Influence of electron-electron scattering on shot noise in diffusive contacts

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Shot noise in contacts with a small elastic mean free path in the presence of electron-electron scattering is calculated using the Boltzmann-Langevin approach and the approximation of effective electron temperature. In the case of strong electron-electron scattering, the ratio between the shot noise S_I and its classical value $2e \mid I \mid$ increases from $\frac{1}{3}$ to $\sqrt{3}/4$ owing to a broadening band of partially occupied states. The influence of electron-phonon scattering on the shot noise at finite temperatures is also considered. As this scattering decreases the energy of the electron gas, it suppresses the shot noise at high voltages and increases the low-voltage boundary of the shot-noise range at finite temperatures. However, the noise-to-current ratio decreases with increasing contact length more slowly than its reciprocal.

If the electrical current I flowing through a device consists of random independent pulses, this device exhibits shot noise with a spectral density $S_I = 2e|I|$. Unlike equilibrium Nyquist noise, shot noise is not a universal phenomenon. It is present, e.g., in vacuum diodes and tunnel junctions and is absent in macroscopic wires.

Until recently, it was believed that the shot noise vanishes if the length of the wire is much larger than the mean free path of the electrons. Recent investigations,[1–3] however, showed that in diffusive-conduction wires with purely elastic scattering, $S_I = \frac{2}{3}e|I|$, i.e., this noise is 1/3 of the noise of a classical Poisson process.

The question yet to be elucidated is why the shot noise present in mesoscopic wires vanishes in the macroscopic limit. There is a general comprehension (see, e.g., Refs. 1 and 4) that the reason for the shot noise vanishing in macroscopic wires is inelastic scattering. However, it was not exactly known which types of inelastic scattering suppress the shot noise and which do not. To the best of our knowledge, quantitative results have been obtained for only two particular cases. For the particular case of zero temperature, it was shown[2] that the spontaneous emission of phonons by the electrons decreases the nonequilibrium noise. In Ref. 4, the result was obtained that spin-flip scattering does not affect the shot noise and it was predicted qualitatively that electron-phonon scattering should suppress it.

Actual measurements of shot noise in diffusive mesoscopic contacts have been performed, however, under the conditions of strong electron-electron scattering.[5] These measurements revealed certain deviations of the $S_I/e|I|$ ratio from the predicted value of 2/3 towards both higher and lower values depending on the experimental conditions. Hence it is of interest to investigate the influence of the electron-electron scattering on the shot noise in high-impurity-content contacts.

Another related problem is to take into account the scattering of electrons by thermal phonons. This is important for refining the conditions under which the shot noise can be observed.

In this paper, we consider the shot noise in a diffusive contact in the presence of electron-electron and electron-phonon scattering. As in Ref. 2, we calculate the fluctuations using the Boltzmann-Langevin equation. This approach is alternative to the scattering-matrix formalism used in Refs. 1 and 3. We obtain the result that electron-electron scattering increases the shot noise rather than suppresses it because of the increasing number of electron states that contribute to the noise. In contrast, electron-phonon scattering contracts the range of electron energies that contribute to the noise and, therefore, decreases this noise. The noise, however, decreases with the length of the wire more slowly than predicted in Refs. 1 and 4 because the energy relaxation of electrons is a substantially nonlinear process.

Consider a three-dimensional (3D) metal channel connecting two massive banks with its length L much larger than the elastic mean free path of electrons and the transverse dimensions of the channel. In addition to impurity scattering, the electrons also experience electron-phonon and electron-electron scattering. However, the inelastic scattering is much weaker than the elastic. We also assume that the quasiparticle description of electrons is valid, i.e., the electron-level broadening resulting from the inelastic scattering $\Delta\epsilon \sim \tau_\varphi^{-1}$ is much smaller than the characteristic energy scales at which the electron distribution function essentially varies. Moreover, we assume that the different scattering processes are independent, i.e., interference between the electron-impurity and electron-electron or electron-phonon scattering is absent. Hence the electrons are described by the quasiclassical distribution function $f(\vec{p}, \vec{r}, t)$ scalar which obeys the traditional Boltzmann equation. In the case of strong impurity scattering, the distribution function is almost isotropic in momentum space and, therefore, it may be considered as a function of the total electron energy $\epsilon = p^2/2m + e\phi(r) - \epsilon_F$. In a quasiclassical electron gas, fluctuations originate from the electron-impurity, electron-phonon, and electron-electron collisions, which are random events. In this case, the nonequilibrium noise may be calculated by the method of Kogan and Shul'man.[6] In Ref. 2, this method was used for calculating the spectral density of current fluctuations in a high-impurity-content mesoscopic contact. In the case

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where impurity scattering alone is present, the spectral density of current fluctuations was obtained in the form

$$S = \frac{4}{RL}\int_{-L/2}^{+L/2} dx \int_{-\infty}^{+\infty} d\epsilon\, f(\epsilon,x)\,[1-f(\epsilon,x)]\,, \qquad (1)$$

where R is the contact resistance and the x axis is directed along the contact. Following the derivation of (1) in Ref. 2, one may note that it also remains valid when impurity scattering coexists with electron-electron or weak electron-phonon scattering. Taken alone, electron-electron collisions cannot generate current fluctuations at all (provided that the elastic scattering time is energy independent near the Fermi surface) because they do not change the total momentum of the electron gas. As for the electron-phonon collisions, their contribution to fluctuations of the electron momentum is small in comparison with that of impurity scattering because of their low intensity.

Both types of inelastic collisions, however, participate in forming the distribution function $f(\epsilon,x)$, which enters into (1), because they determine the inelastic relaxation. Inside the contact, the distribution function $f(\epsilon,x)$ obeys the equation

$$D\frac{d^2}{dx^2}f(\epsilon,x) + I_{ee}(\epsilon,x) + I_{\rm ph}(\epsilon,x) = 0, \qquad (2)$$

where $D = v_F^2\tau_{\rm imp}/3$ is the diffusion coefficient of electrons,

$$\begin{aligned} I_{ee}(\epsilon) = -\frac{\pi^2}{64}\epsilon_F^{-1}\frac{k}{p_F}\int_{-\infty}^{+\infty} d\epsilon' \int_{-\infty}^{+\infty} d\omega \\ \times\{f(\epsilon)f(\epsilon'-\omega)[1-f(\epsilon-\omega)][1-f(\epsilon')] \\ -f(\epsilon-\omega)f(\epsilon')[1-f(\epsilon)][1-f(\epsilon'-\omega)]\}, \qquad (3) \end{aligned}$$

and k is the inverse screening length;

$$\begin{aligned} I_{\rm ph}(\epsilon) = \frac{\alpha_{\rm ph}}{\theta_D^2}\int_0^{\infty} d\omega\,\omega^2\{[1-f(\epsilon)]f(\epsilon+\omega)[1+N(\omega)] \\ +[1-f(\epsilon)]f(\epsilon-\omega)N(\omega) \\ -f(\epsilon)[1-f(\epsilon-\omega)][1+N(\omega)] \\ -f(\epsilon)[1-f(\epsilon+\omega)]N(\omega)\}, \qquad (4) \end{aligned}$$

where $\alpha_{\rm ph}$ is the dimensionless parameter of electron-phonon interaction, θ_D is the Debye temperature, and $N(\omega)$ is the phonon distribution function. The boundary conditions for Eq. (2) at the left and the right ends of the contacts are

$$\begin{aligned} f(\epsilon,-L/2) = f_F(\epsilon - eV/2), \\ f(\epsilon,L/2) = f_F(\epsilon + eV/2), \qquad (5) \end{aligned}$$

where $f_F(\epsilon) = 1/[1+\exp(\epsilon/T)]$ is the Fermi distribution function and V is the voltage drop across the contact.

First, consider the case of weak electron-electron scattering when

$$(k/p_F)(eV)^2/\epsilon_F \ll D/L^2.$$

In this case, the collision integral I_{ee} may be treated as a perturbation. As the zeroth approximation, we use the solution of the equation $\partial^2 f/\partial x^2 = 0$ with the boundary conditions (5). At zero temperature, this solution takes the form

$$f_0(\epsilon) = \begin{cases} 0 & \text{for } \epsilon > eV/2 \\ 1/2 - x/L & \text{for } eV/2 > \epsilon > -eV/2 \\ 1 & \text{for } \epsilon < -eV/2. \end{cases} \qquad (6)$$

In the first approximation in I_{ee}, the correction to f_0 obeys the equation

$$\frac{\partial^2 f_1}{\partial x^2} = -I_{ee}\{f_0\}, \qquad f_1(\pm L/2,\epsilon) = 0. \qquad (7)$$

Upon solving Eq. (7) and substituting $f = f_0 + f_1$ into (1), one obtains

$$S_I(V) = \frac{2}{3}\frac{eV}{R}\left[1 + \frac{11\pi^2}{40\,320}\frac{k}{p_F}\frac{(eVL)^2}{D\epsilon_F}\right]. \qquad (8)$$

Hence the correction to the spectral density of the noise from electron-electron scattering is positive. Note that, according to Refs. 2 and 4, the similar correction from the electron-phonon scattering at $T = 0$ was negative.

Now, consider the case of strong electron-electron scattering

$$(k/p_F)(eV)^2/\epsilon_F \gg D/L^2.$$

It may be easily seen that the collision integral (3) vanishes if the distribution function is of the form $f = f_F(\epsilon - \epsilon_0)$ with arbitrary ϵ_0 and temperature T. Hence it may be assumed that in the case of strong electron-electron scattering the electron distribution is described by the local temperature T_e, which differs from the lattice temperature. Therefore,

$$f(\epsilon,x) = \left[1 + \exp\left(\frac{\epsilon - e\phi(x)}{T_e(x)}\right)\right]^{-1}, \qquad (9)$$

where $\phi(x) = -(x/L)eV$ is the electrical potential. To obtain an equation for the effective temperature of the electrons, multiply Eq. (2) by ϵ and integrate it with respect to energy from $-\infty$ to $+\infty$. By using the explicit form of $f(\epsilon,x)$ (9), it may be easily obtained that

$$\int_{-\infty}^{+\infty} d\epsilon\,\epsilon f(\epsilon,x) = \frac{\pi^2}{6}T_e^2 + \frac{1}{2}(e\phi)^2. \qquad (10)$$

By introducing the new variable $\Delta = T_e^2$ and making use of the explicit form of $\phi(x)$, one may easily obtain an equation for Δ:

$$\frac{d^2\Delta}{dx^2} = -\frac{6}{\pi^2}\left(\frac{eV}{L}\right)^2 \qquad (11)$$

with the boundary condition $\Delta(\pm L/2) = T^2$. In terms of Δ, the expression for the spectral density of the noise (1) takes the form

$$S_I = \frac{4}{RL}\int_{-\frac{L}{2}}^{+\frac{L}{2}} dx\, \Delta^{1/2}. \quad (12)$$

Consider the case where $T = 0$ and electron-phonon scattering is absent. The solution of (11) is of the form

$$\Delta(x) = \frac{3}{\pi^2}(eV)^2\left(\frac{1}{4} - \frac{x^2}{L^2}\right). \quad (13)$$

Hence

$$S_I = \frac{\sqrt{3}}{2}\frac{eV}{R}. \quad (14)$$

Therefore strong electron-electron scattering increases the noise in comparison with its value in the absence of inelastic scattering, $S_I = \frac{2}{3}eV/R$. The reason for this increase is that in the former case the electron-electron scattering gives rise to partially occupied states at $\epsilon > eV/2$ and $\epsilon < -eV/2$. This result is independent of the dimensionality of the electron system because the explicit form of the collision integral was not used in deriving (14). An increase of the shot noise in excess of $\frac{2}{3}eV/R$ was experimentally observed in Ref. 5.

Now we consider the case where strong electron-electron scattering coexists with weaker electron-phonon scattering, which implies that

$$(k/p_F)(eV)^2/\epsilon_F \gg \alpha_{\rm ph}(eV)^3/\theta_D^2.$$

Unlike electron-electron scattering, electron-phonon scattering does not conserve the total energy of the electron system and may change the effective temperature of electrons, T_e. Because the electron distribution is far from equilibrium, its relaxation is an essentially nonlinear process and cannot be described by the standard approximation of the energy-relaxation time $\tau_\epsilon(T)$. In general, describing this relaxation requires solving the kinetic equation for electrons with the collision integrals (3) and (4) combined with the similar equation for phonons. In turn, this requires specifying the thermal conductivity of the substrate, the acoustic transmission of the metal-substrate interface, and other parameters describing the phonon kinetics. Instead, we assume that the phonon distribution is equilibrium, i.e.,

$$N(\omega) = [\exp(\omega/T) - 1]^{-1}, \quad (15)$$

where T is the contact temperature corrected for possible Joule heating. Because the electron-electron scattering is assumed to be strong, the approximation of local electron temperature is valid. Substituting the distribution functions (9) and (15) into Eq. (2), multiplying both its parts by ϵ, and then integrating them with respect to ϵ, one obtains an equation for $\Delta = T_e^2$ in the form

$$\frac{\pi^2}{6}L^2\frac{d^2\Delta}{dx^2} = -(eV)^2 + \frac{\gamma}{\theta_D^3}(\Delta^{5/2} - T^5), \quad (16)$$

where $\gamma = 24\zeta(5)\alpha_{\rm ph}L^2\theta_D/D$ is a dimensionless parameter describing the energy relaxation of electrons in the contacts and $\zeta(5)$ is Riemann's zeta function. In the right-hand side of (16), the second term corresponds to spontaneous emission of phonons by the electrons and the third term corresponds to the scattering of electrons by thermal phonons. This equation should be supplemented by the same boundary conditions as (11), and the noise is again given by (12).

Unlike (11), Eq. (16) is nonlinear. Introducing a new dimensionless variable $\psi = \gamma^{2/3}\theta_D^{-2}\Delta$, it is seen that the behavior of the solution depends on the ratios of eV and the characteristic energies $\epsilon_1 = \gamma^{1/2}\theta_D^{-3/2}T^{5/2}$ and $\epsilon_2 = \gamma^{-1/3}\theta_D$. If $eV \ll \epsilon_1$, $\psi \approx \gamma^{2/3}T^2/\theta_D^2$ almost through the whole length of the contact and the noise differs inessentially from the equilibrium noise. If $\epsilon_1 \ll eV \ll \epsilon_2$, the second and the third terms in the right-hand side of (16) may be neglected and, therefore, one obtains the shot noise (14). If, lastly, $eV \gg \epsilon_1$ and $eV \gg \epsilon_2$, the $\psi(x)$ dependence takes a nearly rectangular shape: along almost the whole contact length, $\psi = (\gamma^{-1/3}eV/\theta_D)^{4/5}$ so that the right-hand side of (16) is zero, and it is only near the ends of the contact that ψ decreases to its values in the banks. Hence $S_I = 4\gamma^{-1/5}\theta_D^{3/5}(eV)^{2/5}/R$. This is precisely the same power-law dependence that was obtained in Ref. 2 in the absence of electron-electron interaction yet with a different numerical coefficient. At a constant voltage, the ratio $S_I/e|I|$ decreases with increasing contact length according to the law $L^{-2/5}$, which differs from the L^{-1} law predicted in Refs. 1 and 4. This implies that the contact cannot be considered as a series of mesoscopic resistors of a certain characteristic length l_c, which are independent sources of shot noise.[1] This simple model does not work because the energy relaxation of electrons is substantially nonlinear and cannot be described by a characteristic relaxation length. Note also that the exponent 2/5 depends on the particular form of the electron-phonon collision integral and may be different, e.g., for a different dimensionality of the phonon system.

From this consideration, it follows that, as the contact length L or the parameter of the electron-phonon interaction $\alpha_{\rm ph}$ increases, the shot-noise range of voltages shrinks. Moreover, the larger the energy-relaxation parameter γ, the more difficult is the condition $eV \gg \gamma^{1/2}\theta_D^{-3/2}T^{5/2}$ to implement because of the Joule heating of the lattice in the contact. Although we assumed T and V to be independent variables throughout this paper, in fact, $T \propto V^2$ at high voltages and the required relationship between eV and T finally is violated. For this reason, nonequilibrium noise is not observed in macroscopic samples.

In summary, it was shown that different inelastic scattering processes differently affect the shot noise in dirty microcontacts depending on whether they conserve the total energy of the electron system or not. For example, electron-electron scattering increases the shot noise because of the broadening energy band of partially occupied states in the contact and results in the universal ratio $S_I/e \mid I \mid= \frac{\sqrt{3}}{2}$, which holds for both 3D and 2D systems.

The reverse is true in the case of electron-phonon scattering, which takes up the energy of the electrons and decreases their effective temperature to the lattice temperature. This type of scattering suppresses the shot noise and decreases the voltage range where the shot noise is observed from both sides. For long contacts, the noise-to-current ratio decreases with increasing contact length more slowly than its reciprocal. Eventually, the increasing electron-phonon scattering results in the nonequilibrium noise sinking to the equilibrium value.

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