Departement Physik

Quantum Interference Effects: Aharonov-Bohm effect, weak localization and conductance fluctuations

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Quantum Transport lecture, 31.3.2015

Aharonov – Bohm effect

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 115, NO. 3 AUGUST 1, 1959

Significance of Electromagnetic Potentials in the Quantum Theory

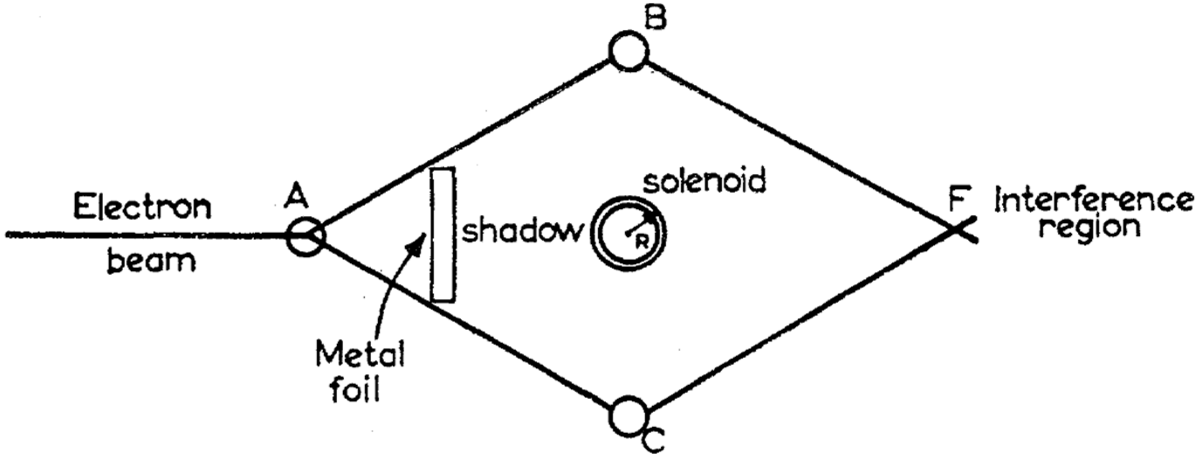
Y. AHARONOV AND D. BOHM

H. H. Wills Physics Laboratory, University of Bristol, Bristol, England

(Received May 28, 1959; revised manuscript received June 16, 1959)

In this paper, we discuss some interesting properties of the electromagnetic potentials in the quantum domain. We shall show that, contrary to the conclusions of classical mechanics, there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish. We shall then discuss possible experiments to test these conclusions; and, finally, we shall suggest further possible developments in the interpretation of the potentials.

Aharonov – Bohm effect



B = 0 everywhere, except inside solenoid of radius R

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

A : vector potential

$$\boldsymbol{A} = \frac{\Phi}{2\pi r}\,\hat{\phi}$$

$\hat{\phi}$: polar unit vector (counterclockwise)
r > R : distance from solenoid center
(exercise: show this is correct)

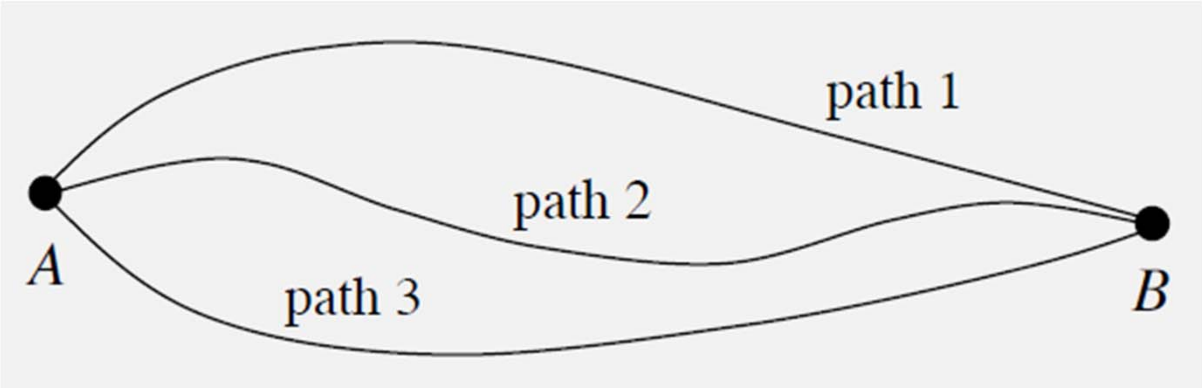
$$\Phi = \int_{S_C} d\vec{S} \cdot \vec{B} = \pi R^2 B$$

Φ : **magnetic flux** (surface S_C)

$$\Phi = \iint_{S_C} d\boldsymbol{S} \cdot \boldsymbol{B} = \iint_{S_C} d\boldsymbol{S} \cdot \nabla \times \boldsymbol{A} = \oint_C d\boldsymbol{l} \cdot \boldsymbol{A}$$

Stokes theorem

topology



$$\vec{B} = 0 = \vec{\nabla} \wedge \vec{A}$$

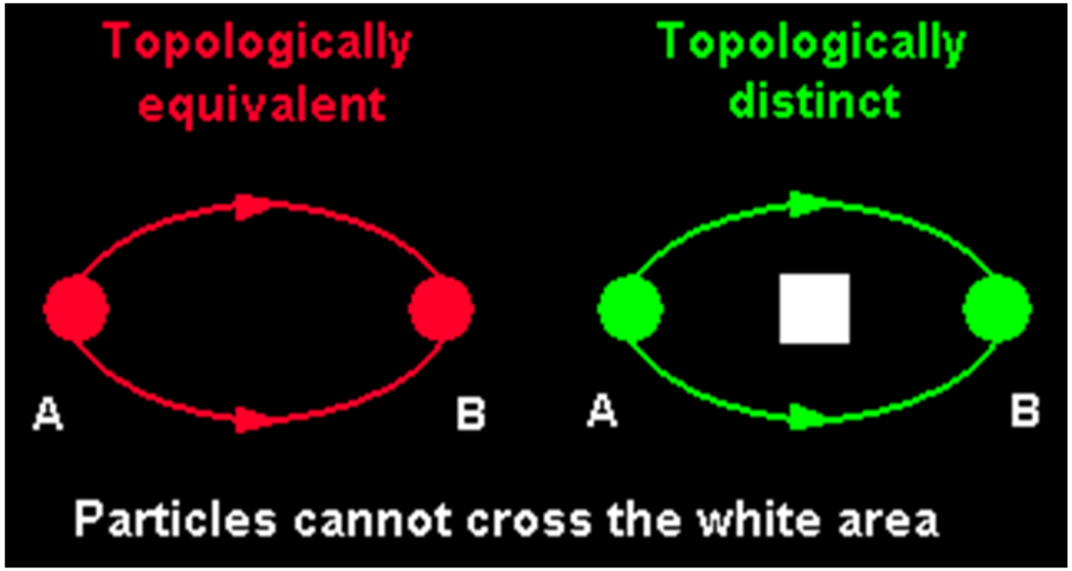
$$\int_A^B d\vec{r_1} \cdot \vec{A} + \int_B^A d\vec{r_2} \cdot \vec{A} = \oint_C d\vec{r} \cdot \vec{A} = \int_{S_C} d\vec{S} \cdot \vec{\nabla} \wedge \vec{A} = 0$$

integral depends only on points A and B, not the path chosen

simply connected region: path surrounding it can be continuously deformed to a point without changing value of integral.

if flux is inside region, the integral depends on whether or not the flux region is enclosed (not simply connected).

topology (2)



Topologically equivalent
Topologically distinct
A
B
A
B
Particles cannot cross the white area

vector potential: phase factor

$$\left(\frac{1}{2m}\left(-i\hbar\nabla - (q/c)\mathbf{A}\right)^2 \right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

time dep. Schrödinger eq.
A ≠ 0, but B = 0
scalar potential $\phi = 0$

$$\psi = \psi_0 e^{ig(\boldsymbol{r})}$$

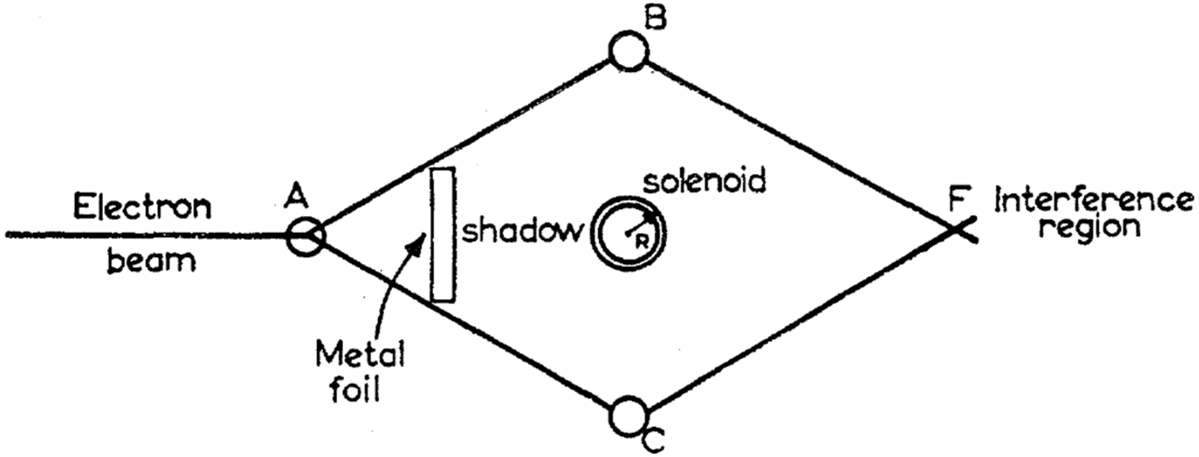
ψ_0 = solution with A = 0
g(**r**) : phase factor

$$g(\boldsymbol{r}) = (q/\hbar c)\int_{\boldsymbol{r}_0}^{\boldsymbol{r}} d\boldsymbol{r}'\cdot\mathbf{A}(\boldsymbol{r}')$$

independent of path
where B = 0
r_0 : arbitrary origin

(exercise: demonstrate that this is correct (plugging into SE))

Aharonov – Bohm effect



B = 0 everywhere, except inside solenoid of radius R

$$\psi = \psi_1{}^0 e^{-iS_1/\hbar} + \psi_2{}^0 e^{-iS_2/\hbar},$$

interference $|\psi|^2 = |\psi_1 + \psi_2|^2$

oscillating term $\propto \cos(\Delta S/\hbar) = \cos((S_2 - S_1)/\hbar) = \cos(\beta)$

$$\beta = (q/\hbar c)\left\{\int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} d\boldsymbol{r}\cdot\boldsymbol{A}_{\text{lower}} - \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} d\boldsymbol{r}\cdot\boldsymbol{A}_{\text{upper}}\right\}$$

phase difference β = lower - upper

$$\beta = \frac{e}{\hbar}\oint d\vec{r}\cdot\vec{A} = 2\pi\frac{\Phi}{\Phi_0} = 2\pi\frac{AB}{\Phi_0}$$

quantum interference
$\phi_0 = h/e = 4.12\ \mathrm{mT}\ \mu\mathrm{m}^2$

Aharonov – Bohm effect: significance (1)

1. AB effect demonstrates that the potentials A and ϕ are **physical quantities**

Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1)$$

$$\nabla \cdot \vec{B} = 0 \quad (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (4)$$

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B}) \quad (5)$$

introduce potentials A and ϕ such that

$$\vec{B} = \nabla \times \vec{A}.$$

$$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t}$$

- automatically fulfills eq. 2 and 3
- easier to work with
- no physical significance

potentials: classical gauge invariance

potentials not uniquely defined

$$\vec{A}' = \vec{A} + \nabla\chi$$

$$\phi' = \phi - \frac{\partial\chi}{\partial t},$$

gauge transformation
χ : gauge field

give the same E and B fields (show this)
Maxwell's equation are gauge invariant.

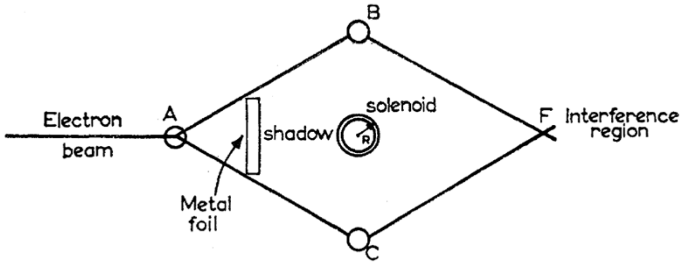
thus, in Maxwells (classical) theory, the potentials are a purely mathematical construct without any physical significance.

Aharonov – Bohm effect : significance (2)

1. AB effect demonstrates that the potentials A and ϕ are **physical quantities**

in ***quantum interference***, the vector potential appears, even when B = 0 everywhere along trajectories

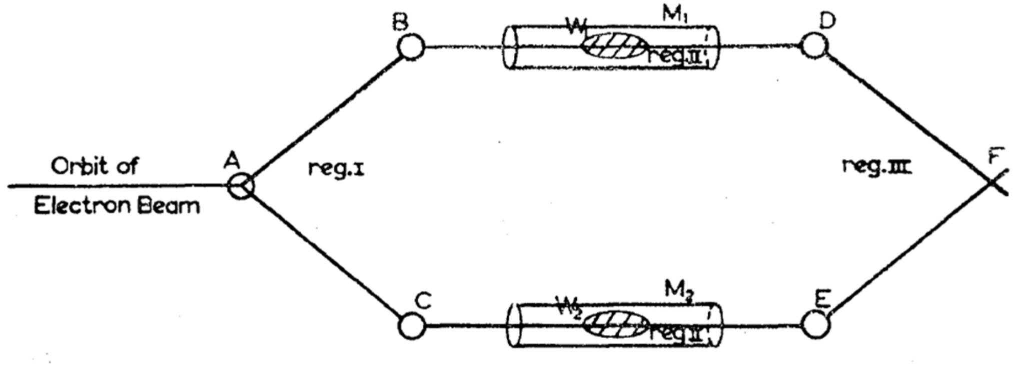
$$\beta = \frac{e}{\hbar}\oint d\vec{r}\cdot\vec{A} = 2\pi\frac{\Phi}{\Phi_0} = 2\pi\frac{AB}{\Phi_0}$$



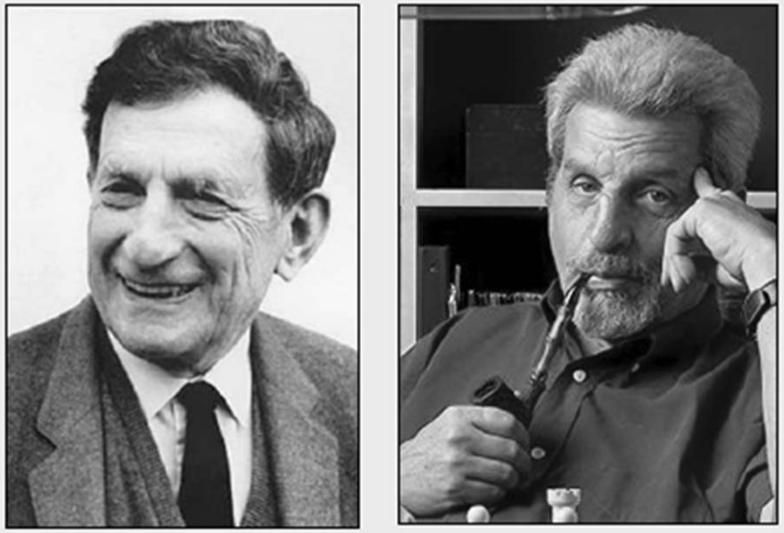
2. AB quantum inferference effect oscillating with magnetic flux

also true when trajectories are in field B ≠ 0
(a common source of quantum interference effects)

3. similar arguments can also be made for the scalar potential ϕ (using time dependent potentials, see e.g. original AB paper)



Aharonov – Bohm effect : some historical remarks



David Bohm Yakir Aharonov

Aharonov: PhD student of Bohm (Bristol)

- paper published (Physical Review) in 1959.
- shortly after, they learned that Ehrenberg and Siday published equivalent results in 1949, 10 years before
  W. Ehrenberg R. E. Siday, Proc. Phys. Soc. B62, 8 (1949)
- consequently, Bohm referred to it as the ESAB effect
- this did not stick, and now carries the name AB effect
- paper has over 3’000 citations !!

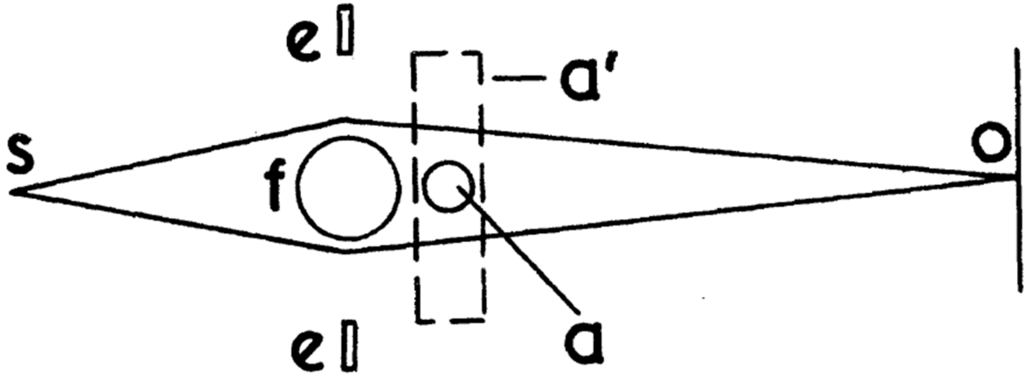
first experiment:
1960!!

AB effect : Chambers experiment: electron beam

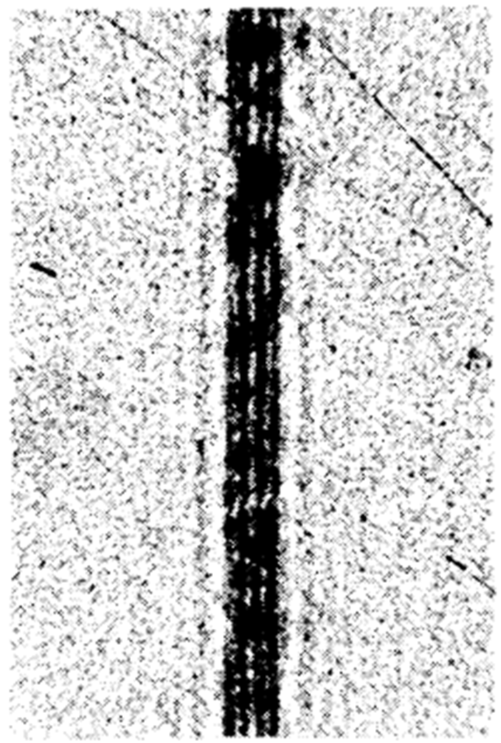
SHIFT OF AN ELECTRON INTERFERENCE PATTERN BY ENCLOSED MAGNETIC FLUX

R. G. Chambers
H. H. Wills Physics Laboratory, University of Bristol, Bristol, England
(Received May 27, 1960)

Phys. Rev. Lett. 3, 5



cm^2. It has since been pointed out[2] that the same conclusion had previously been reached by Ehrenberg and Siday,[3] using semiclassical arguments, but these authors perhaps did not sufficiently stress the remarkable nature of the result, and their work appears to have attracted little attention.



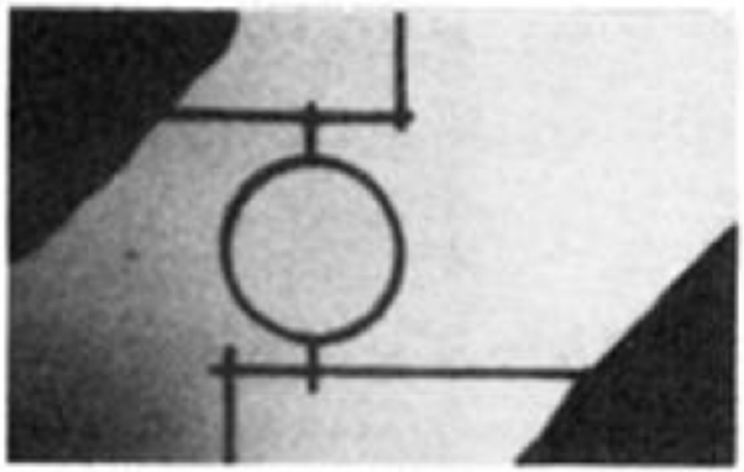
AB effect : solid state experiment

VOLUME 54, NUMBER 25 PHYSICAL REVIEW LETTERS 24 JUNE 1985

Observation of h/e Aharonov-Bohm Oscillations in Normal-Metal Rings

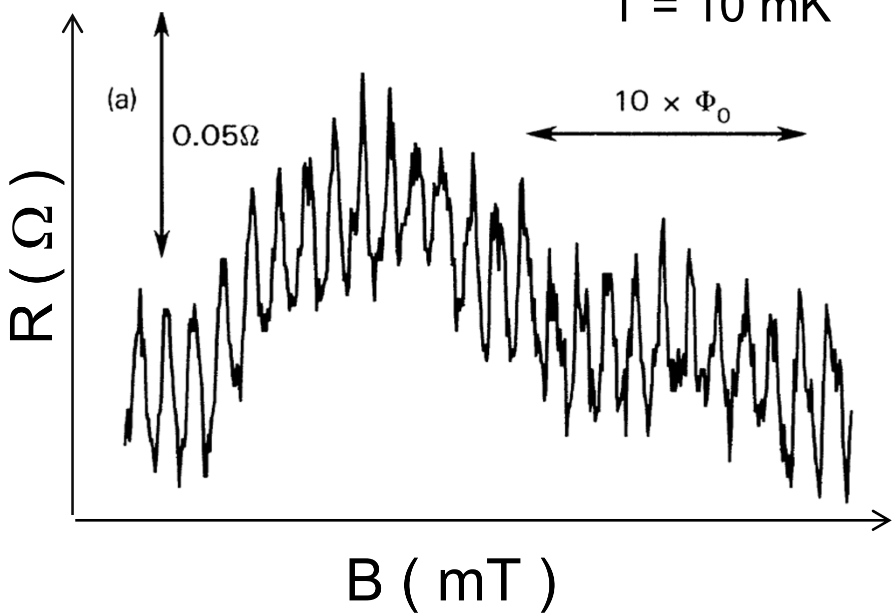
R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 27 March 1985)

Magnetoresistance oscillations periodic with respect to the flux h/e have been observed in submicron-diameter Au rings, along with weaker $h/2e$ oscillations. The h/e oscillations persist to very large magnetic fields. The background structure in the magnetoresistance was *not* symmetric about zero field. The temperature dependence of both the amplitude of the oscillations and the background are consistent with the recent theory by Stone.

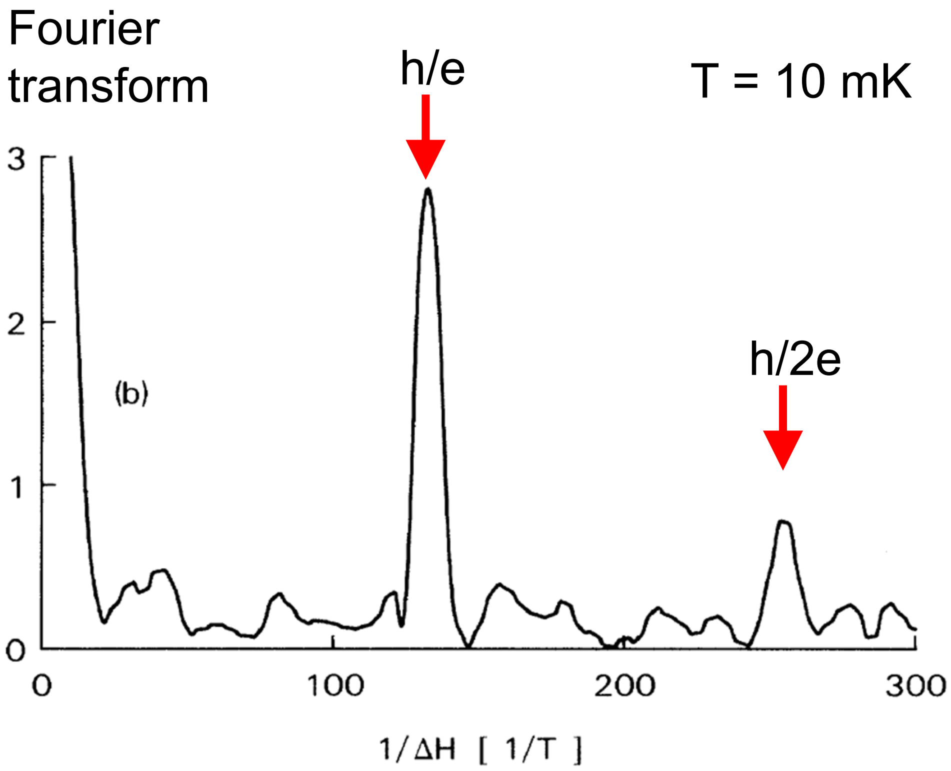


sample: Au ring,
ID 784 nm, width wires 41 nm

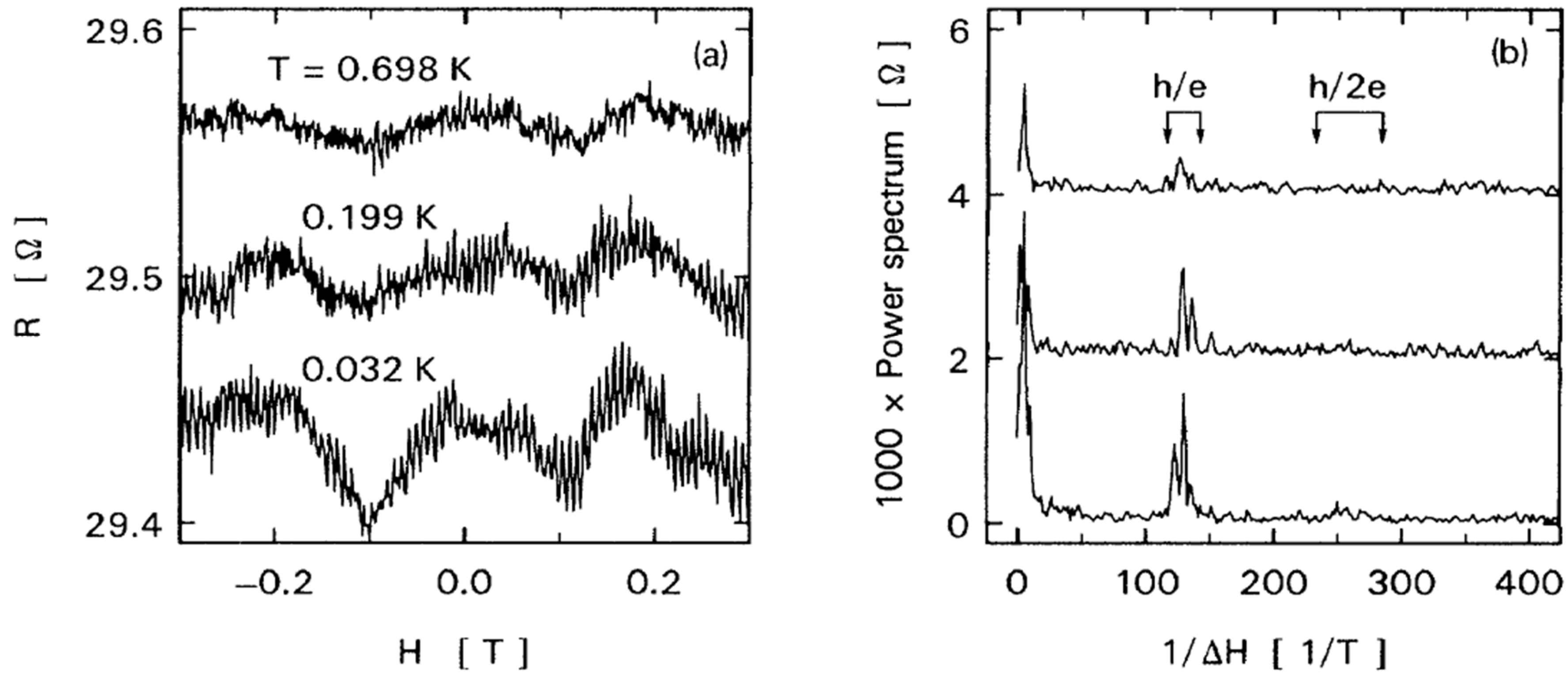
T = 10 mK



AB effect : solid state experiment (2)



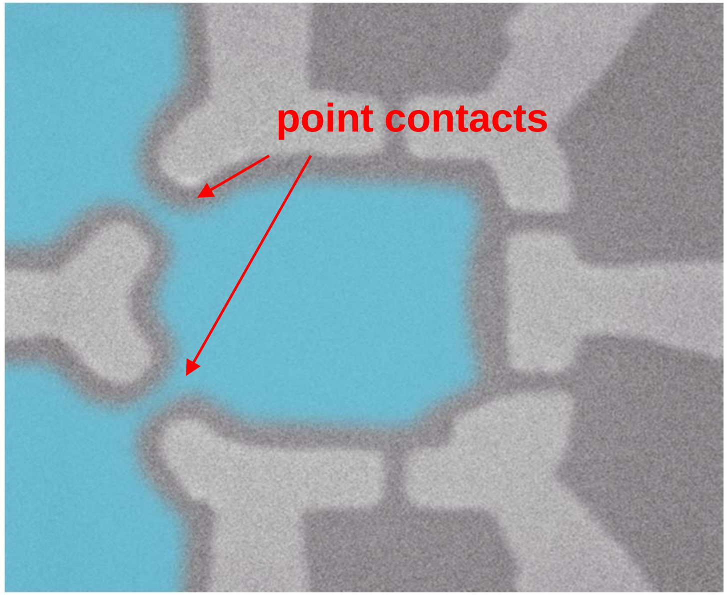
AB effect : solid state experiment (3): temperature

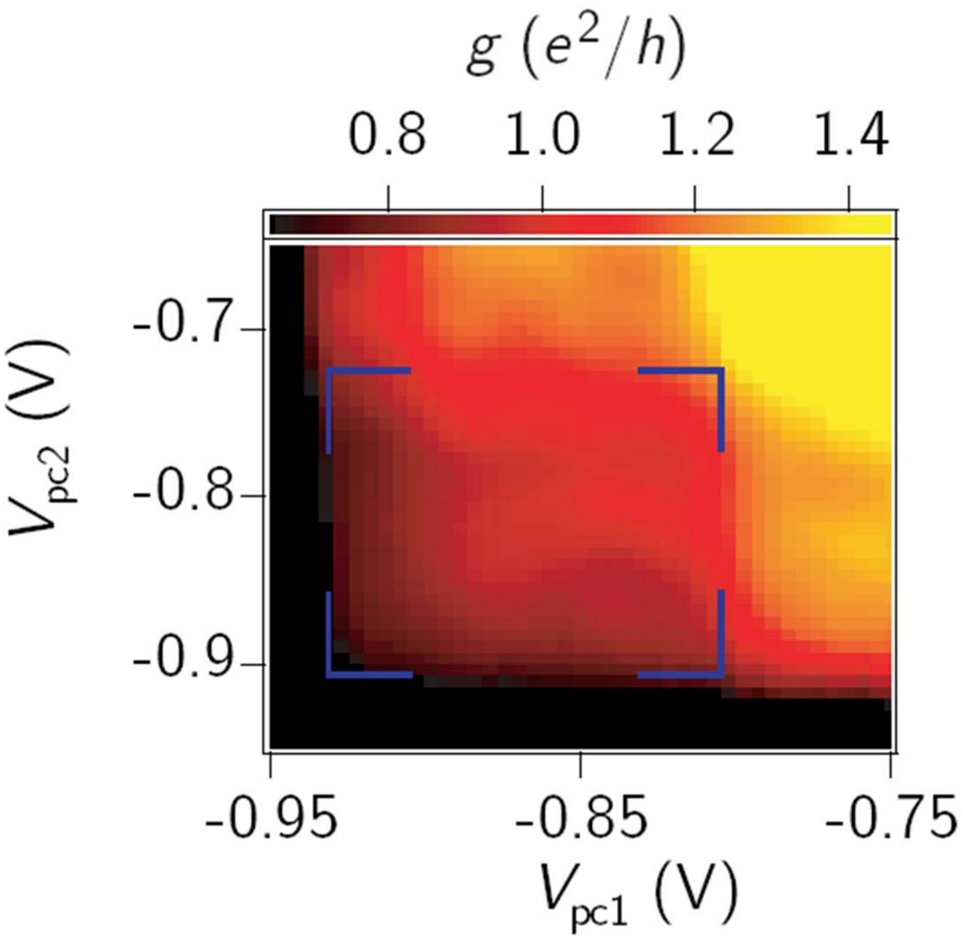


elevated temperatures destroy quantum interference: decoherence

Quantum interference in open GaAs quantum dots

Open Dot



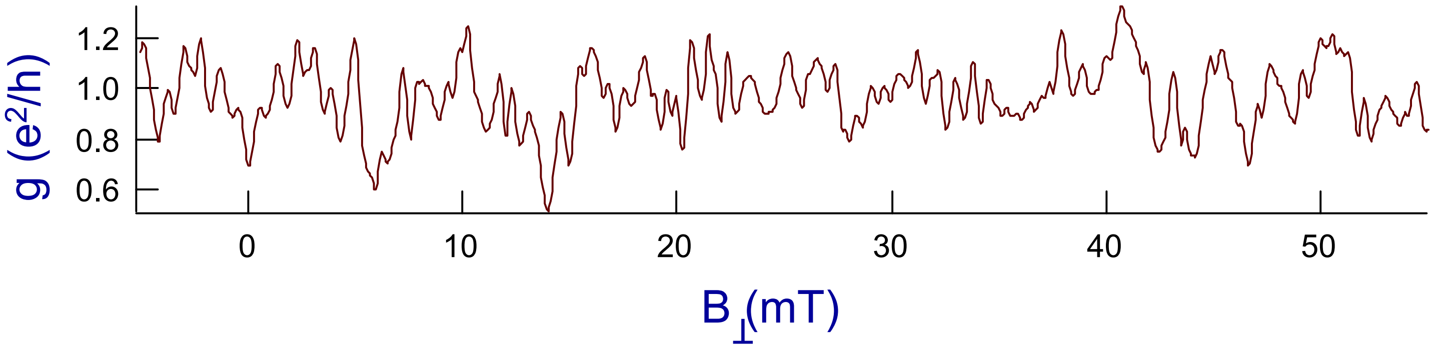


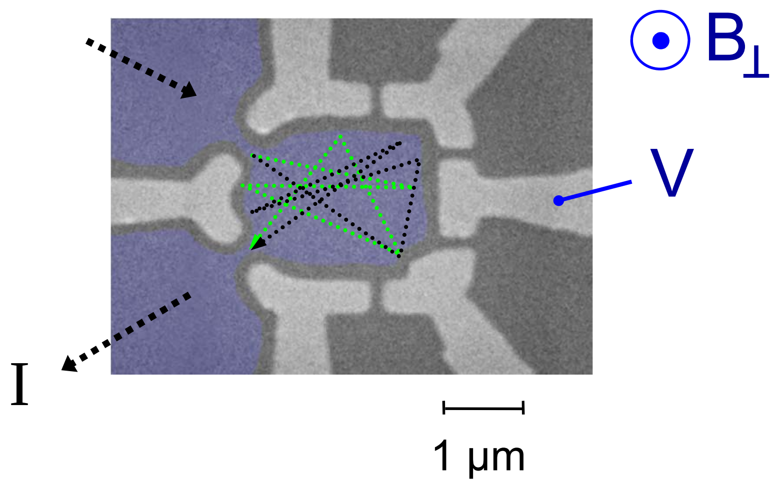
- V_{gate} set to allow ≥ $2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit conductance fluctuations and weak localization

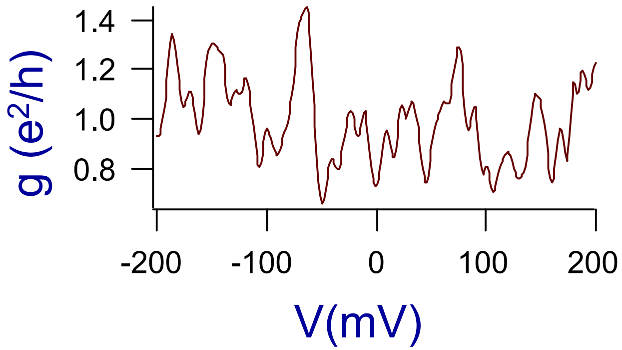
many open dot slides: A. Huibers and J. Folk

Open Dot Regime: Conductance Fluctuations

$N_L = N_R = 1$



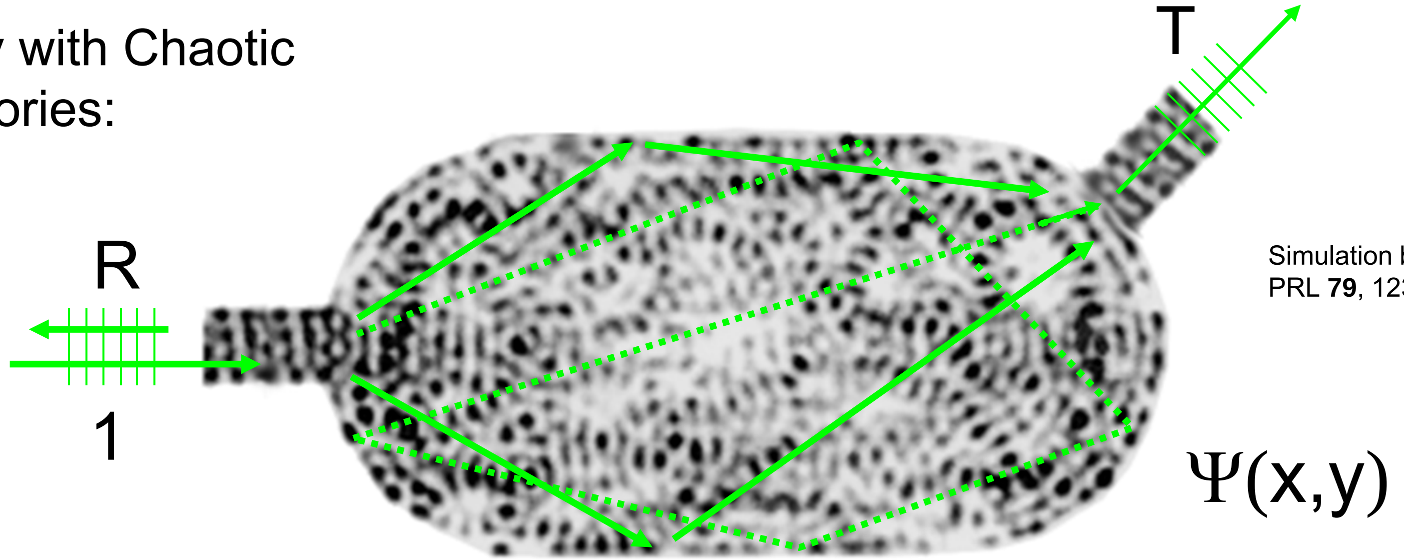




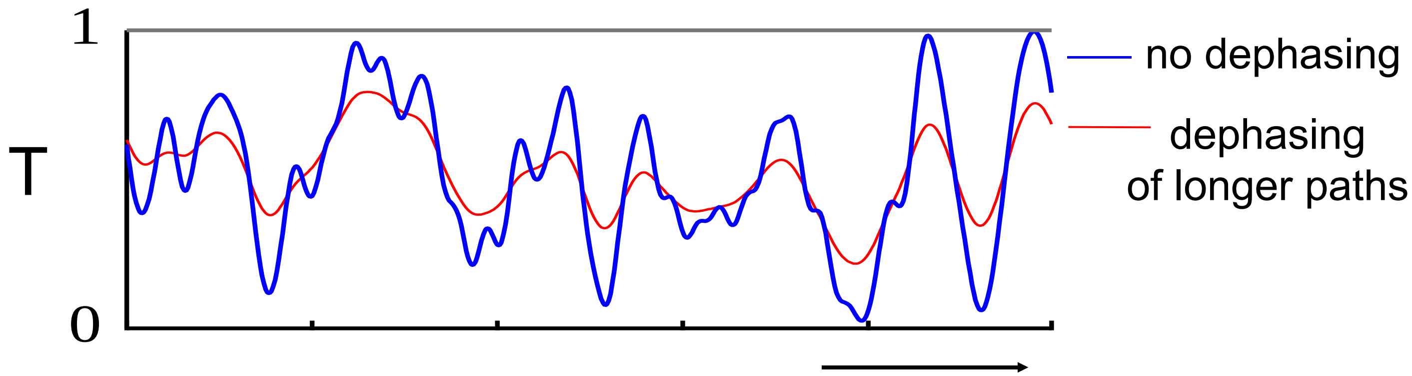
Repeatable random intereference fluctuations as function of dot parameters

Two-Dimensional Quantum Dot

2D Cavity with Chaotic trajectories:



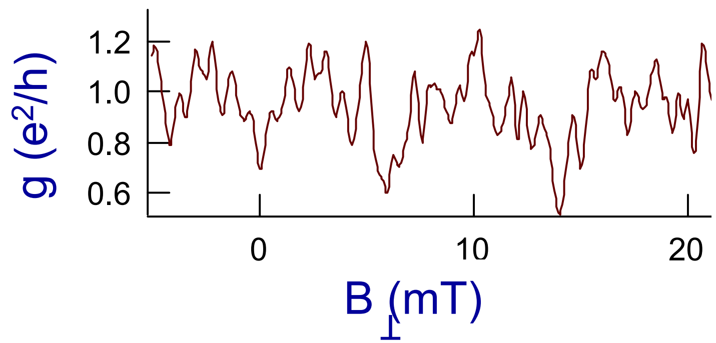
Simulation by R. Akis, PRL **79**, 123 (1997)

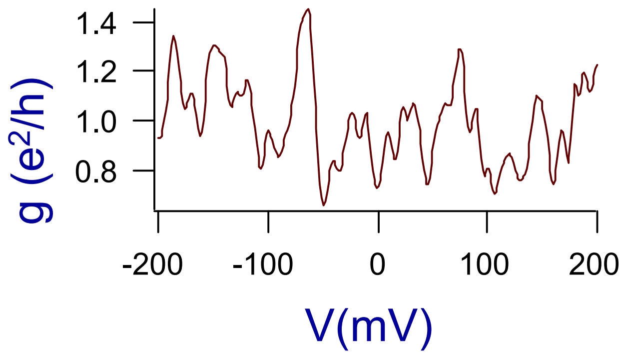


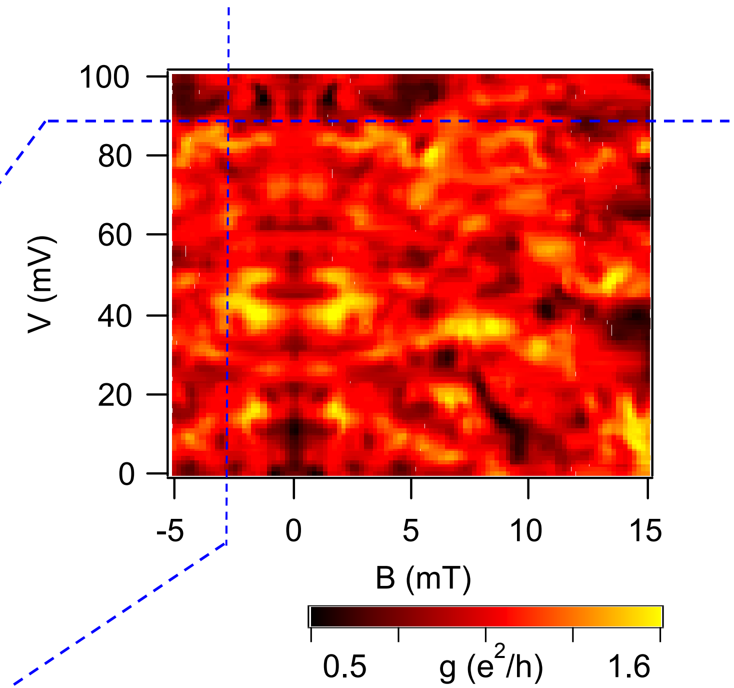
Goal: use quantum dot as a probe of quantum phase coherence

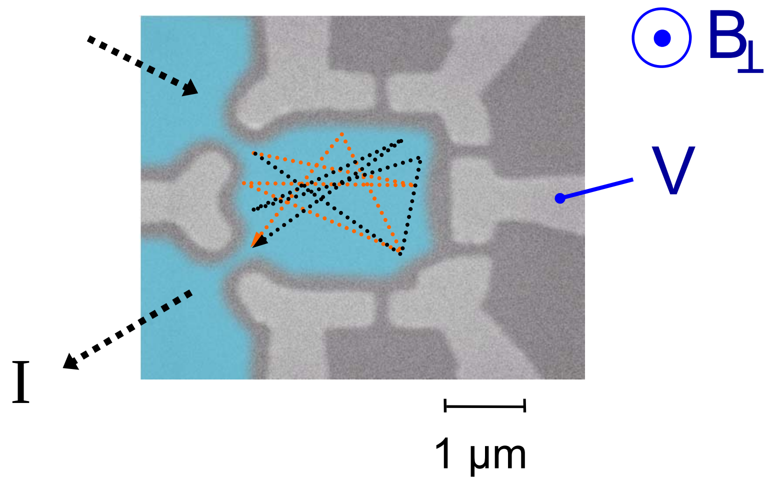
Quantum Interference in Open Dots

Interference between all possible trajectories gives rise to <u>repeatable</u> random intereference fluctuations as function of dot parameters

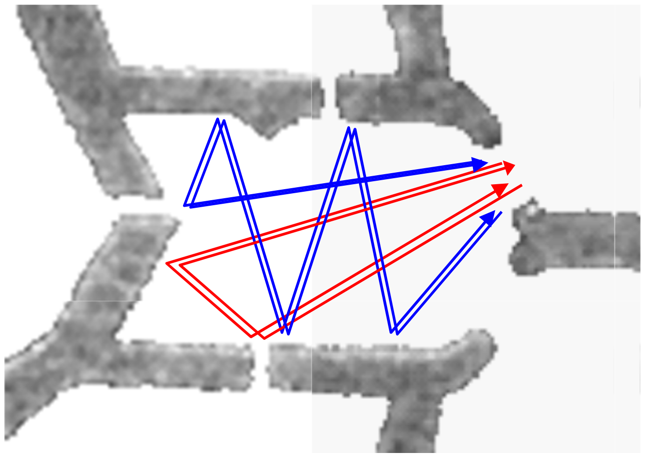








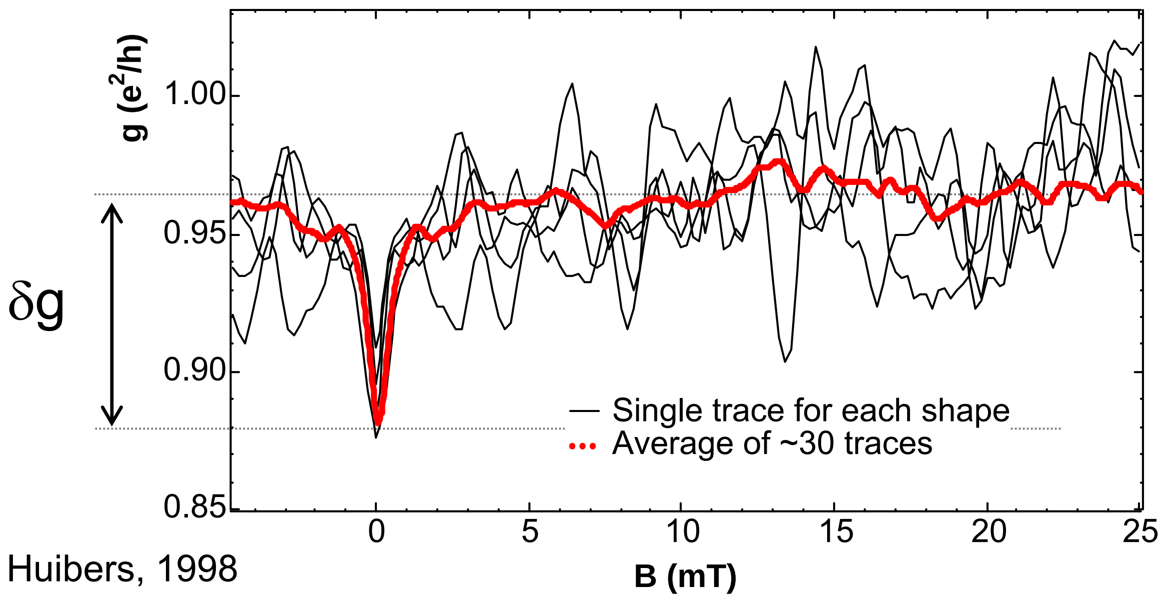
Weak Localization

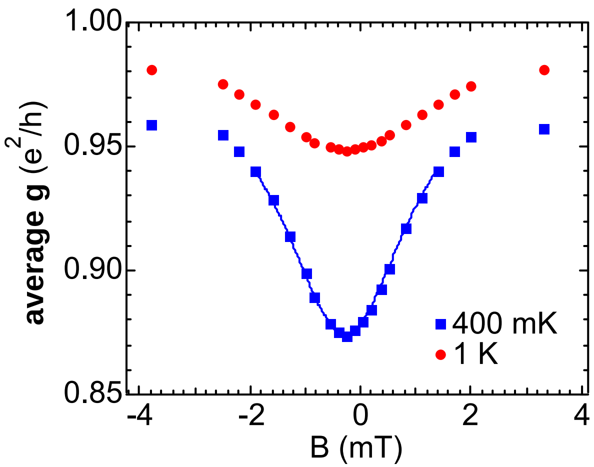


At B=0, phase-coherent backscattering results in "weak localization"

↓

Conductance dip at B=0

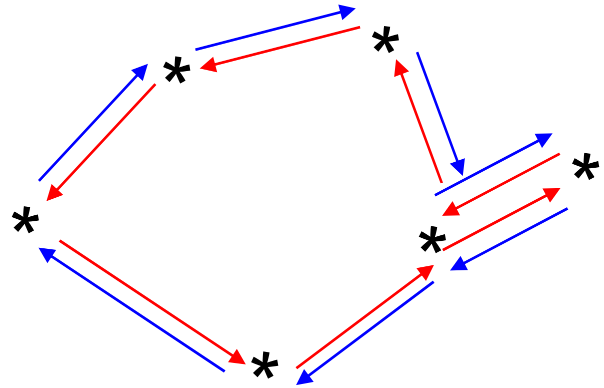




Huibers, 1998

Quantum Correction: Weak Localization

constructive interference of coherently backscattered, time reversed trajectories decreases conductivity



$$\frac{\delta\sigma_{loc}}{\sigma} \propto -\frac{1}{k_F\lambda}\ln\left(1+\frac{\tau_\varphi}{\tau}\right) \qquad \text{2D } (L_\varphi \ll W)$$

$$\frac{\delta\sigma_{loc}}{\sigma} \propto -\frac{L_\varphi}{W}\frac{1}{k_F\lambda}\left(1-\left(1+\frac{\tau_\varphi}{\tau}\right)^{-1/2}\right) \qquad \text{1D } (L_\varphi \gg W)$$

magnetic field: AB-flux, cut off trajectories of area $A>\phi_0/B$
magnetoconductance

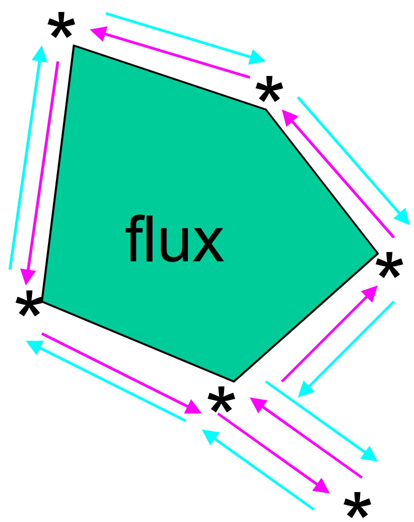
(assuming spinless electrons)

Weak Localization in Magnetic Fields

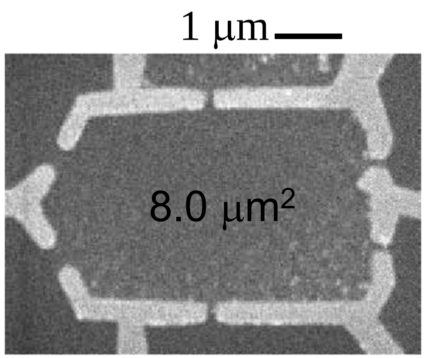
in a given magnetic field B, trajectories enclosing flux acquire additional Aharonov-Bohm phase:

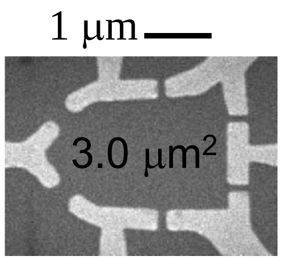
$$\phi = \frac{2e}{\eta}\int (\nabla \times A)\cdot d\vec{S} = \frac{2eBS}{\eta}$$

when summing over all trajectories, this ϕ will effectively eliminate trajectories of area $A \gg \phi_0/B$. $(\phi_0 = h/e)$



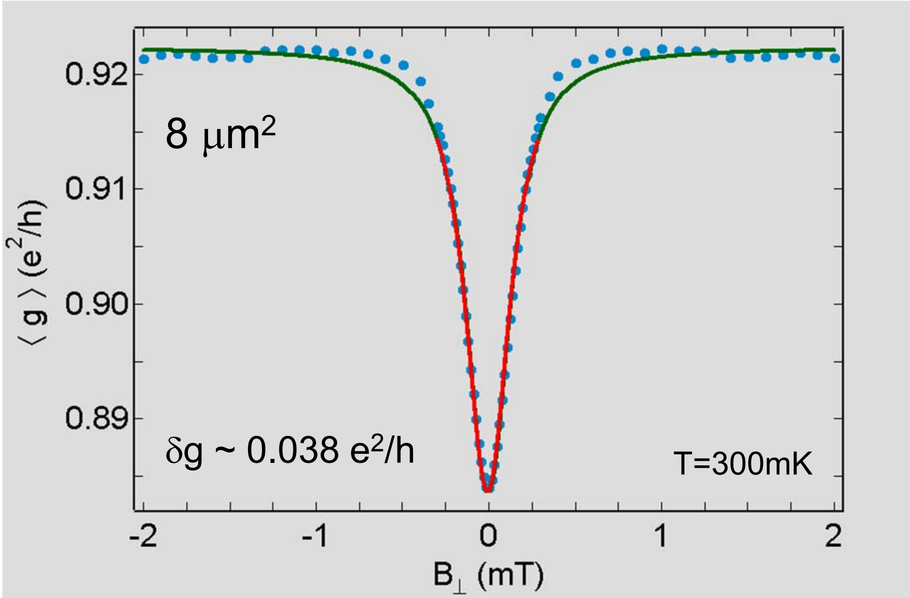
Weak Localization: Measure of Dephasing

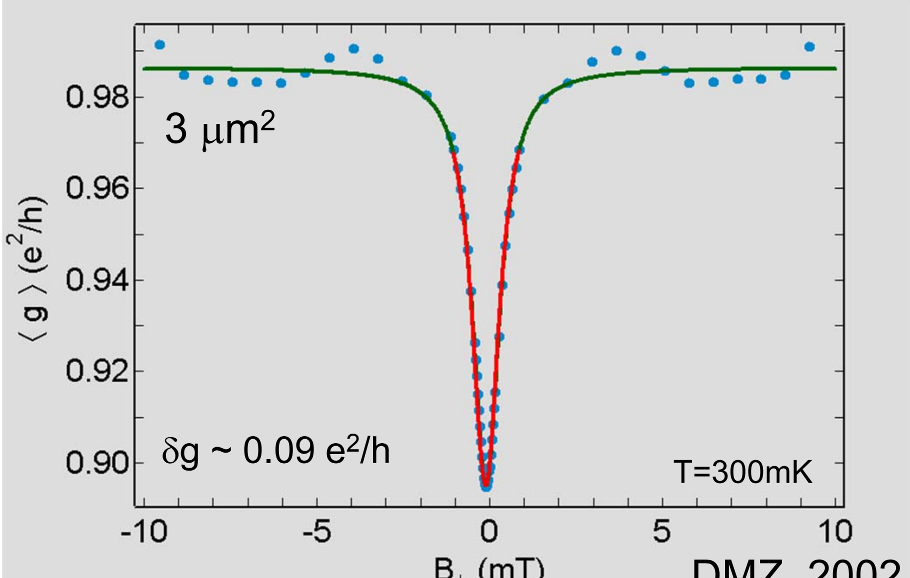




random matrix theory

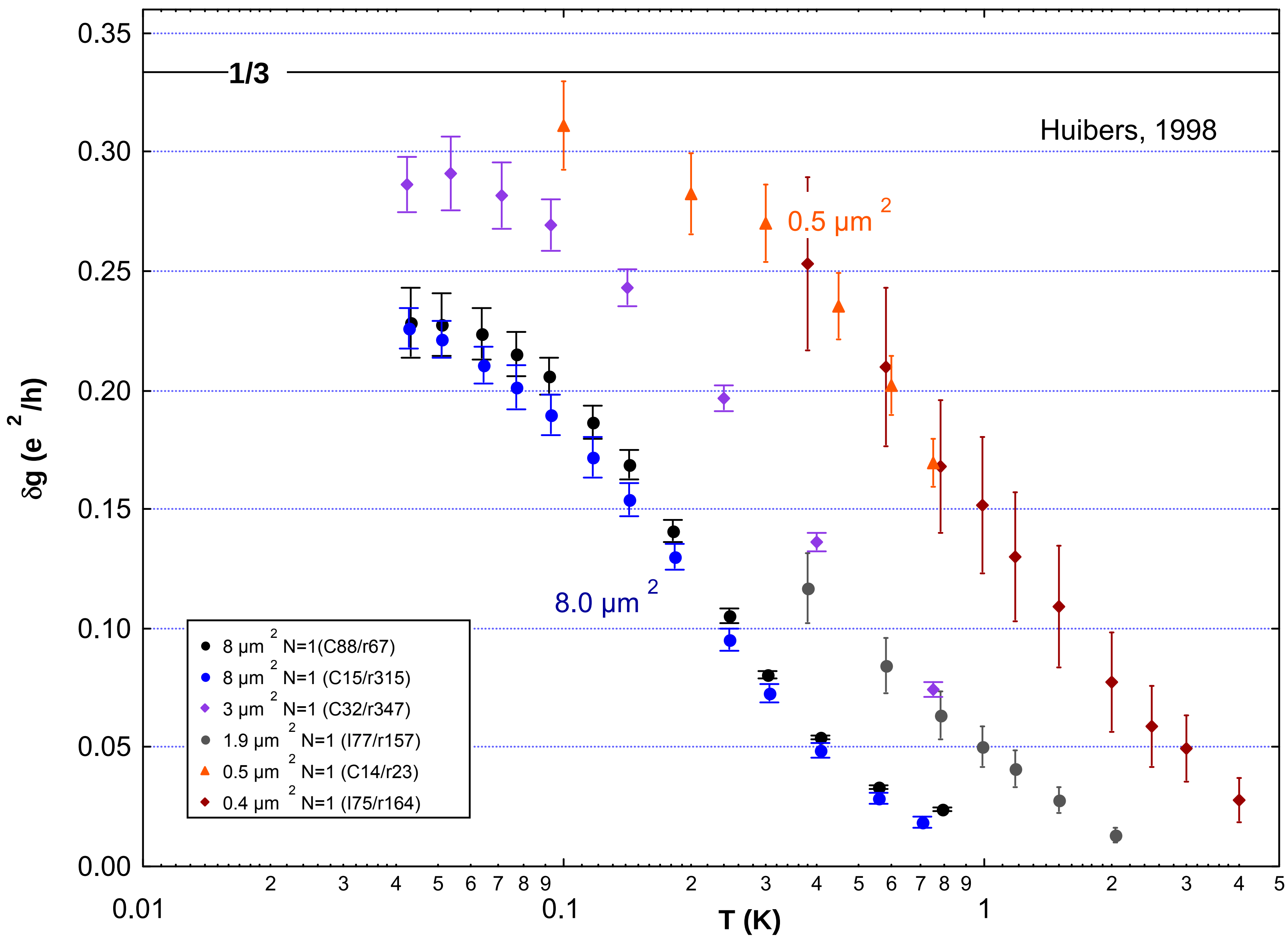
$$\delta g = \frac{1}{2N + 1 + \gamma_\phi} \qquad \tau_\phi^{-1} = \frac{\Delta}{h}\gamma_\phi$$





DMZ, 2002

Weak Localization vs T



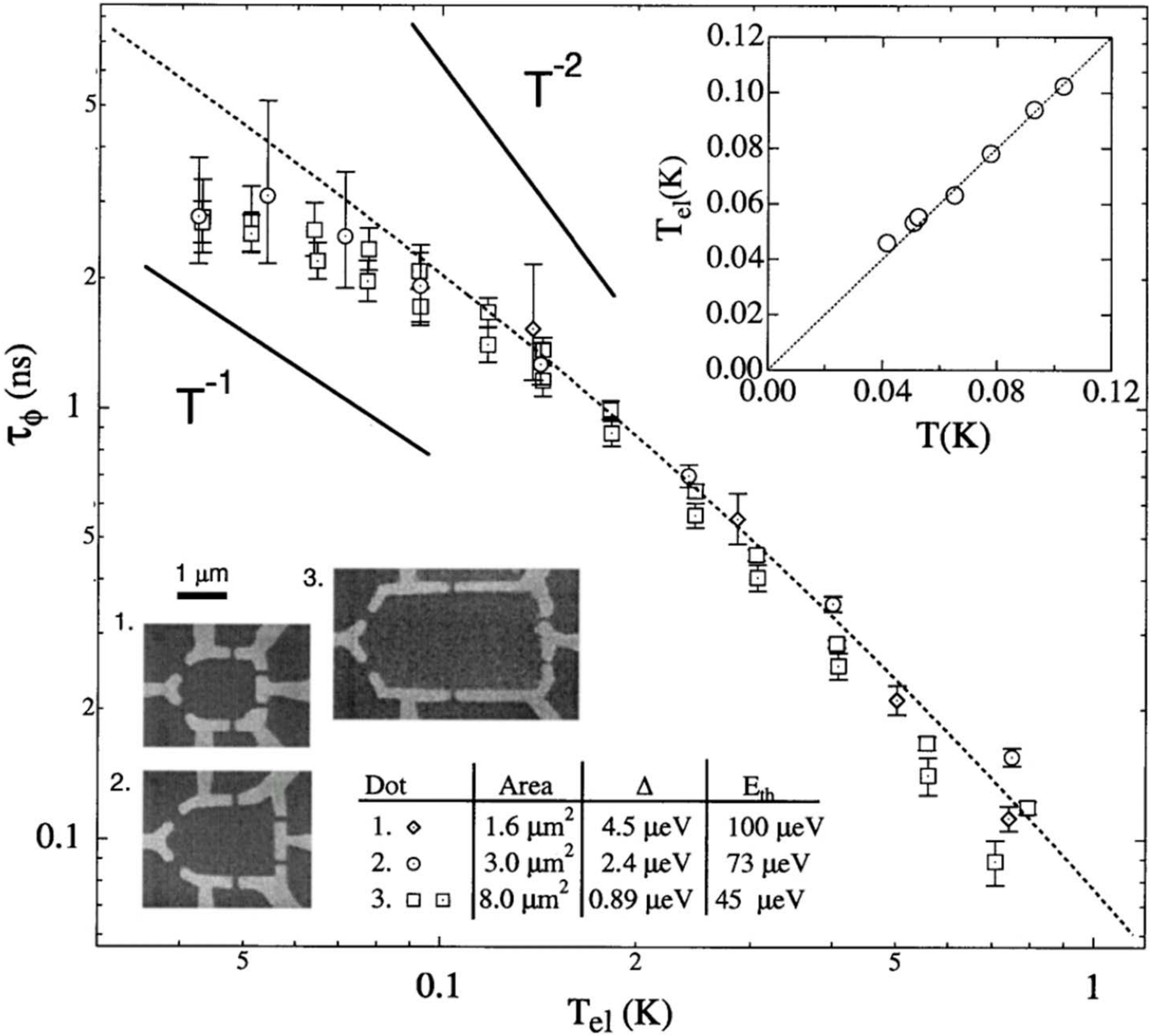
Low Temperature Saturation?

using

$$\delta g = \frac{1}{2N + 1 + \gamma_\phi}$$

obtain

$$\tau_\phi^{-1} = \frac{\Delta}{h}\gamma_\phi$$



Dot	Area	Δ	E_{th}
1. ◇	1.6 µm²	4.5 µeV	100 µeV
2. ○	3.0 µm²	2.4 µeV	73 µeV
3. □ ⊡	8.0 µm²	0.89 µeV	45 µeV

Huibers et al., PRL83, 5090 (1999)

Spin-Orbit Coupling

electrons move with the Fermi velocity, electric fields in material appear as magnetic fields in the rest frame of the electron

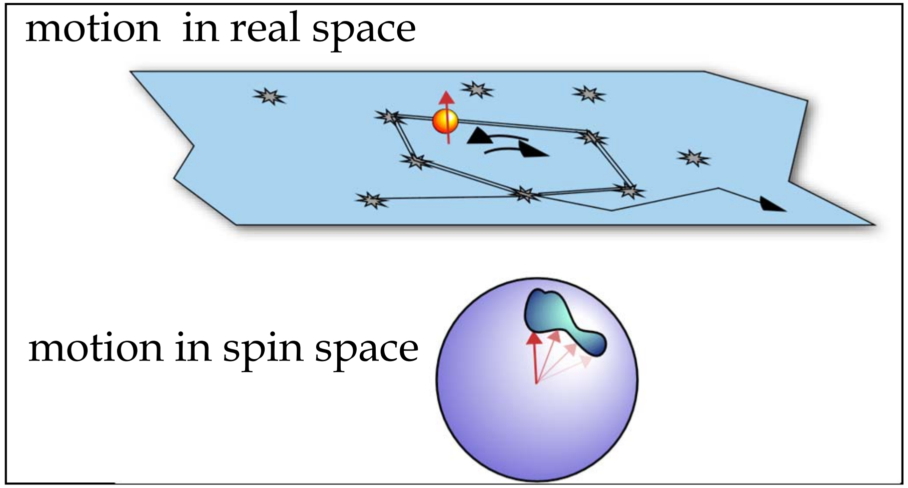
these magnetic fields
- depend on magnitude of electron velocity (density dependence)
- couple to the electron spin via Zeeman coupling

⟶ spin-precessions

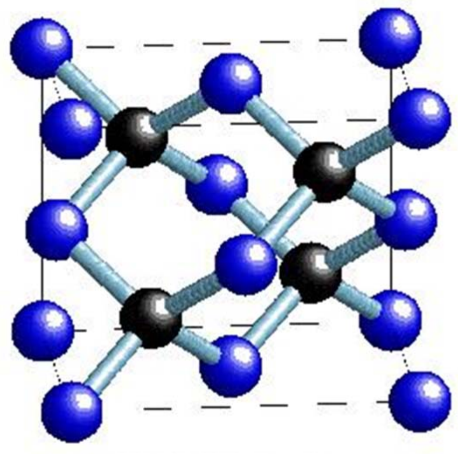
electric fields due to:

- heterointerface (***Rashba***)
- crystalline anisotropy in III-V zincblende crystal (***Dresselhaus***)

spin precession affects phase interference
(2π in spin space gives -1 to phase)



Spin-Orbit Coupling due to Crystal Anisotropy



III-V Semiconductor

Zinkblende crystall structure:
two interpenetrating fcc lattices
with only Ga atoms on one lattice,
only As on the other

absence of inversion symmetry

symmetry considerations:

$$H_{so} = \gamma(\sigma_x k_x (k_y^2 - k_z^2) + \text{cycl.})$$

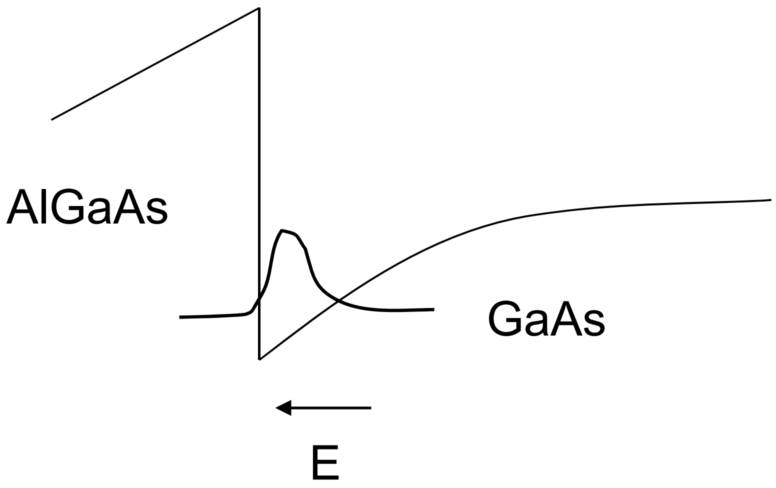
G. Dresselhaus,
Phys. Rev. 100, 580 (1955)

after size quantization (2D): $\langle k_z \rangle = 0$ $\alpha = \gamma \langle k_z^2 \rangle$

$$H_D^{(1)} = \alpha(\sigma_x k_x - \sigma_y k_y)$$ k-linear Dresselhaus term

$$H_D^{(3)} = \gamma(\sigma_y k_y k_x^2 - \sigma_x k_x k_y^2)$$ k-cubic Dresselhaus term

Spin-Orbit Coupling due to Heterointerface



electric field at heterointerface
perpendicular to 2D plane

$$\vec{B}_{so} \propto (k_y E, -k_x E, 0) \perp \vec{k}$$

$$H_R = \beta(\sigma_x k_y - \sigma_y k_x)$$

Rashba term (k-linear)

coupling strength parameters β and γ can be determined
from Band structure, for example in $k \cdot p$ approximation

Weak Antilocalization

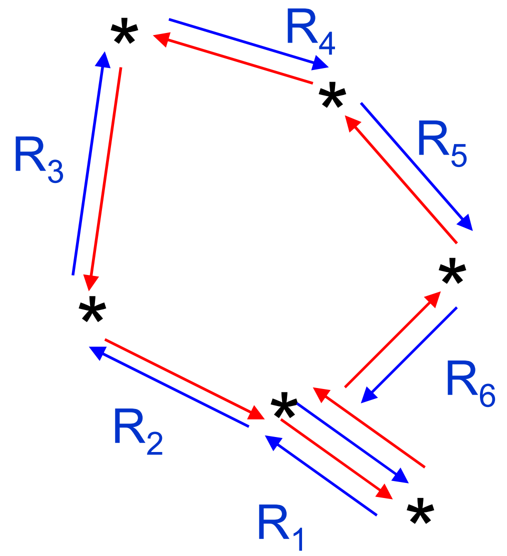
initial state: $|i\rangle$

final (forward): $|f_f\rangle = R_N \ldots R_2 R_1 |i\rangle = R|i\rangle$

final (backward): $|f_b\rangle = R_1^{-1} R_2^{-1} \ldots R_N^{-1} |i\rangle = R^{-1}|i\rangle$ (TRS)

R_i: spin rotations $R = R_N \ldots R_2 R_1$ $R^{\dagger} R = 1$ $R^{-1} = R^{\dagger}$

interference term $\langle f_b | f_f \rangle = \langle i | R^2 | i \rangle$



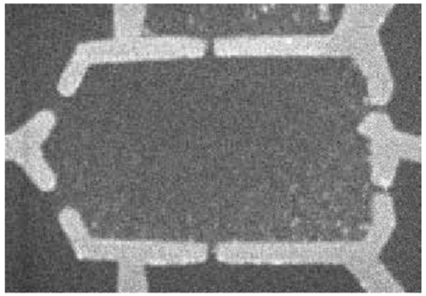
assuming strong spin-orbit coupling,
summing over all trajectories is
equivalent to averaging R^2 over sphere

$$\overline{\langle f_f | f_b \rangle} = -\frac{1}{2}$$

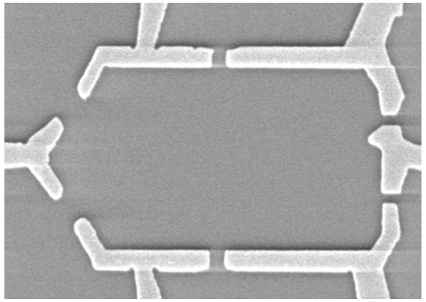
destructive interference, opposite sign
manifestation of fermionic nature of electron

exercise: show this

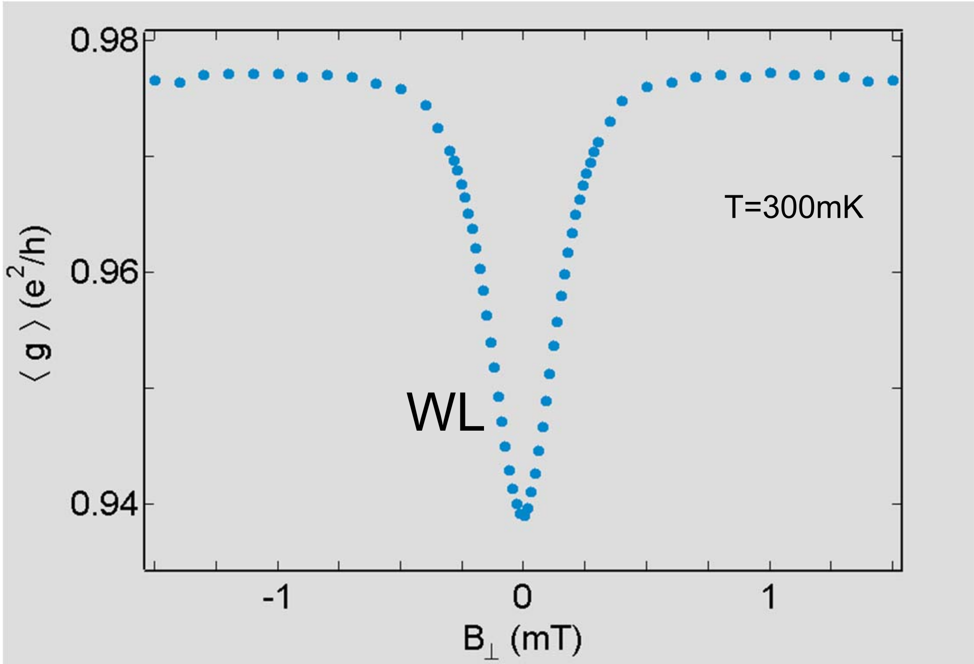
low density
weaker SO coupling
weak localization (WL)

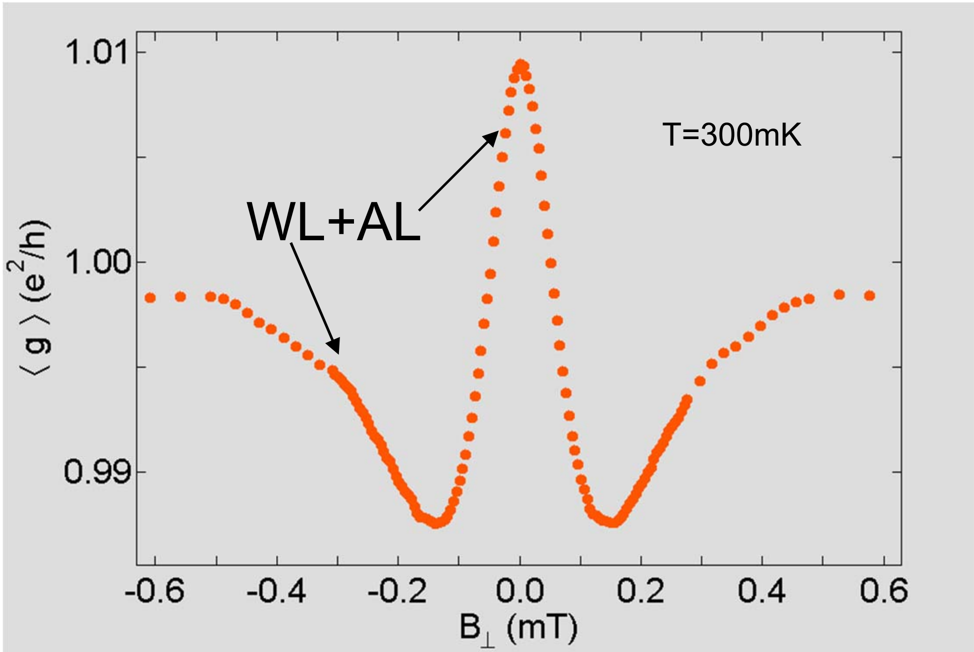


dots are on **different** wafers



high density
stronger SO coupling
antilocalization (AL)





Spin-Orbit (SO) interaction

- a preeminent effect governing spins in solids
- **novel quantum states**: helical states, topological insulators, Majorana fermions etc
- **resource for spin based quantum technologies** spintronics, quantum information
- useful: all electrical **spin manipulation** adverse: **spin dephasing, relaxation**

understanding and control of SO coupling profoundly important

Summary

- **Aharonov Bohm effect**
  potentials are physical quantities observable in quantum interference
  oscillations with magnetic field with period ϕ_0/A
- **weak localization**
  suppression of conductance at B=0 due to quantum interference
  size of effect of order e^2/h for full coherence
  can be used to extract quantum phase coherence
- **weak antilocalization**
  enhancement of conductance at B=0 (dito, plus strong spin-orbit coupling)
  size of order e^2/h (half of WL)
  can be used to extract SO parameters
- **conductance flucuations**
  quantum interference correction to conductivity
  as a function of a mesoscopic parameter, such as B-field, gate, chem. pot. etc
  of order e^2/h for fully coherent system
  can also be used to extract phase coherence