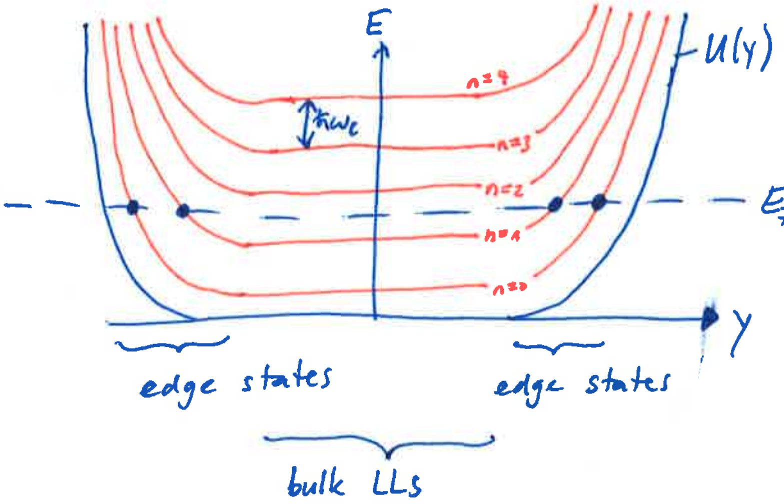
First-order perturbation:

$$E(n,k) = \underbrace{E_S + (n+1/2)\hbar\omega_c}_{\text{unperturbed}} + \langle n,k|U(y)|n,k\rangle$$

Note that $\Psi_{n,k}(x,y)$ is centered at $y_k = \frac{\hbar k}{eB}$ and we assume that $U(y)$ is smooth on the length $\ell_B = \sqrt{\frac{\hbar}{eB}}$ (magnetic length), i.e. the extent of the wave function.

$\Rightarrow$ $U(y) \approx$ const. for each state

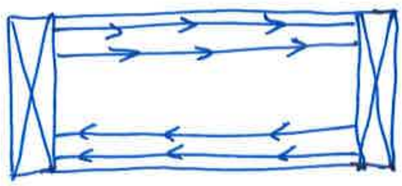
$\Rightarrow$ $E(n,k) = E_S + (n+1/2)\hbar\omega_c + U(y_k)$



E_F between LLs: edge states!

↳ equipotential lines: $E(n,k) = E_F$

velocities: $v = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \frac{1}{\hbar}\frac{\partial U}{\partial k} = \frac{1}{\hbar}\frac{\partial U}{\partial y_k}\cdot\frac{\partial y_k}{\partial k} = \frac{1}{eB}\cdot\underbrace{\frac{\partial U(y)}{\partial y}}_{\substack{\text{opposite sign on} \\ \text{opposite edges!}}}$



number of edge states $= \nu$ (bulk filling factor)

$I_{2t} = \frac{2e}{h}\cdot M\cdot T\,\Delta\mu$; T: since the $+k$ and $-k$ states are spatially separated by sometimes macroscopic distances (width of Hallbar), „scattering" requires an electron to tunnel over a large distance $\Rightarrow T=1$

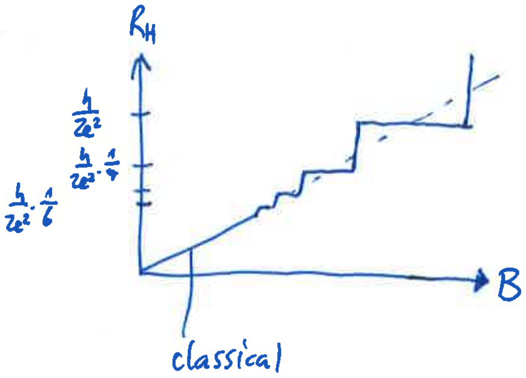
↳ No momentum scattering $\Rightarrow$ one edge carries the electrochemical potential of the left, the other edge carries μ of the right side

$\Rightarrow$ the voltage drops only in the contacts, like in a ballistic conductor.

$\hookrightarrow$ inside the Hallbar $V_L \equiv 0 \Rightarrow R_L = \frac{V_L}{I} = 0$ (no longitudinal resistance)

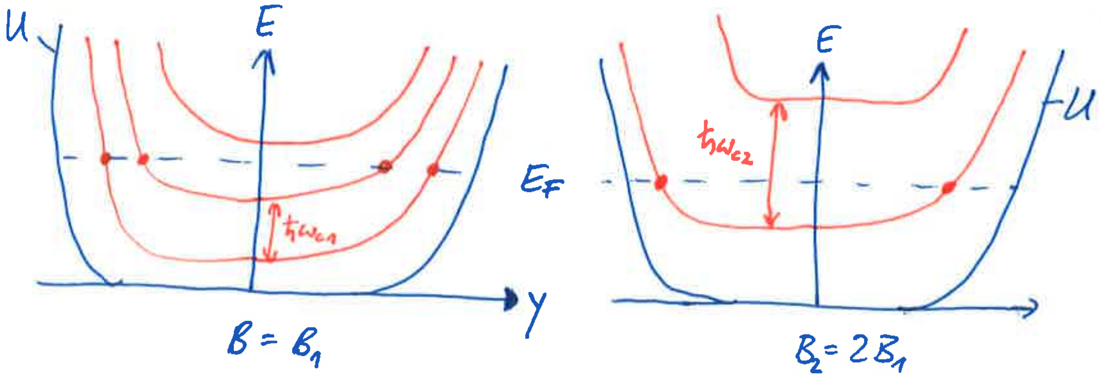
$\hookrightarrow R_H = \frac{V_H}{I} = \frac{1/e \cdot \cancel{(\mu_L - \mu_R)}}{2e^2/h \cdot M \cdot \cancel{\Delta\mu}} = \frac{h}{2e^2} \cdot \frac{1}{M}$ $\Rightarrow$ steps in R_H!

M: number of edge channels $\equiv \nu$



The plateaus in R_H depend only on fundamental constants $\Rightarrow$ metrology

$\nu \equiv M$ depends on B (and n):



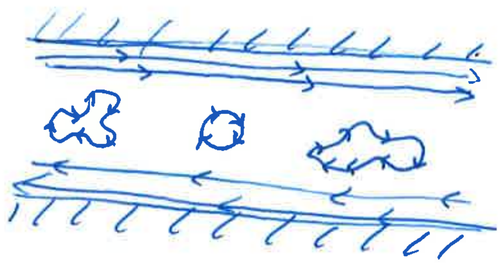
(E_F given by contacts)

But degeneracy also x2 $\rightarrow$ constant electron density

Some fineprint: Why can E_F lie between two LL (density of states = 0!?)

$\hookrightarrow \Delta n = N \cdot \Delta\mu$: small change in n can lead to large change in μ if N is small $\Rightarrow \mu$ (E_F) pinned to large DoS

$\Rightarrow$ localized states at potential fluctuations:



Compressible and incompressible strips:

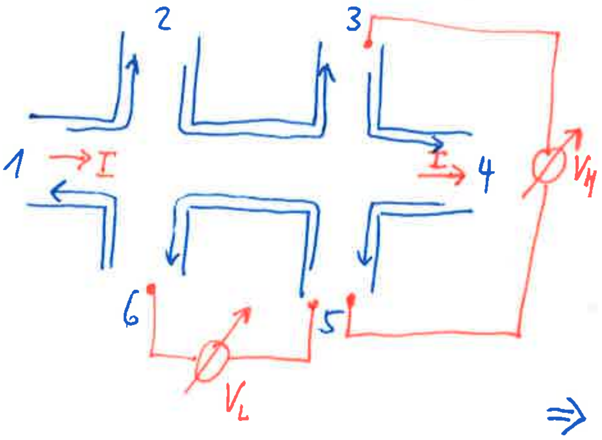
At the positions where a LL pierces E_F the electron density changes abruptly by $n_L = \frac{1}{2\pi \ell_m^2}$ (the density of a LL)

$\Rightarrow$ very large density gradient $\Rightarrow$ unphysical ($\mu = \text{const} = E_F = \mu_{ch} + \mu_{el}$)

$\hookrightarrow$ very large electric field $\Rightarrow$ screened by mobile charges

$\Rightarrow$ „edge reconstruction": see slides

Conductance matrix of a Hallbar:



transmissions: $T_{21} = T_{32} = T_{43} = T_{54} = T_{65} = T_{16} = \lfloor \nu \rfloor = M$ (integer part of filling factor)

$T_{other} = 0$

sum of all rows and colums $= 0$ $\Rightarrow$ diagonal elements

$$\Rightarrow \underline{\underline{G}} = \frac{2e^2}{h} \begin{pmatrix} M & 0 & 0 & 0 & 0 & -M \\ -M & M & 0 & 0 & 0 & 0 \\ 0 & -M & M & 0 & 0 & 0 \\ 0 & 0 & -M & M & 0 & 0 \\ 0 & 0 & 0 & -M & M & 0 \\ 0 & 0 & 0 & 0 & -M & M \end{pmatrix}$$

We can choose $I_4 = -\sum_{\text{all others}} I_i$ and $V_4 \equiv 0$ and invert the resulting $\underline{\underline{\tilde{G}}}$.

Here it is simpler „by hand":

Boundary conditions: $I_1 = -I_4$ (current terminals) and $I_2 = I_3 = \cancel{I_4} = I_5 = I_6 = 0$ (voltage terminals)

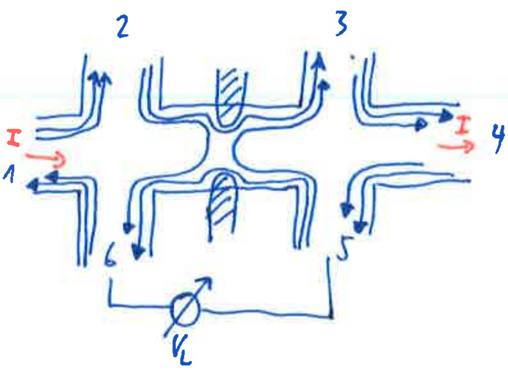
We see directly:
$$\left.\begin{aligned} V_2 &= V_1 \\ V_3 &= V_1 \\ V_5 &= 0 \ (V_4) \\ V_6 &= 0 \ (V_4) \end{aligned}\right\} \text{edge states carry } \mu_1 \text{ (or } \mu_4)$$

$$\Rightarrow I = I_1 = -I_4 = \frac{2e}{h} \cdot M(\mu_1 - \mu_6) = \frac{2e^2}{h} \cdot M \cdot V_1$$

$$\Rightarrow R_L = \frac{V_2 - V_3}{I} = 0 \quad \text{(zero resistance)}$$

$$R_H = \frac{V_2 - V_6}{I} = \frac{V_2}{\frac{2e^2}{h} \cdot M V_1} = \frac{h}{2e^2} \cdot \frac{1}{M} \Rightarrow \text{plateaus}$$

Backscattering in a Hallbar: we now investigate the effect on the Hall and longitudinal voltage when a constriction („split-gate" or a scatterer) is introduced in the Hallbar.



We assume that of the M channels N are reflected at the constriction.

$$\Rightarrow \quad T_{21} = T_{43} = T_{54} = T_{16} = M$$
$$T_{23} = M-N$$
$$T_{35} = N$$
$$T_{65} = M-N$$
$$T_{62} = N$$
$$T_{rest} = 0$$

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} = \frac{2e^2}{h} \begin{pmatrix} -M & 0 & 0 & 0 & 0 & M \\ M & -M & 0 & 0 & 0 & 0 \\ 0 & N & -M & 0 & M-N & 0 \\ 0 & 0 & M & -M & 0 & 0 \\ 0 & 0 & 0 & M & -M & 0 \\ 0 & M-N & 0 & 0 & N & -M \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} \quad \begin{matrix} a) \\ b) \\ \\ \\ c) \\ d) \end{matrix} \qquad \text{or} \qquad \underline{I} = \underline{\underline{G}} \cdot \underline{V}$$

Again, we omit parts with terminal 4 (for example), since $I_4 = -\sum_{i \in rest} I_i$ and $V_4 \equiv 0$. The resulting matrix $\tilde{\underline{\underline{G}}}$ can be inverted, which gives $\underline{V}$ for given $\underline{I}$, $\tilde{\underline{V}} = \tilde{\underline{\underline{G}}} \cdot \tilde{\underline{I}}$. For example for the boundary condition

$$I_1 = I$$
$$I_4 = -I$$
$$I_{rest} = 0 \ \ldots$$

Again, we calculate directly:

From a) $\quad I = M G_0 V_1 - G_0 M V_6 = G_0 M (V_1 - V_6)$

b) $\quad 0 = G_0 M (V_2 - V_1) \quad \Rightarrow V_1 = V_2$ (intuitive!)

c) $\quad 0 = G_0 M V_5 \quad \Rightarrow V_5 = V_4 \equiv 0$ (intuitive!)

d) $\quad 0 = -\cancel{G_0}[V_2(M-N) + \cancel{V_5}N - MV_6] \quad \Rightarrow V_6 = \frac{M-N}{M} V_2 = \frac{M-N}{M} V_1$

$\hookrightarrow$ a) $\quad I = G_0 M V_1 \left[1 - \frac{M-N}{M}\right] = G_0 M V_1 \cdot N/M = G_0 N V_1$

$$\Rightarrow \underline{\underline{R_H}} = \frac{V_2 - V_6}{I} = \frac{\cancel{V_1}\left[1 - \frac{M-N}{M}\right]}{G_0 N \cancel{V_1}} = \underline{\underline{\frac{h}{2e^2} \cdot \frac{1}{M}}}$$

$\Rightarrow$ Hall resistance plateau <u>unaffected</u> of scatterer inside the Hallbar!

$$\underline{\underline{R_L}} = \frac{V_6 - V_5}{I} = \frac{\cancel{V_1}\left[\frac{M-N}{M} - 0\right]}{G_0 N \cancel{V_1}} = \frac{h}{2e^2} \frac{\frac{M-N}{M}}{N} = \underline{\underline{\frac{h}{2e^2}\left(\frac{1}{N} - \frac{1}{M}\right)}} \neq 0 \,!$$