Noise

tutorial

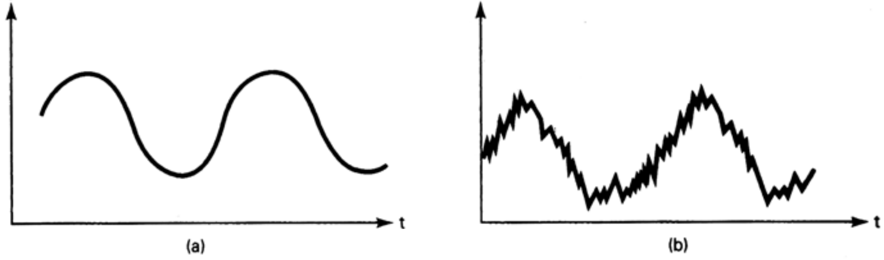
by Christian Schönenberger

3. April 2015

A definition (not mine)

Electrical Noise

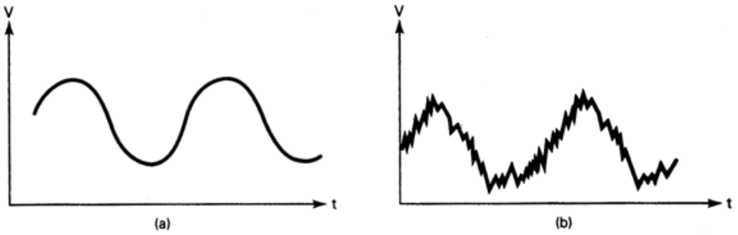
- Electrical noise is defined as **any undesirable** electrical energy (?)



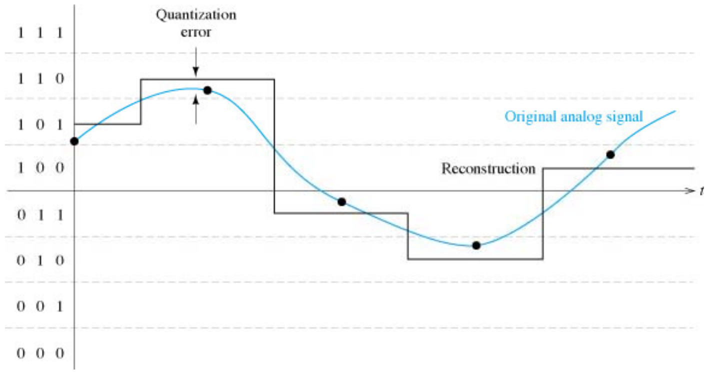
Effect of noise on a signal. (a) Without noise (b) With noise

Noise may have been added to by ...

Electrical Noise



- it may be in the **“source”** signal from the start
- it may have been introduced by the **electronics**
- it may have been added to by the **envirnoment**
- it may have been generated in your **computer**



for the latter, e.g.
sampling noise

Introduction: Noise is widespread

- Noise (audio, sound, HiFi, encoding, MPEG)
- Noise (industrial, pollution)
- Noise (in electronic circuits)
- Noise (images, video, encoding)
- Noise (environment, pollution)
- Noise (radio)
- Noise (economic => theory of pricing with fluctuating source terms)
- Noise (astronomy, big-bang, cosmic background)
- Noise figure
- Shot noise
- Thermal noise
- Quantum noise
- Neuronal noise
- Standard quantum limit
- and more ...

Introduction (Wikipedia)

Electronic Noise (from Wikipedia)

Electronic noise exists in any electronic circuit as a result of **random variations** in current or voltage caused by the **random movement** of the electrons carrying the current as they are jolted around by thermal energy.

The lower the temperature the lower is this **thermal noise**.

This same phenomenon limits the **minimum signal level** that any radio receiver can usefully respond to, because there will always be a small but significant amount of thermal noise arising in its input circuits. This is why *radio telescopes*, which search for very low levels of signal from stars, use front-end circuits, usually mounted on the aerial dish, cooled in *liquid nitrogen to a very low temperature*.

Introduction

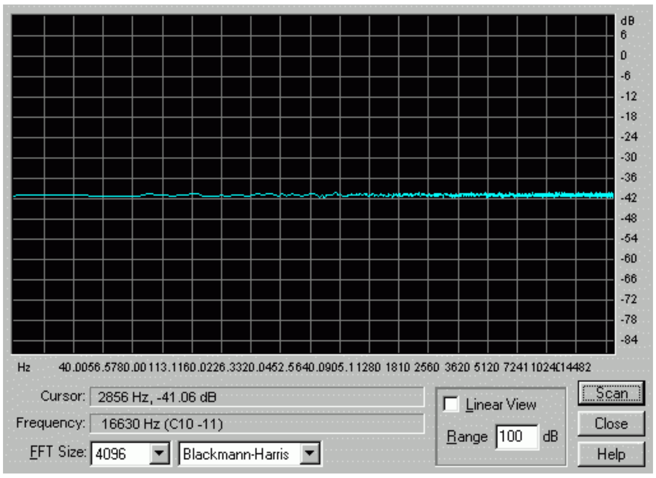
Electronic noise is quantified as a spectral density, i.e. is a frequency dependent quantity that (for a small frequency span) is proportional to bandwidth.

Explicitly: assume we measure the voltage at the output of an amplifier. Then, there is a certain **voltage noise S(f)**, known as the **spectral voltage noise power density**. It has units V^2/Hz.

In terms of voltage, the units are e.g. μV/√Hz; for a good amplifier it is rather like nV/√Hz

→ **the noise voltage grows with bandwidth**

→ explicit f-dependence is a characteristics, e.g. white-noise, pink-noise ...

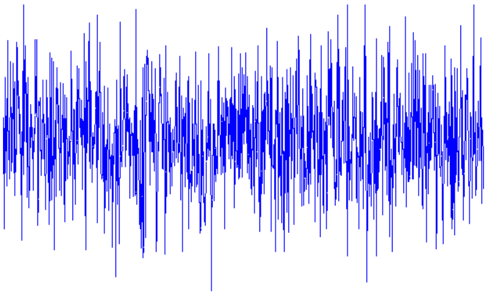


Cursor: 2856 Hz, -41.06 dB

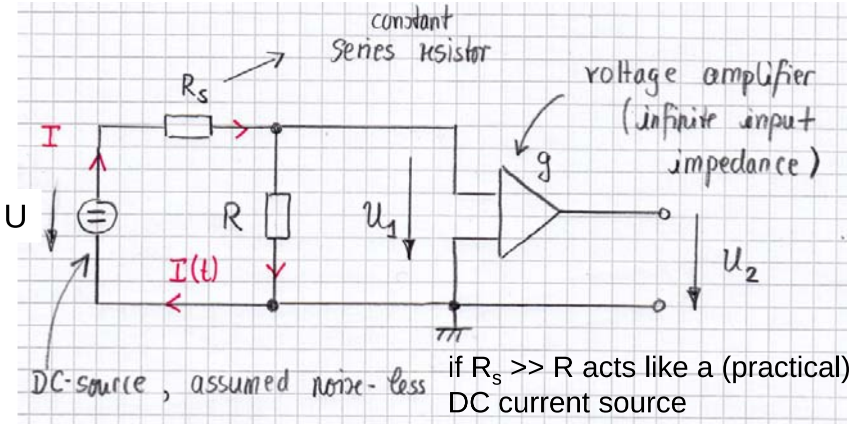
Frequency: 16630 Hz (C10 -11)

FFT Size: 4096 ▼ Blackmann-Harris ▼

Range 100 dB

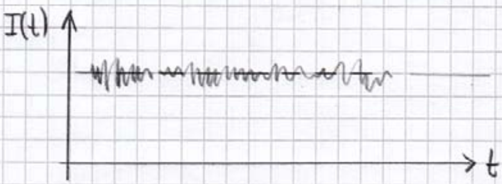


A practical example from the lab



if R_s >> R acts like a (practical) DC current source

R is "the device" and the main source of noise here (we assume that R_s is noise less).



$$I(t) = \bar{I} + \delta I(t)$$

$$\bar{I} = \frac{u}{R_s + R}$$

$\delta u_1(t) = \delta I \cdot R$ and $\delta u_2 = g\, \delta u_1$

spectral noise powers $S_{u_2} = g^2 S_{u_1} = g^2 R^2 S_I$

in reality, all elements add noise, often multiple ones. E.g. an amplifier may have input noise and gain noise; a resistor may have intrinsic "thermal noise" but also 1/f noise... One has to add all noises up. One then assumes that they are statistically independent. See **exercise 1.**

Introduction

Thermal Noise (from Wikipedia)

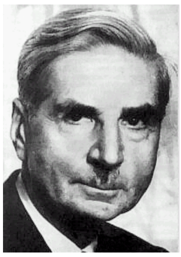
Johnson-Nyquist noise (thermal noise, Johnson noise, or Nyquist noise) is the noise generated by the **equilibrium fluctuations** of the electric current inside an electrical conductor, which happens regardless of any applied voltage, due to the **random thermal motion** of the charge carriers (the electrons).

It was first measured by **J. B. Johnson** at Bell Labs in 1928. He described his findings to **H. Nyquist**, also at Bell Labs, who was able to explain the results by deriving a **fluctuation-dissipation relationship**.


Nyquist

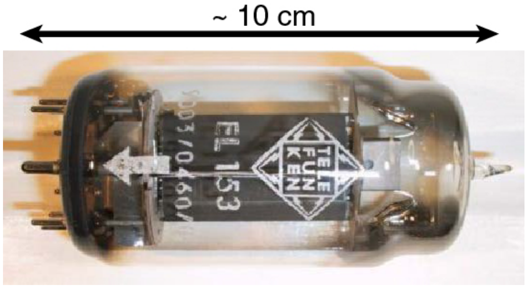

Johnson

Thermal noise is to be distinguished from *shot noise*, which consists of additional current fluctuations that occur when a voltage is applied and a macroscopic current starts to flow


Schottky

shot noise is a very general phenomena. It always appears if the medium that is transported consist of **"quanta"**. Then, the number of "quanta's" transmitted in a given time interval shows **number fluctuations**. This is shot noise. It appears fro electrons, photons, for money etc. Shot-noise was introduced by Johnson and theoretically by Schottky

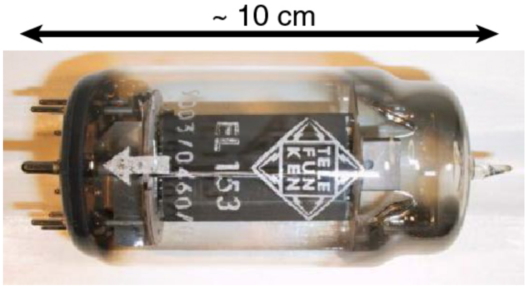
Introduction



shot-noise measured first in vacuum tube

Shot noise is a type of noise that occurs when the **finite number** of particles that carry energy, such as electrons in an electronic circuit or photons in an optical device, gives rise to detectable statistical fluctuations in a measurement. It is important in electronics, telecommunications and fundamental physics (Wikipedia)

Vacuum Tube: its implication to flicker



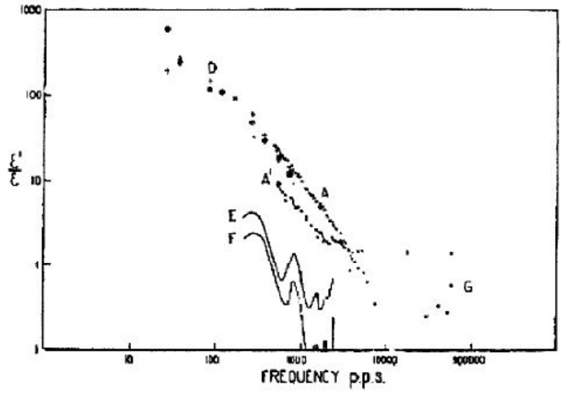


Fig. 6. Frequency variation for tube No. 2, coated filament; same data as in Fig. plotted to a frequency scale; curves E and F give Hartmann's results for 2 m-a. a 20 m-a.; points G were obtained with less steady measuring circuit.

"The **triode** was invented by Lee de Forest in 1907, and soon afterwards the first amplifiers were built. By 1921 the **'thermionic tube**' amplifiers were so developed that **C.A. Hartmann** made the first attempts to verify **Schottky's** formula for the shot noise spectral density.

Hartmann's attempt failed *(that's not fully true, though*), and it was finally **J.B. Johnson** who successfully measured the predicted **white noise spectrum**.

However, Johnson also measured an unexpected **'flicker noise'** at low frequency... and shortly thereafter W. Schottly tried to provide a theoretical explanation. Schottky's explanation was based on the physics of electron transport inside the vacumm tube, but in the years that followed Johnson's discovery of flicker noise, it was found that this strange noise appeared again and again in many different electrical deviced."

Signal and Noise

Frequency dependence of noise

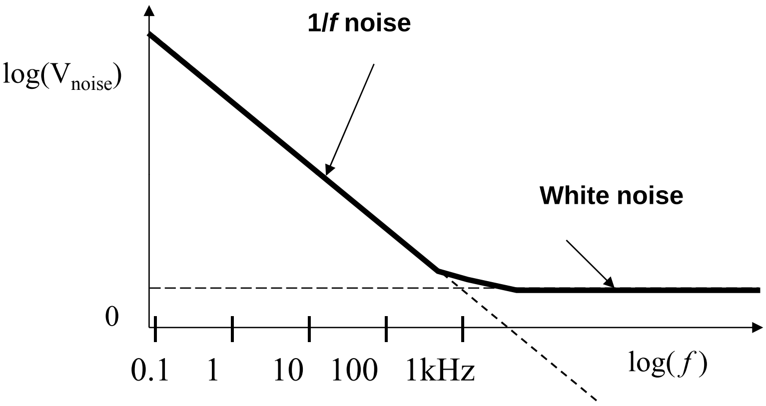
- **Low frequency ~ 1 / *f***
  - example: temperature (0.1 Hz) , pressure (1 Hz), acoustics (10 -- 100 Hz)
- **High frequency ~ constant = white noise**
  - example: shot noise, Johnson noise, spontaneous emission noise
- Total noise depends strongly on signal freq
  - **worst at DC**, best in white noise region

J. Hooge

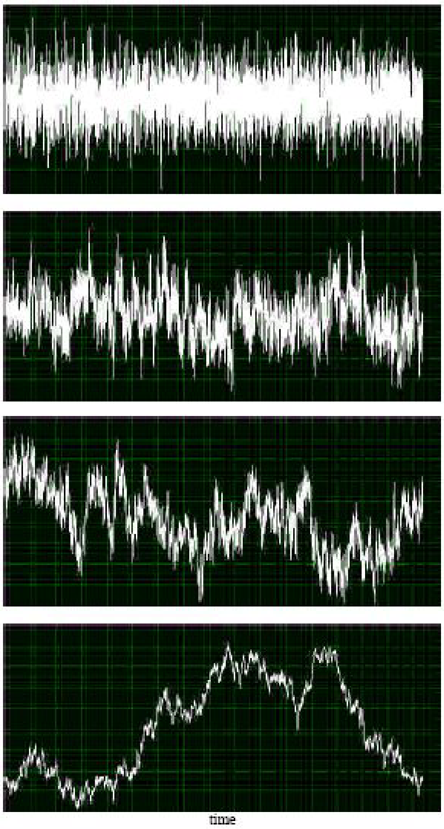
$$\boxed{S_R/R^2 = \frac{\alpha_H}{N_c f}}$$

$\alpha_H \sim 10^{-3}$

Noise amplitude



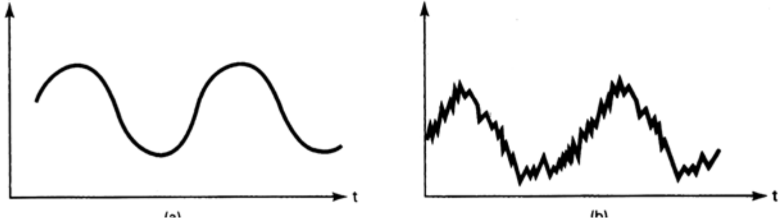
1/f-Noise

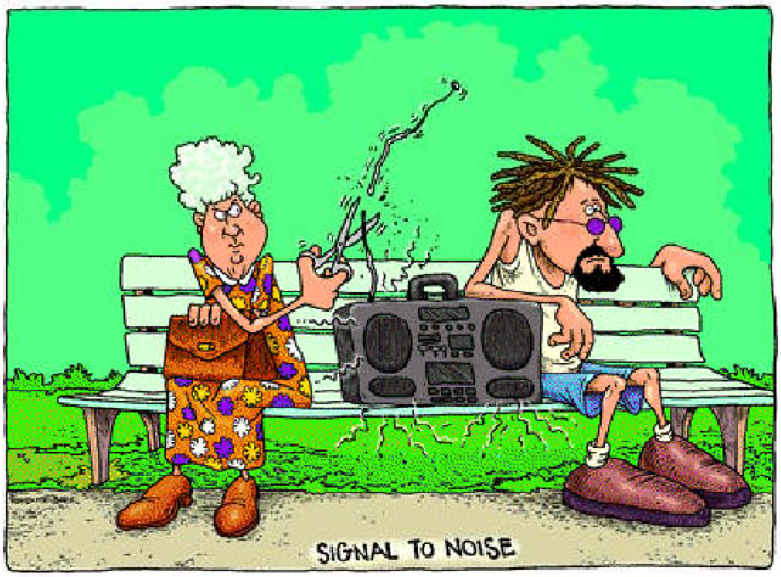


$1/f^{\alpha}$ noise generated with the algorithm described in section 10, amplitude vs. time (both linear scales and arbitrary units); starting from top, $\alpha = 0, 1, 1.5$ and 2.

Noise can be a nuiscance but also the signal

usually undesirable (a nuisance)





Cartoon by Rand Kruback, Agilent Technologies.

Condensed-matter physics

The noise is the signal

Rolf Landauer

NATURE | VOL 392 | 16 APRIL 1998

C. Beenakker and C. Schönenberger
Physics Today, ***56****-5,37-42 (2003)*

Origin of electronic noise (state picture)

noise is due to “uncertainty” and “randomness”

consider a single electron state at energy E. It is occupied with probability *p=f(E)*, where *f* is the Fermi-Diract distribution function

if *p* is either 1 or 0, there is no uncertainty in the state occupation at any time. Hence, there is no noise!

but obviously, if 0<*p*<1, we expect noise.

it turns out that the state adds to the noise a contribution that is proportional to *f(1-f)*

→ *derivation on blackboard*

Formal definition of noise

mean: $\bar{I} = \langle I \rangle$

fluctuation: $\delta I(t) = I(t) - \bar{I}$

auto-correlation: $S_{\delta I}(t,t') = 2\langle \delta I(t) \delta I(t') \rangle$

stationary case: $C_{\delta I}(t) = 2\langle \delta I(\tau) \delta I(\tau + t) \rangle$ is independent of τ

noise power (spectral density): $S_{\delta I}(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t} C_{\delta I}(t) dt$

<u>note</u>: use a **classical definition** here (not operators) → $S(-\omega) = S(\omega)$ $\delta I(t) \in \mathbb{C}$

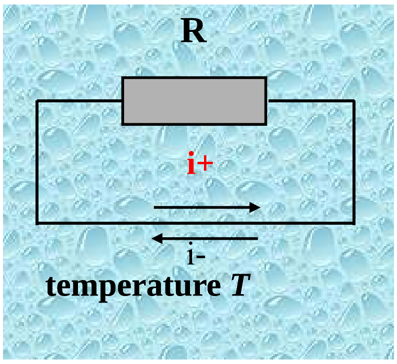
typically: $C_{\delta I}(t) = 2\left\langle (\delta I)^2 \right\rangle e^{-|t|/\tau}$

$$S(\omega) = 2\int_0^\infty \cos(\omega t) C_{\delta I}(t) dt = \left\langle \delta^2 I \right\rangle \frac{4\tau}{1+(\omega\tau)^2}$$

noise is white up to a cut-off given by 1/τ

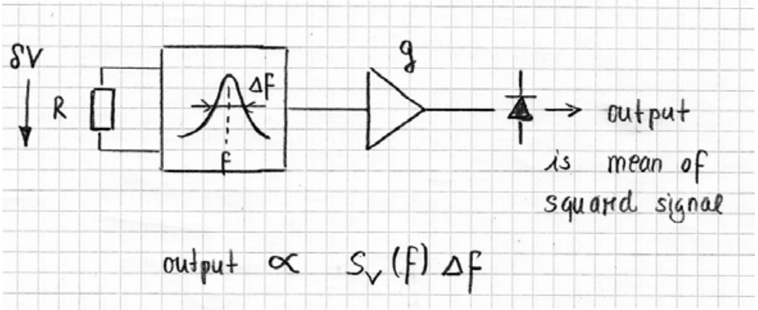
$S(0) = 4\tau \left\langle \delta^2 I \right\rangle$ noise power measures the second moment (up to some factor)

Thermal noise / Johnson-Nyquist noise



$$S_V = 4k_B TR$$

$$S_I = 4k_B T / R = 4k_B TG$$



output ∝ $S_V(f)\,\Delta f$

$$\frac{1}{2} m \left\langle \mathrm{v}_x{}^2 \right\rangle = \frac{1}{2} k_B T$$

(equipartioning)

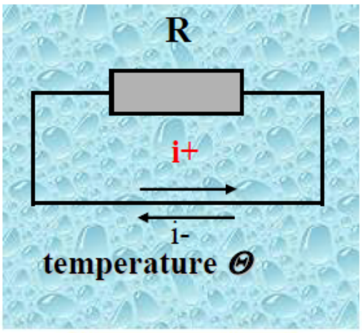
→ *derivation on blackboard*

Thermal noise (also known as equilibrium noise) is much more universal, than this derivation via the velocity distribution in a metallic wire suggests. There is a general **fluctuation-dissipation theorem** that links dissipation (resistance) with fluctuations (we will write down the full equation later).

Thermal noise (backup)

We use the Wiener-Khintchine theorem and start from the velocity correlation function for δv:

$$C_v(t) := \langle \delta v_j(\cdot)\delta v_j(\cdot + t)\rangle = \langle \delta v_j^2 \rangle e^{-|t|/\tau} = \frac{kT}{m} e^{-|t|/\tau}$$



$$\frac{1}{2} m \langle \mathrm{v}_x{}^2 \rangle = \frac{1}{2} k_B T$$

Using the theorem

$$S_v(\omega) = 4\frac{kT}{m}\int_0^\infty e^{-t/\tau} cos(\omega t) = \frac{4kT}{m}\frac{\tau}{1+(\omega\tau)^2}$$

Adding up for the final quantity $S_U(\omega)$ yields

$$S_U(\omega) = \left(\frac{Re}{L}\right)^2 \cdot (NAL)\frac{4kT}{m}\frac{\tau}{1+(\omega\tau)^2}$$

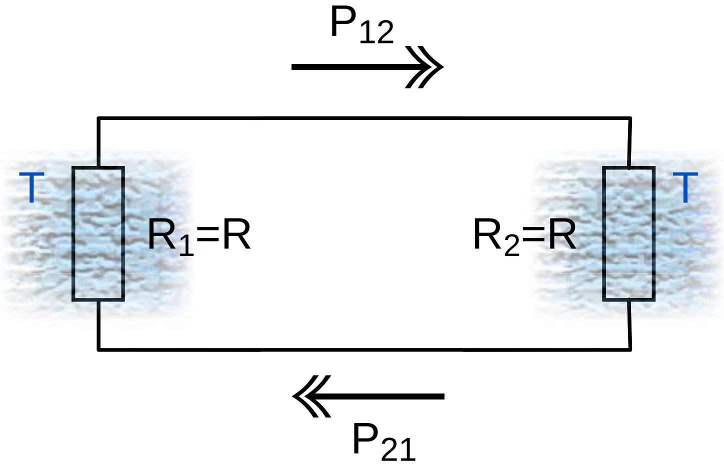
and by noting that $e^2 N\tau/m$ is the Drude conductivity σ_D and $\sigma_D A/L = 1/R$, we arrive at the desired result:

$$S_U(\omega) = 4kTR \cdot \frac{1}{1+(\omega\tau)^2} \sim 4kTR$$

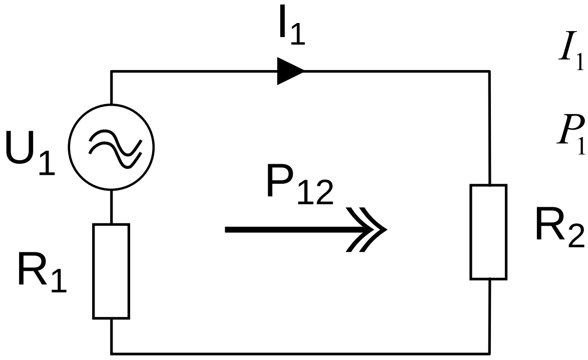
$$S_U(0) = 4kTR$$

J. B. Johnson and H. Nyquist (1928)

Thermal noise by Nyquist

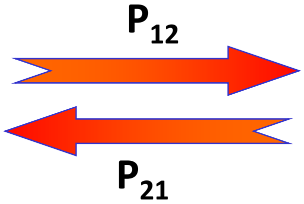


H. Nyquist (1928)



$$I_1 = U_1 / 2R$$

$$P_{12} = U_1^2 / 4R$$



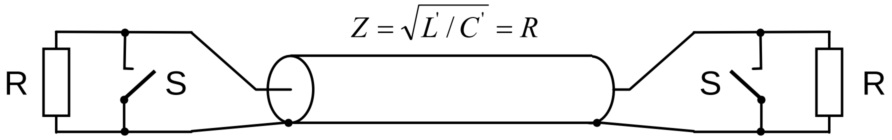
$P_{12} = P_{21}$

(for all frequencies seperately)

Thermal noise by Nyquist

ideal transmission line with

$$Z = \sqrt{L^{'} / C^{'}} = R$$



Gedankenexperiment: After equilibrium has been established, let us close the two switches S. The energy which was moving in and out is now trapped.

In order to calculate it, we need the eigenmode spectrum:

$$f_n = cn / 2L$$ *L* is the lenth of the cavity

According to Planck, the radiation energy within an f-window of Δf is given by

$$E_{\Delta f} = \left(\frac{2L}{c} \Delta f \right) hf_n \left(\frac{1}{2} + \frac{1}{e^{hf_n / kT} - 1} \right)$$

This energy has to be supplied by the two sources R_1 and R_2 during the time *L/c*, so that the power delivered by each resistor (within Δf) is: $P_{1,2} = \frac{1}{2} E_{\Delta f} \frac{c}{L} = U_1^2 / 4R$

$$\boxed{S_U = 4R\, \hbar\omega \left(\frac{1}{2} + \frac{1}{e^{\hbar\omega / kT} - 1} \right) = 2R\hbar\omega \coth\left(\frac{\hbar\omega}{2kT} \right)}$$

$$\left\{ \begin{array}{ll} S_U = 4RkT & (kT >> \hbar\omega) \\ S_U = 4R\left(\frac{1}{2} \hbar\omega \right) & (kT << \hbar\omega) \end{array} \right.$$

Fluctuation dissipation theorem

as it appears in the book of Heikkilä, equation (6.7)

$$S_U(\omega) = \hbar\omega \,\mathrm{Re}(Z) \left[\coth\left(\frac{\hbar\omega}{2kT} \right) + 1 \right]$$

$$S_{\delta I}(t, t') = \left\langle \delta\hat{I}(t) \delta\hat{I}(t') \right\rangle$$

the difference to before is, that the correlation function is now built from **current operators** which do not necessarily commute at different times.

This then results in an asymmetry, i.e. $S_{\delta I}(-\omega) \neq S_{\delta I}(\omega)$

$S_{\delta I}(\omega) \to 0$ for $\omega \to -\infty$ absorption noise

$S_{\delta I}(\omega) \to 2\hbar\omega \,\mathrm{Re}(Z)$ for $\omega \to +\infty$ emission noise

Beyond simple “classical” noise

classical noise power (spectral density): $$\widetilde{S}_{\delta I}(\omega) = 2\int_{-\infty}^{+\infty} e^{i\omega t}\left\langle \delta\, I(t)\;\; \delta\, I(0)\right\rangle dt$$

a modern theorist would however introduce: $$S_{\delta I}(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}\left\langle \delta\, \hat{I}(t)\;\; \delta\, \hat{I}(0)\right\rangle dt$$

or in it’s symmetrized form: $$\widetilde{S}_{\delta I}(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}\left\langle \left\{\; \delta\, \hat{I}(t)\;\; \delta\, \hat{I}(0)\right\}\;\right\rangle dt$$

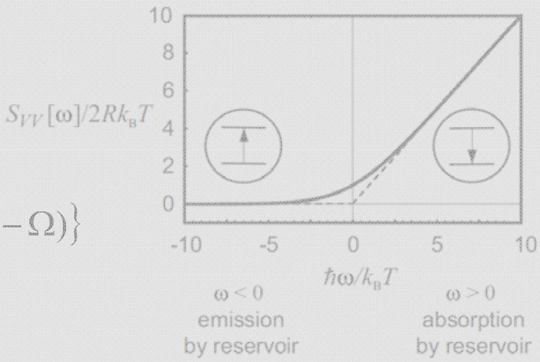
example: harmonic oscillator in 1d with frequ. Ω $$S_x(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}\left\langle \delta\, \hat{x}(t)\;\; \delta\, \hat{x}(0)\right\rangle dt$$

$$\left\langle \hat{x}(t)\hat{x}(0)\right\rangle = \left\langle \hat{x}(0)\hat{x}(0)\right\rangle \cos(\Omega t) + \left\langle \hat{p}(0)\hat{x}(0)\right\rangle \sin(\Omega t)$$

$$\left\langle \hat{x}(t)\hat{x}(0)\right\rangle = x_{ZPF}^2 \left\{\; n_B(\hbar\Omega) e^{i\Omega t} + \left[n_B(\Omega t) + 1\right] e^{-i\Omega t} \right\}$$

$$S_x(\omega) = 2\pi\;\, x_{ZPF}^2 \left\{ n_B(\hbar\Omega)\delta(\omega+\Omega) + \left[n_B(\hbar\Omega)+1\right]\delta(\omega-\Omega)\right\}$$

$x_{ZPF}^2 = \hbar / 2M\Omega$ zero-point fluctuation



Clerk, Devoret, Girvin, Marquart, Schoelkopf Rev. of Mod. Phys. 82, 1155 (2010)

Johnson-Nyquist Noise

$$S_U(0) = 4kTR$$

J. B. Johnson and H. Nyquist (1928)

In early times, this equation was used to **determine the absolute zero** by extrapolating the data points to zero noise.

By fixing the temperature scale at 0 and 100 °C to the Celsius scale, **Boltzmann‘s constant** could be determined as well.

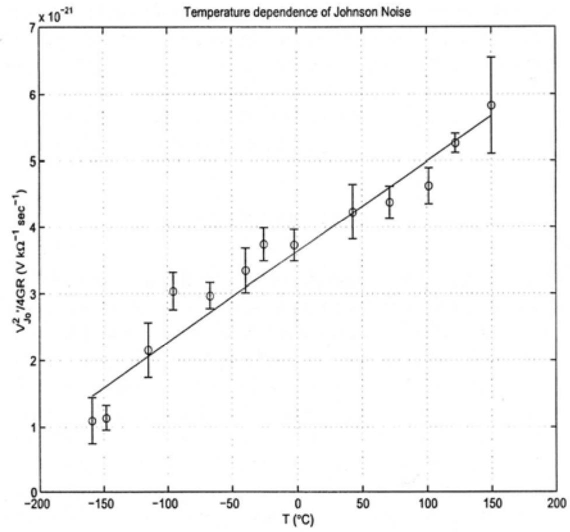
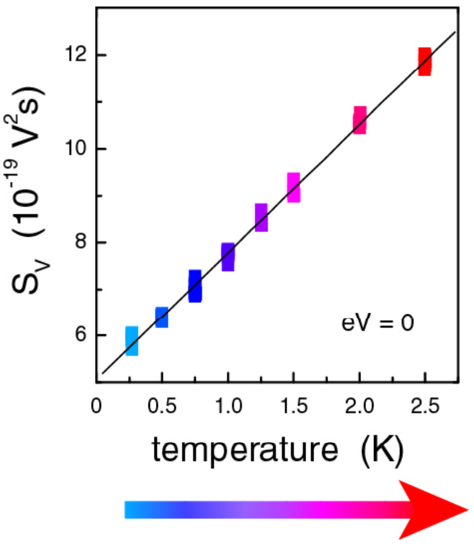
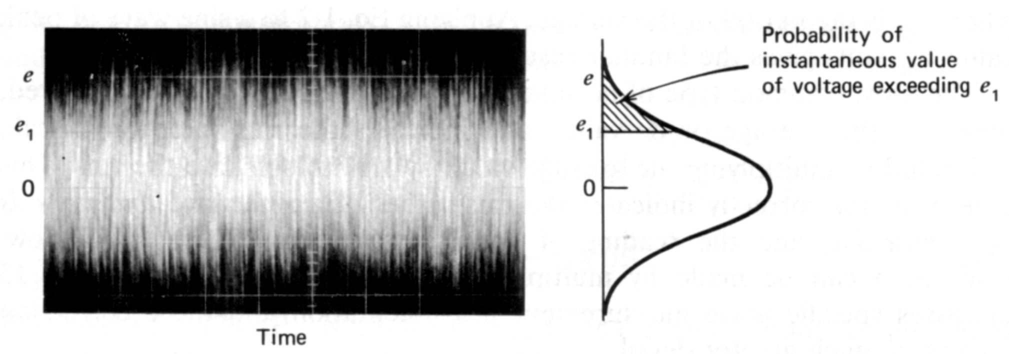


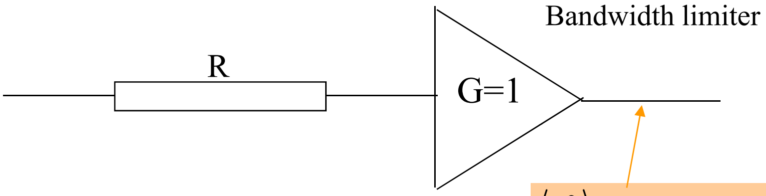
FIG. 10. Temperature dependence of Johnson Noise V'_{Jo}.



Thermal Noise



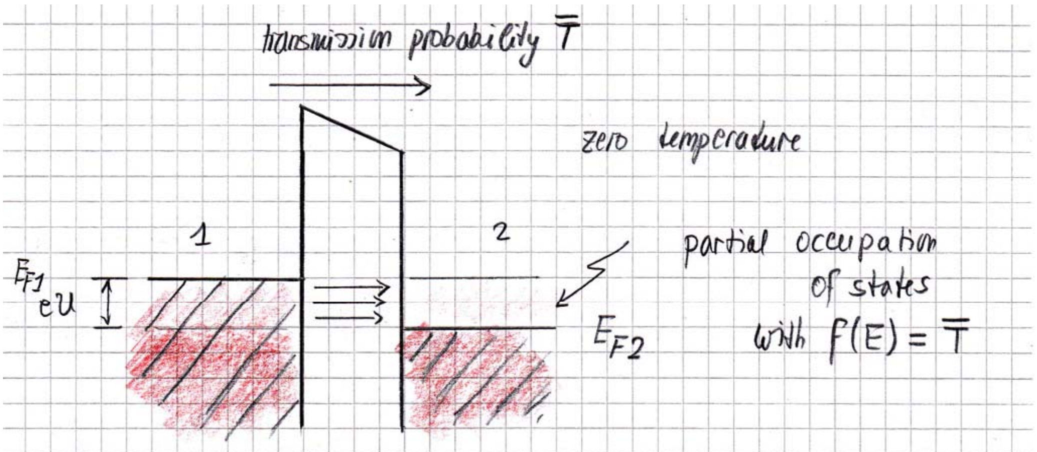
Thermal has a **normal distribution** of amplitude with time.



$$\left\langle v^2 \right\rangle_{tot} = 4kTR.BW_{noise}$$

Shot Noise

let us guess the result first by looking at a **one mode tunnel junction** under bias assuming that transmission eigenvalue T is small.

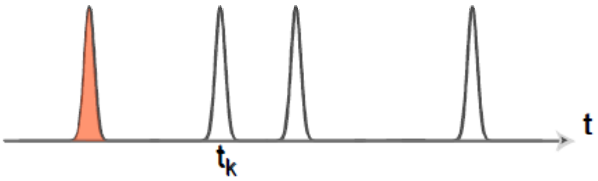


$$\left.\begin{array}{lll} f_1(E) & \rightarrow & 1-T \\ f_2(E) & \rightarrow & T \end{array}\right\} \quad E_{F2} \leq E \leq E_{F1} = E_{F2} + eU$$

$\Longrightarrow$ noise proportional to $\int dE\, f(1-f) = \underline{\underline{eT(1-T)U}}$

for small T : $\underline{\underline{S_I}} \propto TU \propto GU = \underline{\underline{I}}$ (Schottky)

Shot noise



The current is described by a random sequences of pulses, each pulse carrying charge e. Hence,

$$I(t)=\sum\nolimits_k e\delta(t-t_k)$$

The correlation function C$_I$(t) is a Dirac function, as there is no overlap for t≠0, i.e.

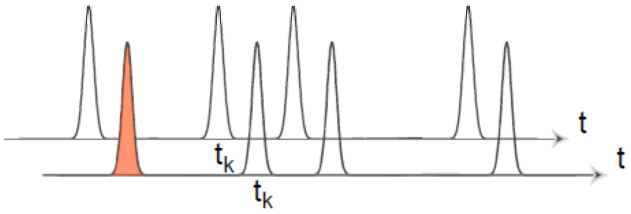
$$C_I(t)=2\langle I(t+\tau)I(t)\rangle_\tau = 2e\bar{I}\delta(t)$$

Since the noise (power spectral noise density) is the Fourier transform of the correlation function, one immediately obtains:

$$S_I(\omega)=\int_{-\infty}^{+\infty} dt\cdot e^{i\omega t}C_I(t)=2e\bar{I}$$

read chapter 6.1.4 in Heikkilä's book where a derivation of Schottky's formula is given

Shot Noise (2nd derivation)



read this as homework

The current is described by a random sequences of pulses, each pulse carrying charge e. At not too large frequenciy, each pulse occurring at $t = T_k$ can be described by a delta function:

$$i(t-T_k)=e\delta(t-T_k)$$

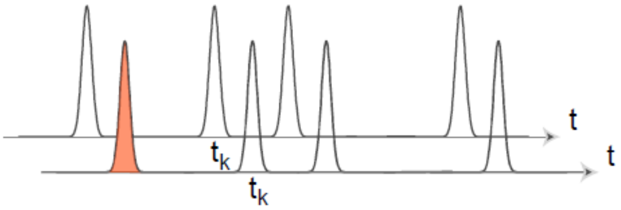
We are using the discrete Fourier series to express the pulse with T much larger than the pulse width. Then,

$$i(t)=\frac{e}{T}+\frac{2e}{T}\sum_0^\infty cos(2\pi nt/T)$$

The first part, the DC component, only adds to the mean current and can be omitted for the fluctuations. Hence,

$$\delta i(t-T_k)=\frac{2e}{T}\sum_0^\infty cos\left(\frac{2\pi n}{T}(t-T_k\right)$$

Shot Noise (2nd derivation)



read this as homework

At a given frequency $\omega_n := 2\pi n/T$, pulses at different instances T_k contribute with a $cos(\omega_n t - \phi_k)$ term with equal amplitude $2e/T$ and random phases ϕ_k. Hence, the intensities of each pulse can be added.
The intensity $\langle \delta i^2 \rangle_n$ per pulse and at frequency with index n is then given by:

$$\langle \delta i^2 \rangle_n = \frac{4e^2}{T^2}\frac{1}{2}$$

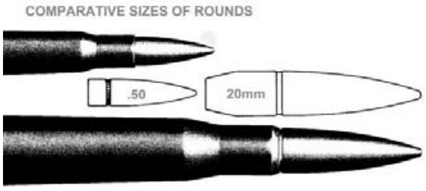
Let r be the number of pulses per unit time. Then, during time T, the total intensity (of all pulses) at frequency with index n amounts to:

$$\langle \delta i^2 \rangle_{n,T} = \frac{2e^2}{T^2} r = \frac{2eI}{T}, \text{ where } I \text{ is the mean current}$$

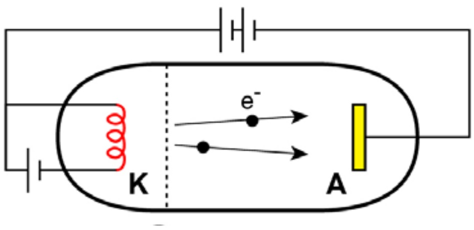
This has to be multiplied with the number of different frequency allowed by the input filter of the measurement setup. If this frequency span is Δf, then $\Delta n = \Delta f T$, so that we obtain as the final result:

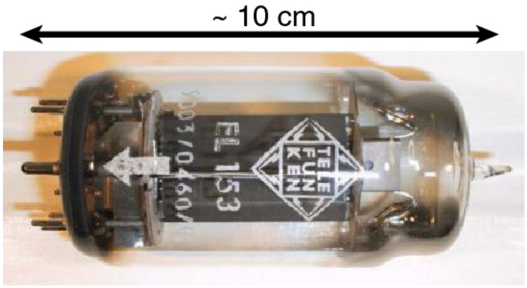
$$\langle \delta i^2 \rangle_{\Delta f} = 2eI\Delta f$$

Shot noise



electrons (quantized charge)
as bullets, i.e. as shots





$$\boxed{S_I = 2e|I|}$$

(Schottky, 1918)

confirmed by: Hull and Williams, Phys.Rev. 25, 147 (1925)

Experiment

Hull and Williams „Determination of Elementary Charge E", Phys. Rev. ***25****, 147 (1925)*

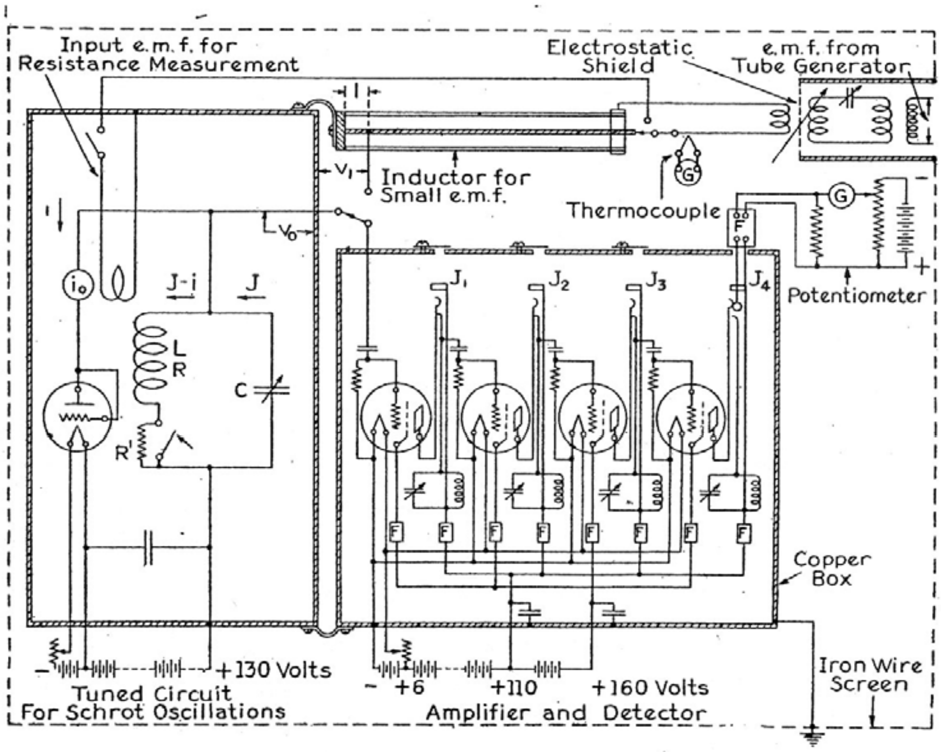
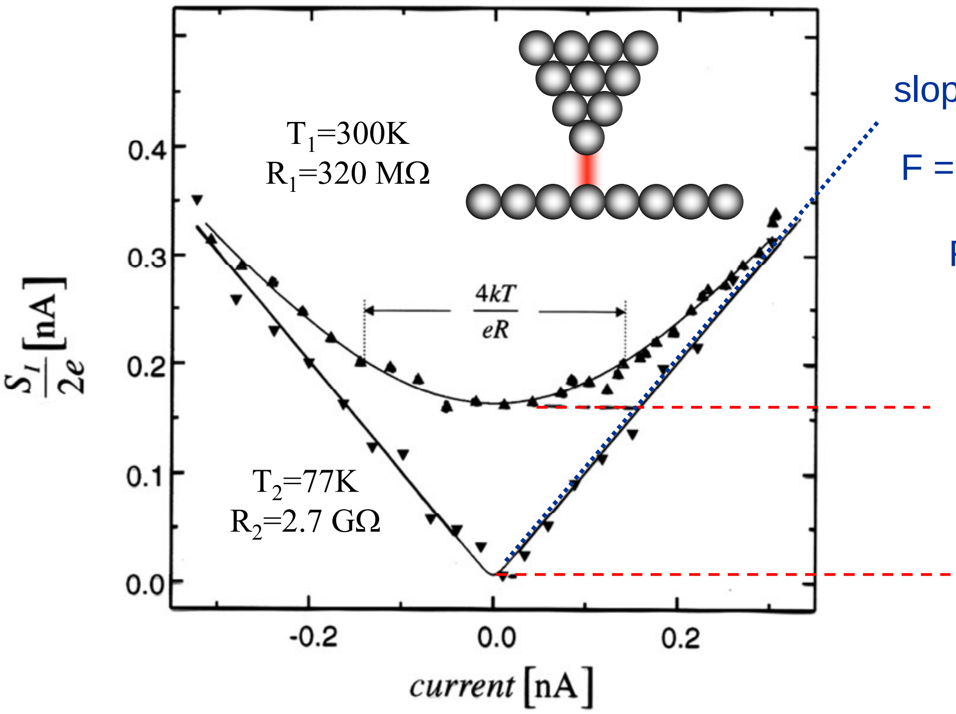


Fig. 2. Complete arrangement of apparatus for measuring shot-effect.

Tunnel junction / crossover

$$S_I(f) = 2eI\coth\left(\frac{eV}{2k_BT}\right)$$

interpolation between thermal noise and shot noise



slope = 1, i.e. F = 1

F = Fano factor

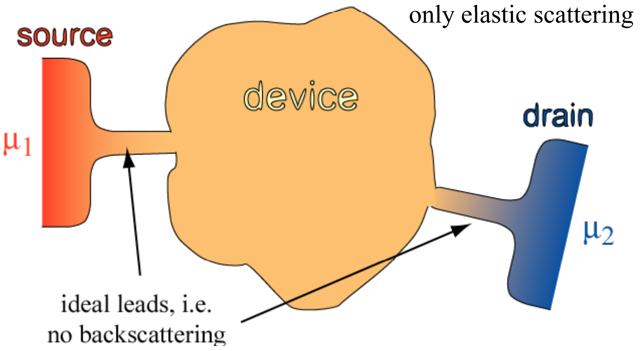
F := S/2eI

$4kT_1/R_1$

$4kT_2/R_2$

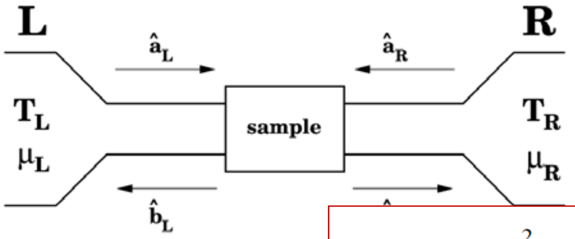
measurement with an STM tunnel junction: H. Birk et al., PRL 75, 1610 (1995)

Generalization Landauer-Büttiker theory



Noise is due to *randomness*. Hence, in a coherent conductor (scattering theory) there are therefore two contributions:

- random emission in lead reservoirs
- random transmission between reservoirs, i.e. from source to drain



reviews:
de Jong & Beenakker, Phys. Rev. B 46 13400 (96)
Blanter and Büttiker, Phys. Reports 336 (2000)

$$\hat{I}_{\mathrm{L}}(t) = \frac{e}{2\pi\hbar}\sum_{n}\int \mathrm{d}E\,\mathrm{d}E'\,\mathrm{e}^{\mathrm{i}(E-E')t/\hbar}[\hat{a}^{\dagger}_{\mathrm{L}}$$

$$\hat{I}_{\mathrm{L}}(t) = \frac{e}{2\pi\hbar}\sum_{\alpha\beta}\sum_{mn}\int \mathrm{d}E\,\mathrm{d}E'\,\mathrm{e}^{\mathrm{i}(E-E')t}$$

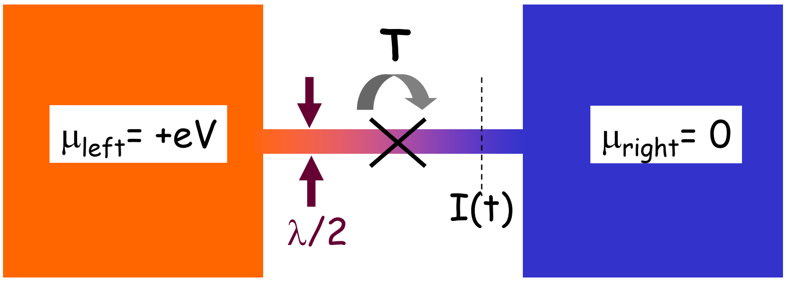
$$A^{mn}_{\alpha\beta}(L;E,E') = \delta_{mn}\delta_{\alpha\mathrm{L}}\delta_{\beta\mathrm{L}} - \sum_{k} s^{\dagger}_{\mathrm{L}\alpha;mk}(E)s_{\mathrm{L}\beta;kn}(E')$$

$$S_{\alpha\beta}(\omega) = \frac{e^2}{2\pi\hbar}\sum_{\gamma\delta}\sum_{mn}\int \mathrm{d}E\,A^{mn}_{\gamma\delta}(\alpha;$$

$$\times\{f_{\gamma}(E)[1 \mp f_{\delta}(E +$$

Generalization Landauer-Büttiker theory

„1 channel wire“ with transmission T



1. Contacts are degenerate (Fermi-Dirac statistics)
2. Transmission is binomial $p_{N,T}(n_t) = \binom{N}{n_t} T^{n_t}(1-T)^{N-n_t}$

→ *derivation as exercise*

$$S_I \propto \left\langle \Delta n_t^{\;2} \right\rangle \propto T(1-T)$$

$$\boxed{S_I(0) = (1-T)2eI}$$

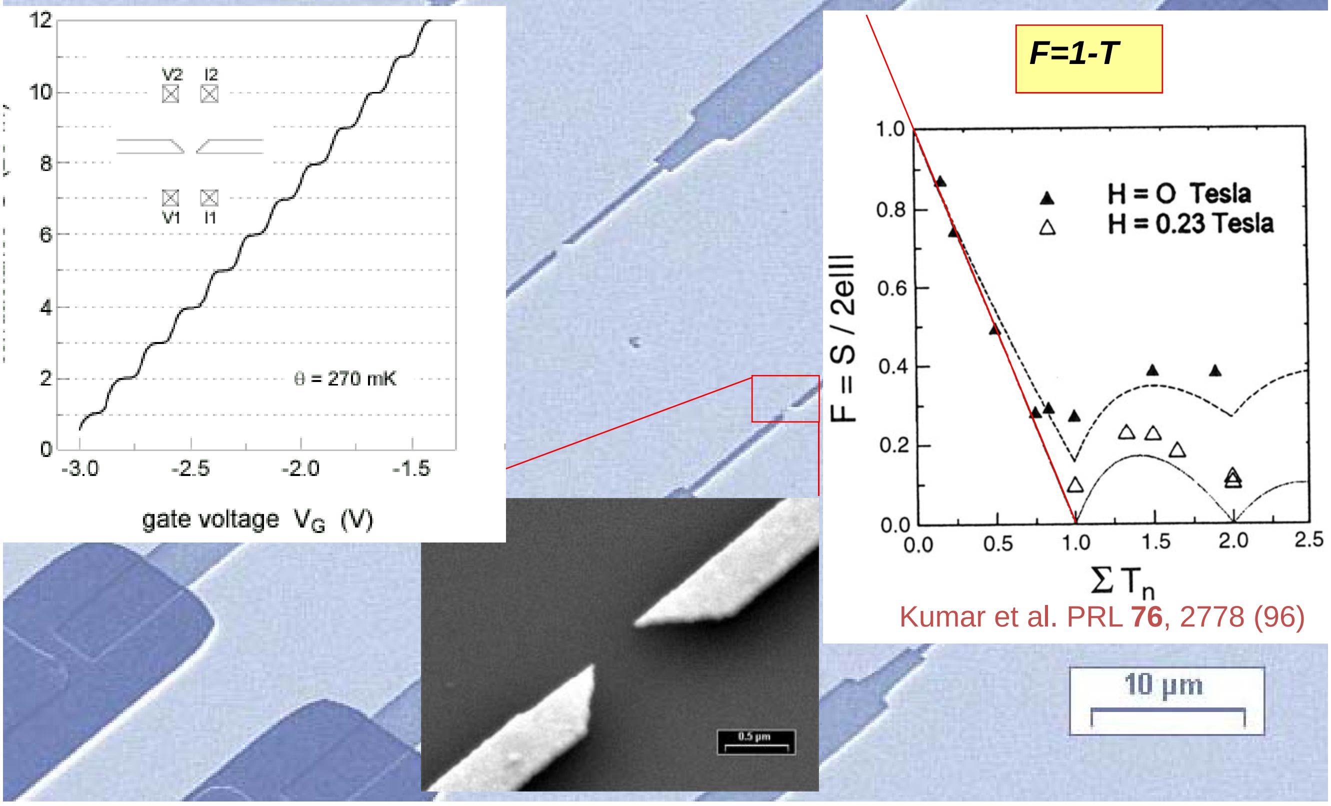
Fano factor S/2eI = (1-T) → „partitioning noise“

3. no „shots“ in a perfect wire (if T=1 → S=0)

Note: there is a direct relation between the **second moment** and **noise**. Higher moments are also of interest today, see chapter 6.7 in Heikkilä’s book on **“full counting statistics”**. 1st moment corresponds to current, 2nd moment to noise, 3rd moment measure skewness, 4th moment contains noise of noise ...

Examples: quantum-point contact

partitioning at a quantum-point contact

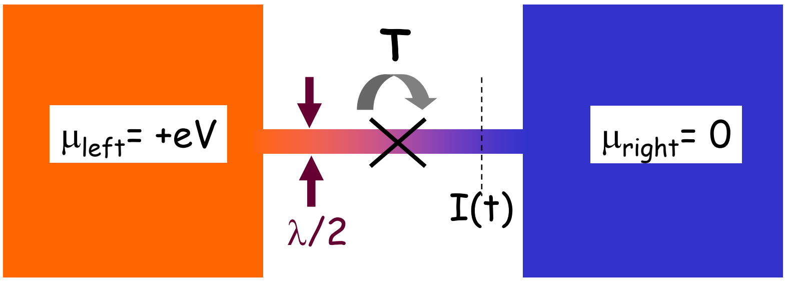


F=1-T

Kumar et al. PRL **76**, 2778 (96)

Generalization Landauer-Büttiker theory

„1 channel wire“ with transmission T



1. Contacts are degenerate (Fermi-Dirac statistics)

2. Transmission is binomial $p_{n,T}(k)=\binom{n}{k}T^k(1-T)^{n-k}$

→ derivation as exercise

$$S_I \propto \left\langle \Delta k^2 \right\rangle \propto T(1-T)$$

$$\boxed{S_I(0)=(1-T)2eI}$$

Fano factor S/2eI = (1-T) → „partitioning noise“

3. no „shots“ in a perfect wire (if T=1 ➔ S=0)

$$\boxed{S_I(0)=G_0eV\sum_k T_k(1-T_k)=G_0eV\int_0^1 p(T)T(1-T)}$$

problem is reduced to the determination of the **distribution of the transmission eigenvalues**

Generalization Landauer-Büttiker theory

Conductance is transmission

$$G = 4\frac{e^2}{h}\sum_{1}^{N} T_n$$

Quantum scattering is noisy

$$S_I = 2eI\,\frac{\sum T_n\,(1-T_n)}{\sum T_n} = 2eI \times \text{"Fano"}$$

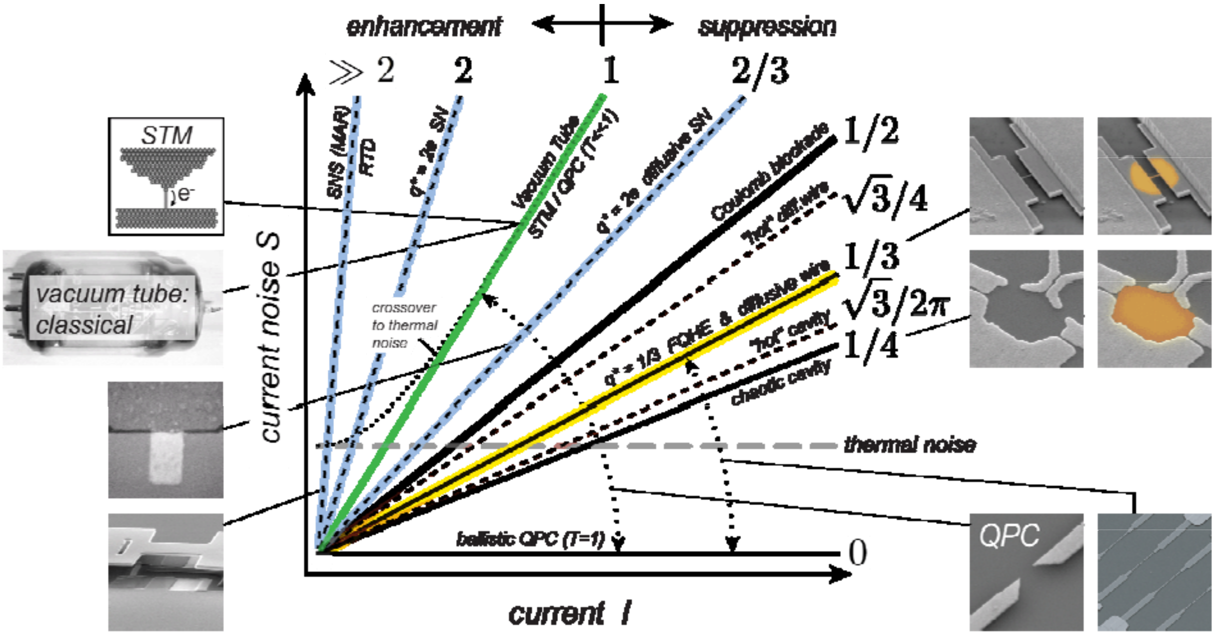
R. Landauer and M. Büttiker

Fano factor F<1 : a measure of noise intensity

Ya.M. Blanter, M. Büttiker / Physics Reports 336 (2000) 1-166

slide from Bernard Placais

Fano factors



Poissonian shot noise 2eI

1	*Tube*	*Hull and Williams, (1925)*
1	*STM*	*Birk et al. (1995)*

effective charge quantum q*

1/3	*FQHE*	*Saminadayar et al. (97)* *de Picciotto et al. (97)*
2/3	*diff. SN*	*Jehl et al. (00)*
2	*SN*	*Lefloch et al. (03)*
>>2	*SNS*	*Hoss et al. (00)*

modulation noise ~ (1-T)

0-1	*QPC*	*Kumar et al. (96)* *Reznikov et al. (95)*

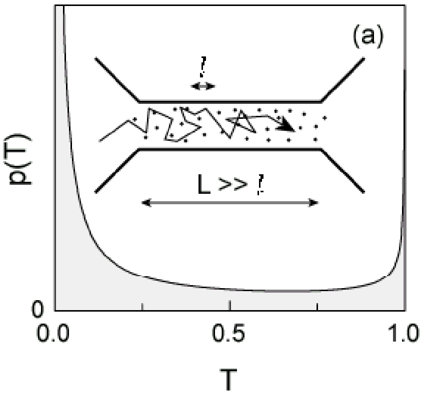
distribution of transmission eigenvalues P(T_n)

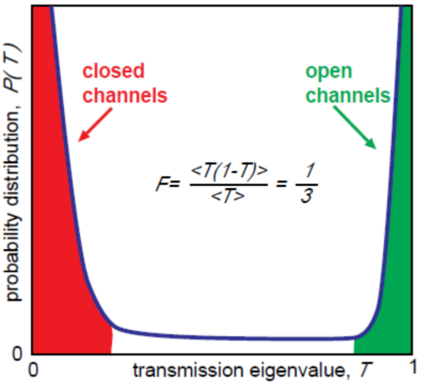
1/4	*cavity*	*Oberholzer et al. (01)*
1/3	*diff.wire*	*Steinbach et al. (96), Schoelkopf et al. (97),* *Henny et al. (99)*
1/2	*SET*	*Birk et al. (95)*
0-1/3	*QPC series*	*Oberholzer et al. (02)*

example: F=1/3rd in coherent, but diffusive wire

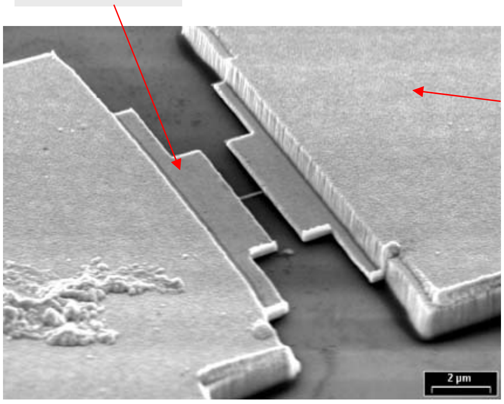
$$F = \frac{\sum T_i(1-T_i)}{\sum T_i}$$

F = 1/3

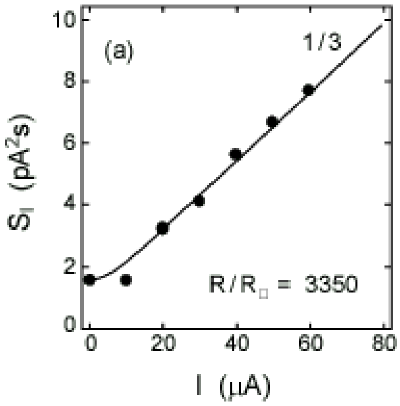




200nm Au



300nm Au
700nm Cu



Henny et al. PRB **59**, 2871 (1999)

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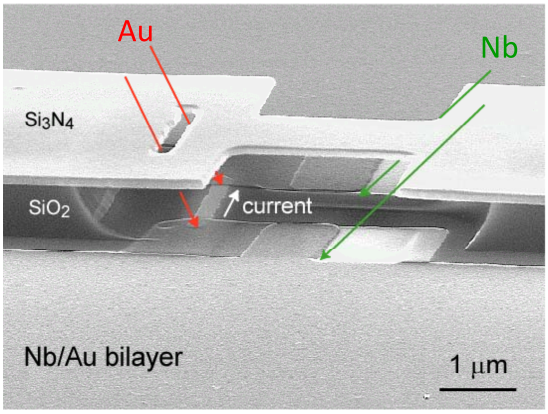
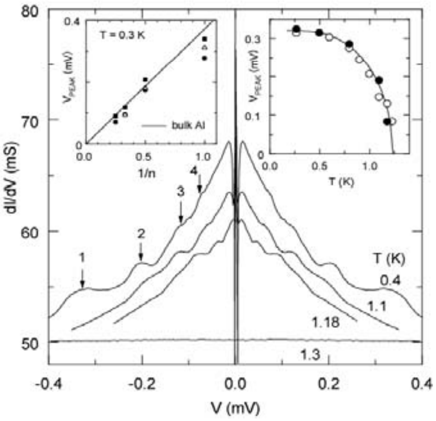
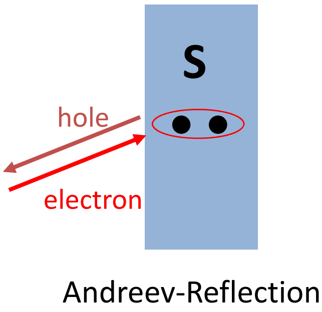
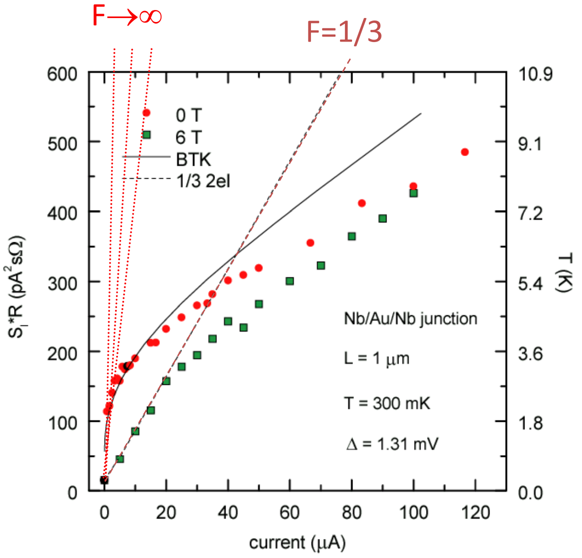
example: F > 1 (S-N-S junction)







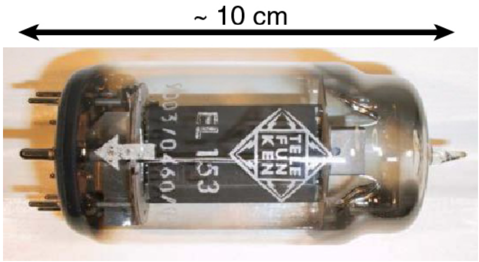
Andreev-Reflection

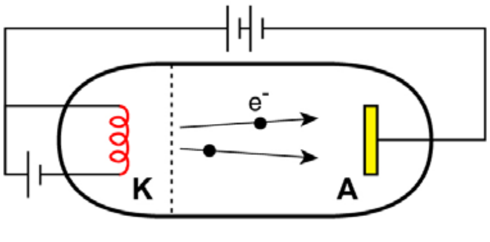


$$F \approx \frac{\Delta}{eV}$$

Hoss et al. PRB **62**, 4079 (2000)

Is noise classical or quantum ?



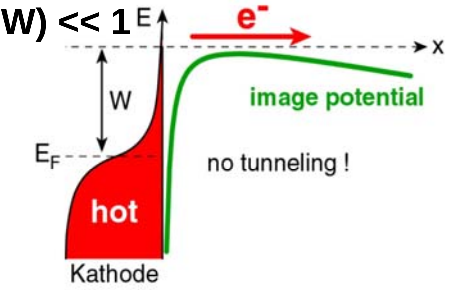


- random emission in cathode? **YES**
- random transmission from cathode to anode ? **NO**

T = 0 or 1

Emission noise due to Boltzmann tail (classical)

f(E_F+W) << 1

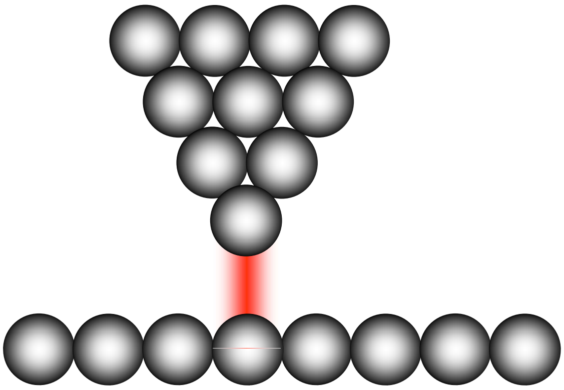


T(E>E_F+W) = 1
T(E<E_F+W) = 0

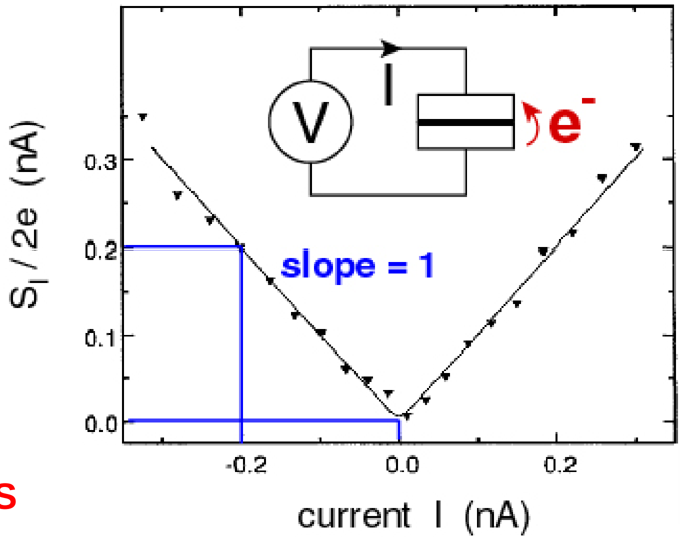
diffraction (tunneling) can be neglected

(Schönenberger et al. cond-mat/0112504)

Is noise classical or quantum ?



*Birk, de Jong and Schönenberger, PRL **75**, 1610 (1995)*



- **random emission in source (tip) ?** **NO**
- **random transmission from tip to substrate ?** **YES**

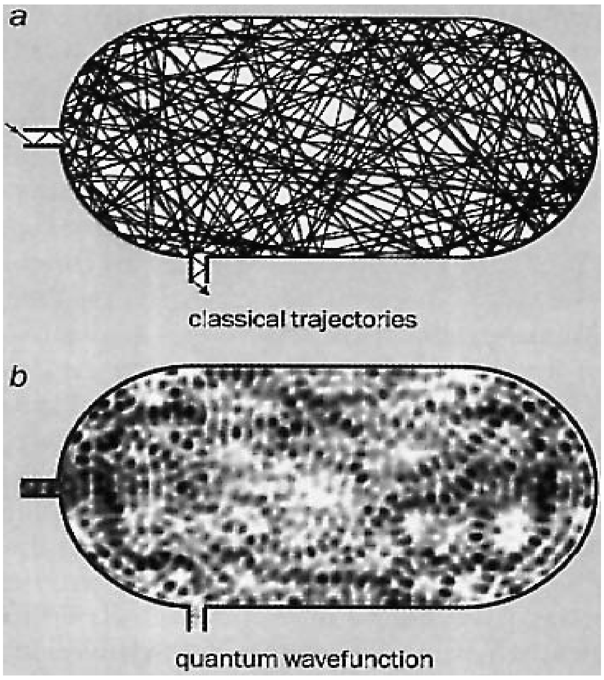
0 < T << 1

$$S_I = 2e|I|$$

(Schottky, 1918)

(Schönenberger et al. cond-mat/0112504)

distinguish waves from particles



chaotic cavity

deterministic scattering
➔ **NO NOISE**

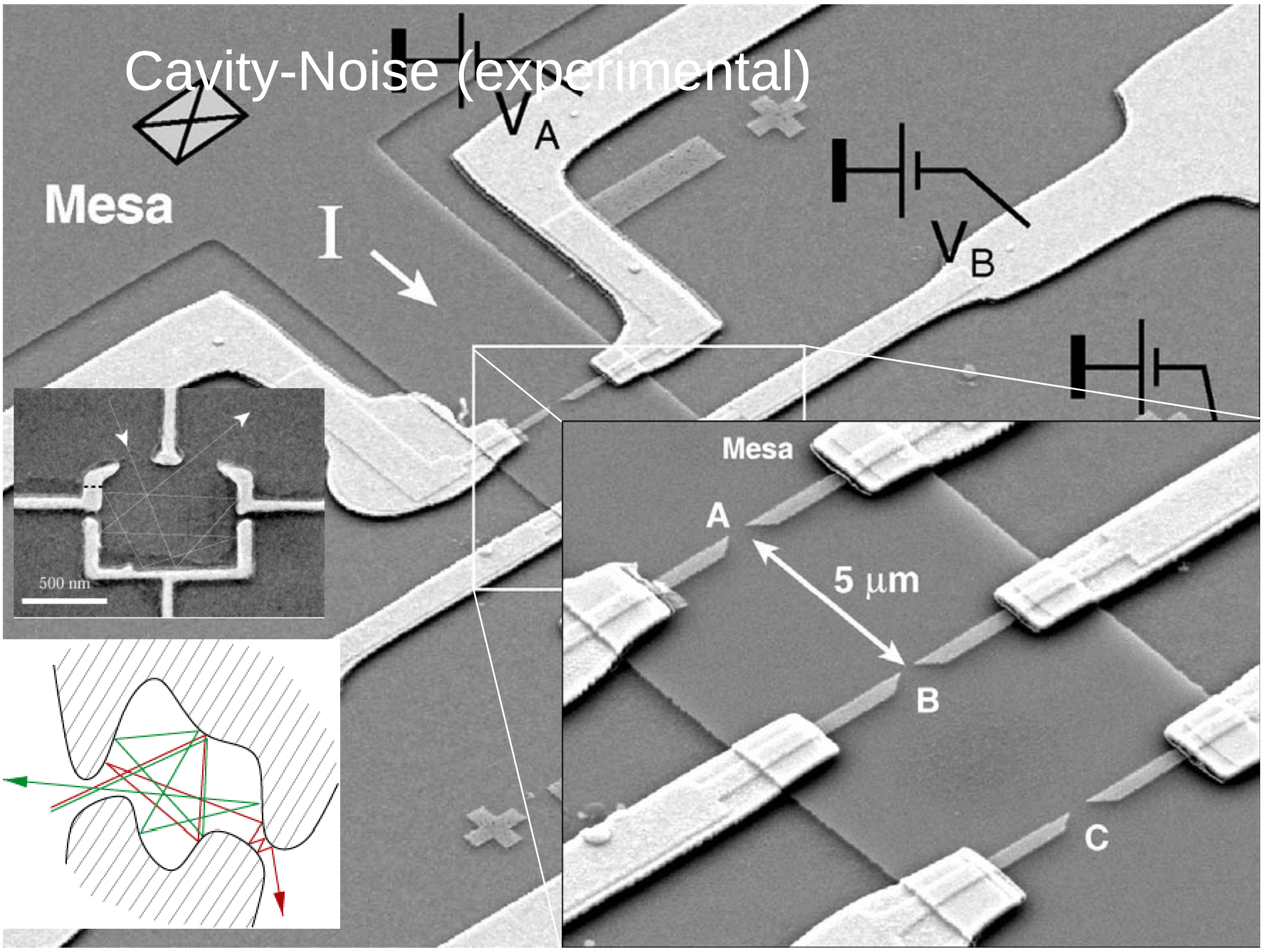
classical: $t_Q > t_D$

stochastic scattering
➔ **NOISE**

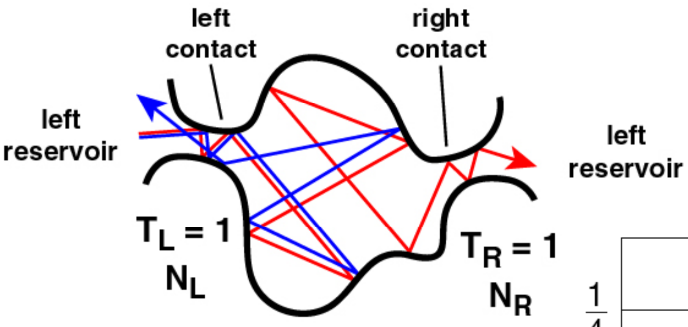
quantum: $t_Q < t_D$

dwell time τ_D **quantum scattering time τ_Q**

Cavity-Noise (experimental)



Quantum to classical crossover



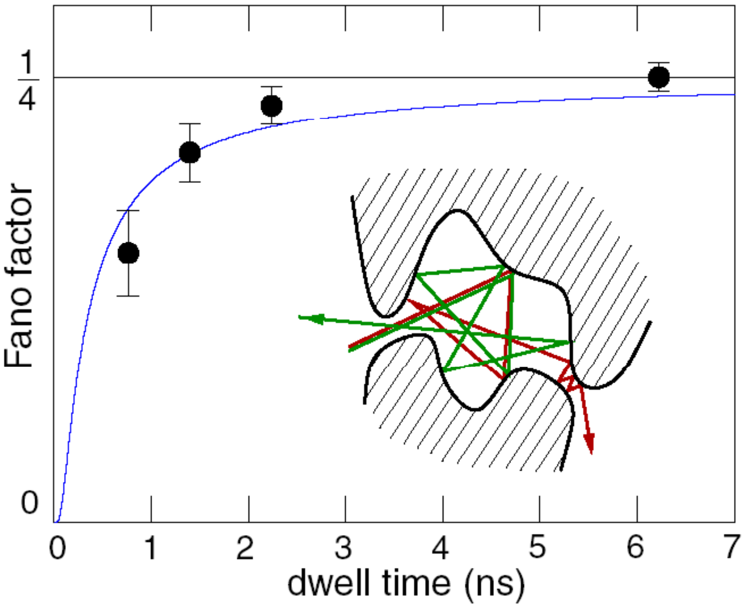
Fano factor F

$$S_I = F \cdot 2e|I|$$

$$F = \frac{1}{4}\exp(-\tau_E/\tau_{\text{dwell}})$$

$$\tau_E = \lambda_L^{-1}\ln(L/\lambda_F)$$

Oberholzer et al. Nature 415, 765 (2002)



Summarize 2-terminal noise formula

→ *derivation on blackboard*

$$S_I = \frac{4e^2}{h}\sum_n \int dE\left[T_n^{\,2}\{f_l(1-f_l)+f_r(1-f_r)\}+T_n(1-T_n)f_l(1-f_r)+T_n(1-T_n)f_r(1-f_l)\right]$$

$$\boxed{S_I = 2\frac{e^2}{h}\left[2k_B\theta\sum T_n^{\,2} + eV\coth\frac{eV}{2k_B\theta}\sum T_n(1-T_n)\right]}$$

see also Heikkilä equation (6.21)

$$\boxed{S_I = 2eI\,\frac{\sum T_n(1-T_n)}{\sum T_n} = 2eI \times \text{"Fano"}}$$

at zero temperature

Boltzmann-Langevin approach (for a wire)

$$S_I(\omega << \omega^*) = \frac{4G_N}{L}\int_0^L dx\left\{\int \bar{f}(E,x)\left(1-\bar{f}(E,x)\right)dE\right\}$$

similar equations can be derived for other geometries

here,
- f is the time averaged distribution function in the diffusive limit
- diffusion is assumed to be one-dimensional with x the coordinate along the wire
- G_N is the conductance of the wire

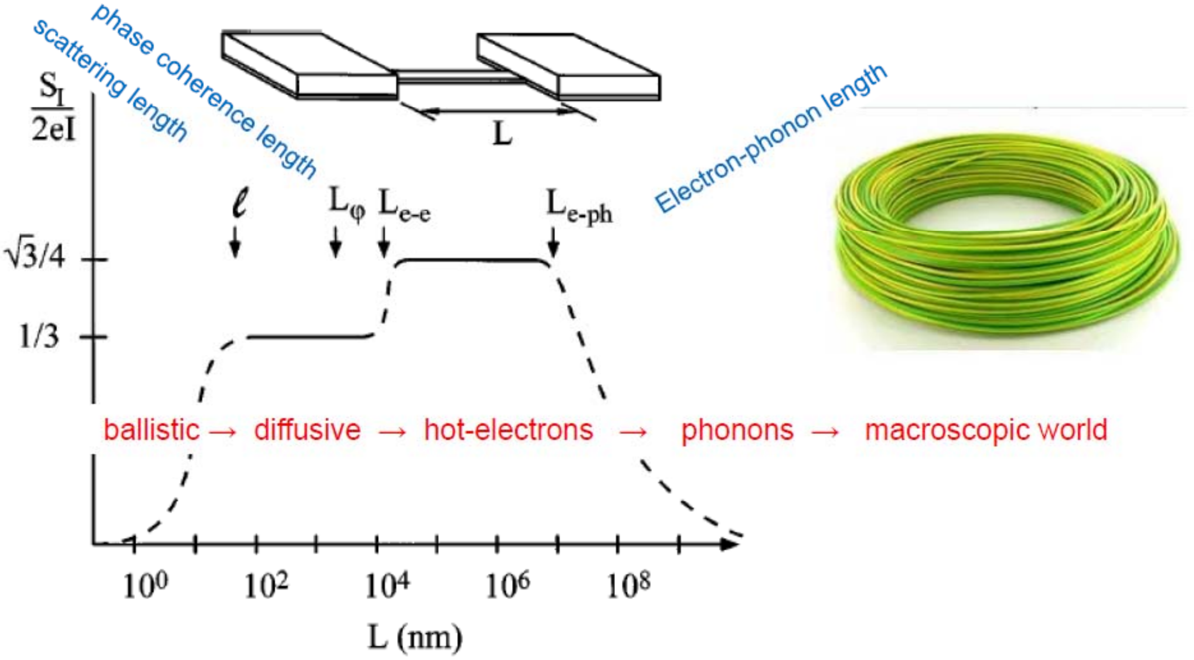
a (relatively) straightforward derivation is given in exercise (6.10) of Heikkilä’s book; it is based on the addition of (uncorrelated) noise

---

Boltzmann-Langevin approach (for a wire)

… on increasing the sample length

Fano factor



Boltzmann-Langevin approach (for a wire)

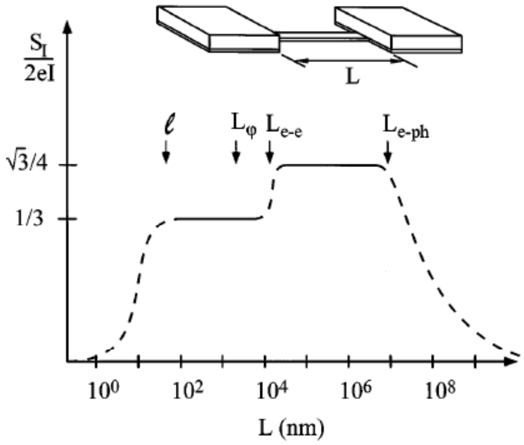
Observation of Hot-Electron Shot Noise in a Metallic Resistor

Andrew H. Steinbach and John M. Martinis
National Institute of Standards and Technology, Boulder, Colorado 80303

Michel H. Devoret
Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique Saclay, F-91191 Gif-sur-Yvette Cedex, France
(Received 12 January 1996)

Phys. Rev. Lett. 76, p3806 (1996)

We have measured the current noise of silver thin-film resistors as a function of current and temperature and for resistor lengths of 7000, 100, 30, and 1 μm. As the resistor becomes shorter than the electron-phonon interaction length, the current noise for large current increases from a nearly current independent value to the interacting hot-electron value $(\sqrt{3}/4)2eI$. However, further reduction in length below the electron-electron interaction length decreases the noise to a value approaching the independent hot-electron value $(1/3)2eI$ first predicted for mesoscopic resistors. [S0031-9007(96)00179-2]



$F = \sqrt{3/4}$

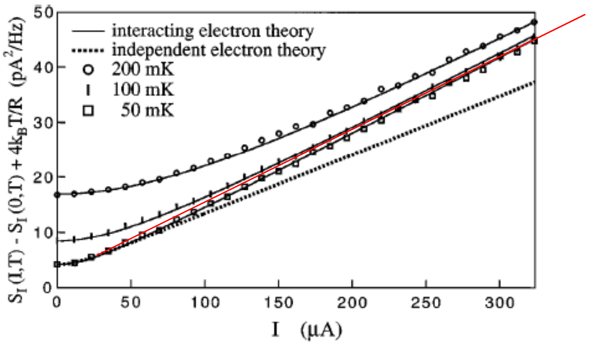


FIG. 3. Variation of $S_I(I,T)$ vs I for the $L = 30$ μm sample at $T = 50$, 100, and 200 mK. Sample resistance $R = 0.68$ Ω. Data show good agreement with the interacting hot-electron theory (solid curves) with no adjustable parameters. The independent hot-electron theory for 50 mK (dotted curve) is also plotted and shows poor agreement.

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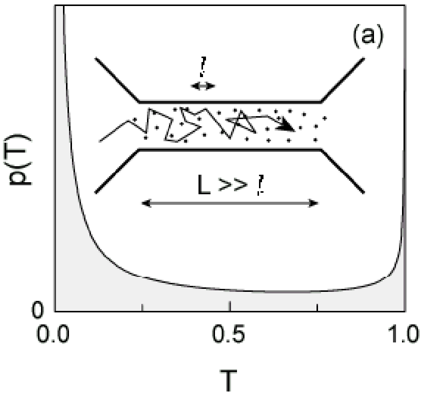
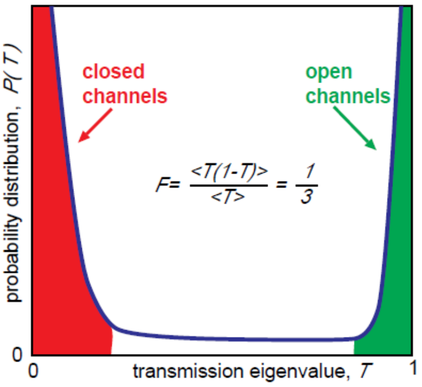
example: F=1/3rd in coherent, but diffusive wire

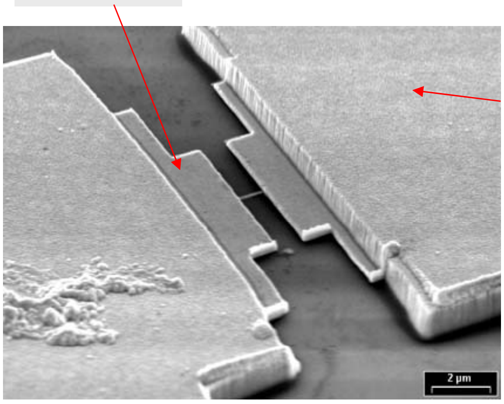
$$F = \frac{\sum T_i(1-T_i)}{\sum T_i}$$

F = 1/3

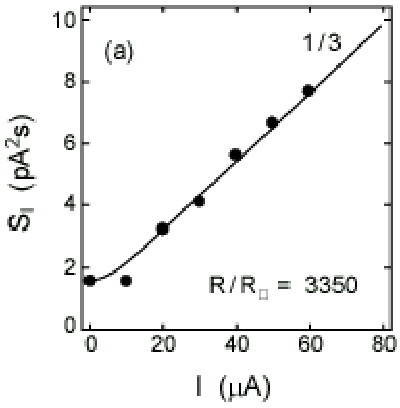




200nm Au

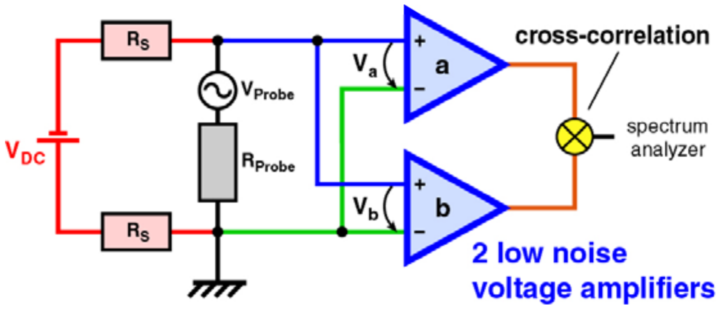


300nm Au
700nm Cu



Henny et al. PRB **59**, 2871 (1999)

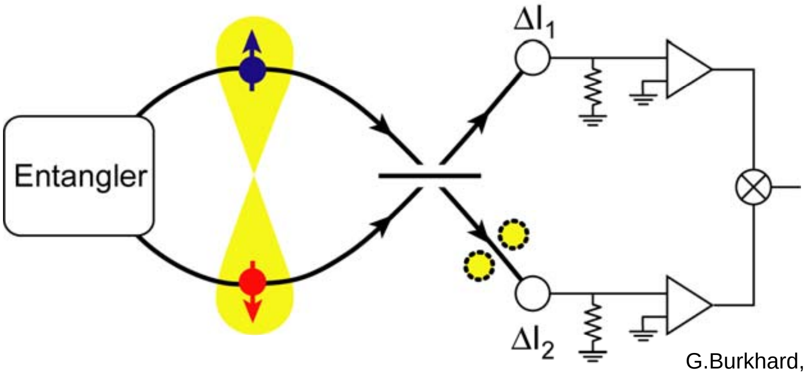
Multiterminal Correlation Spectroscopy



correlation measurement can be already useful even for two terminals

spin entanglement detected via orbital (charge) degrees of freedom

probing symmetry of orbital wave-function by current noise measurements

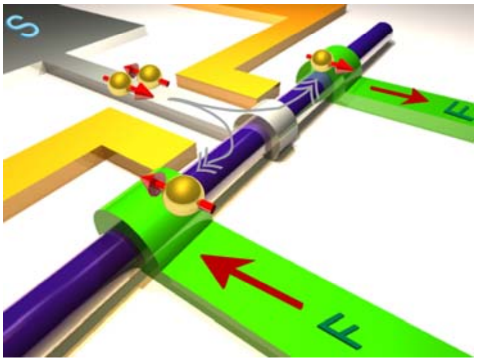


G.Burkhard, D.Loss,E.V.Sukhorukov, PRB 61, R16303 (2000)

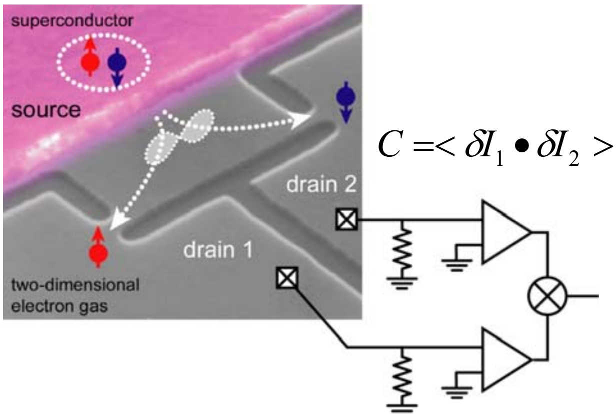
P. Recher, E. V. Sukhorukov, and D. Loss, PRB 63, 165314 (2001).

P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, PRB 70, 115330 (2004)

Cooper-Pair Entangler



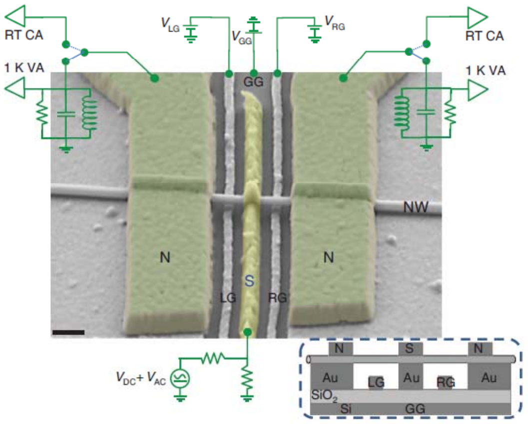
P. Recher, E. V. Sukhorukov, and D. Loss, PRB 63, 165314 (2001).

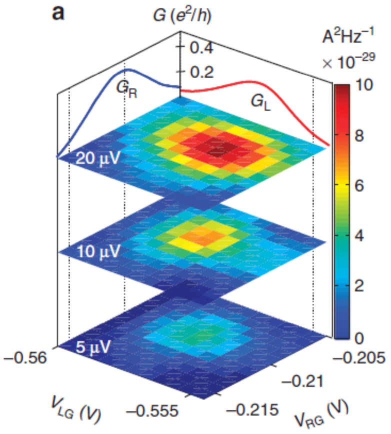


$$C =< \delta I_1 \bullet \delta I_2 >$$

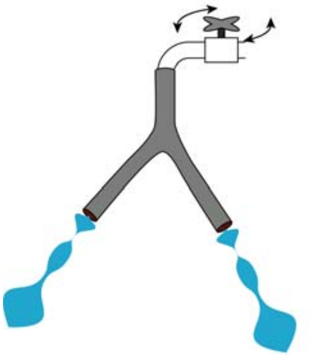
splitting efficiency by positive correlations

A. Das et al. Nature Com. 2012

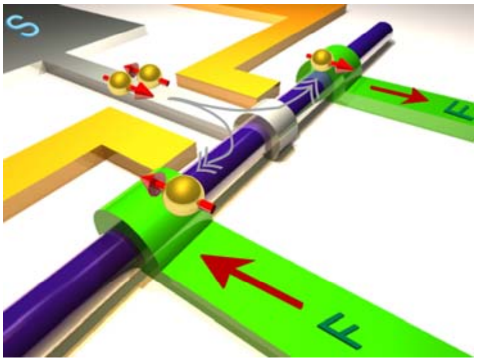




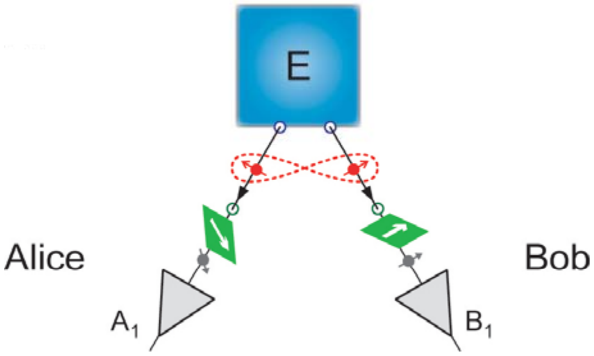
large fluctuations
(positive correlations)



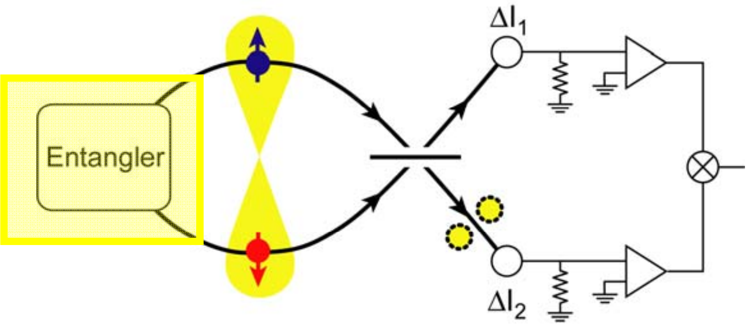
Motivation: Entanglement probes



P. Recher, E. V. Sukhorukov, and D. Loss, PRB 63, 165314 (2001).



Bell-test: need measurement results from Alice and Bob obtained in coincidence for different settings of polarization in detectors
→ **inequality based on noise**, such as S_{12}(a,b)



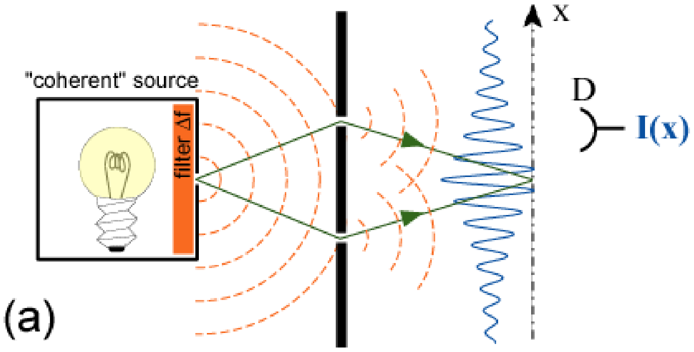
noise-correlation experiment
in this case even with two sources

beam-splitter type experiment

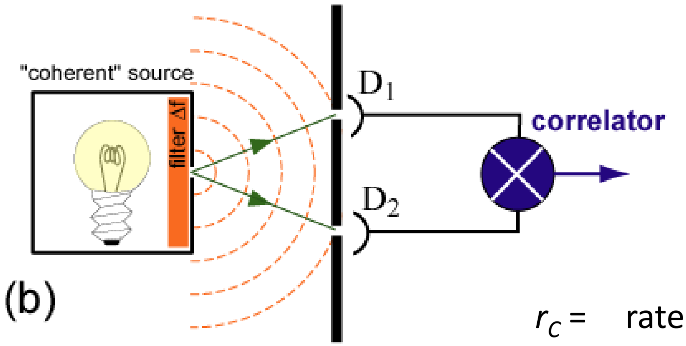
G.Burkhard, D.Loss,E.V.Sukhorukov, PRB 61, R16303 (2000)

P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, PRB 70, 115330 (2004)

History: Hanbury Brown & Twiss (HBT)



Young‘s double slit experiment
measure the g_1 function
(amplitude-amplitude correlation)



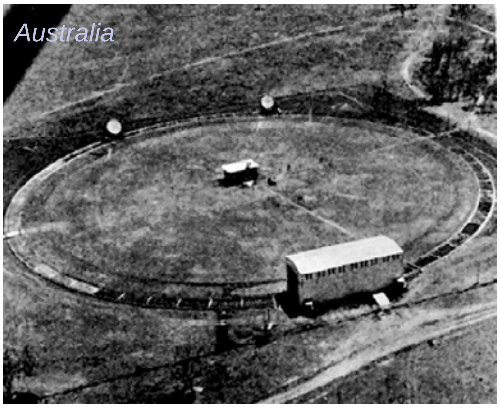
HBT measures the g_2 function
(intensity-intensity correlation)

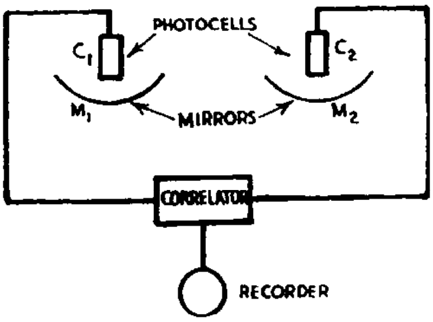
$$C = \frac{\langle \Delta I_{D1} \cdot \Delta I_{D2} \rangle}{\langle I_{D1} \rangle \cdot \langle I_{D2} \rangle}$$

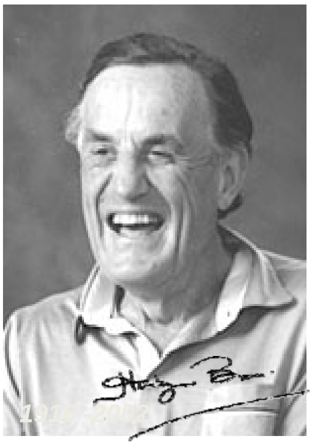
r_C = rate of coincidence
(normalized by $<I_1>\cdot<I_2>$)

$$\mathbf{C := r_C - 1}$$

HBT interferometer





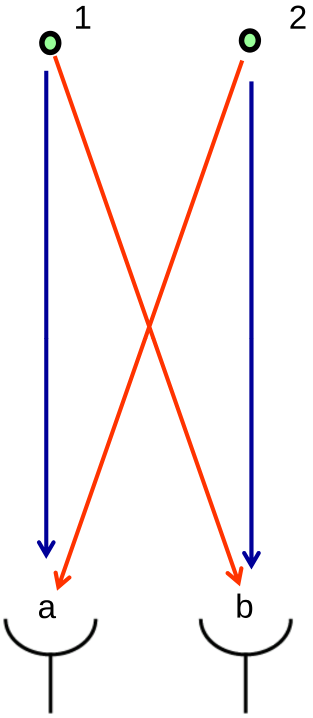


➔ although the phases are dropped, there is a sensible signal !

Radio stars: R. Hanbury Brown and R. Q. Twiss, "A New Type of Interferometer for Use in Radio Astronomy", *Philosophical Magazine* (7) **45** p663 (1954)

*Optical:*R. Hanbury Brown and R. Q. Twiss, "A Test of a New Type of Stellar Interferometer on Sirius", *Nature* **178** p1046 (1956)

HBT interferometer (sign of signal)



intensity correlation for a **two-particle** wavefunction

$\langle a\,|\,1\rangle =$ probability amplitude that particle 1 ends in detector a

probability P to detect two particles simultaneously in the two detectors (rate of coincidence)

classical

$$P=\left|\langle a\,|\,1\rangle\right|^2\left|\langle b\,|\,2\rangle\right|^2+\left|\langle b\,|\,1\rangle\right|^2\left|\langle a\,|\,2\rangle\right|^2=2p^2$$

C=0

Bosons

$$P=\left|\langle a\,|\,1\rangle\cdot\langle b\,|\,2\rangle+\langle a\,|\,2\rangle\cdot\langle b\,|\,1\rangle\right|^2=4p^2$$

C=1 (>0)

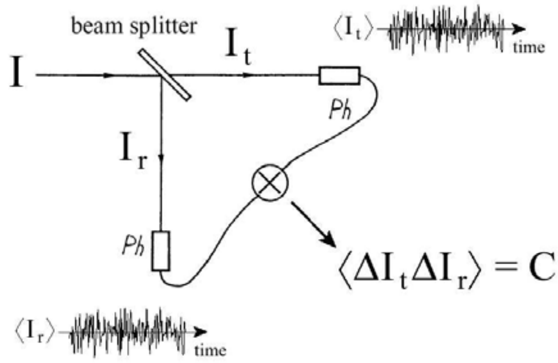
Fermions

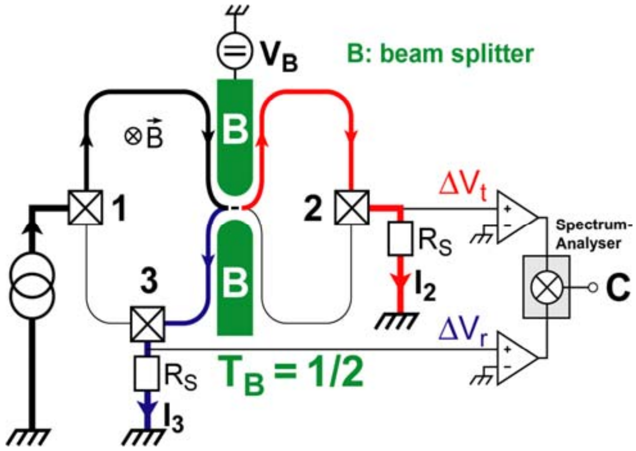
$$P=\left|\langle a\,|\,1\rangle\cdot\langle b\,|\,2\rangle-\langle a\,|\,2\rangle\cdot\langle b\,|\,1\rangle\right|^2=0$$

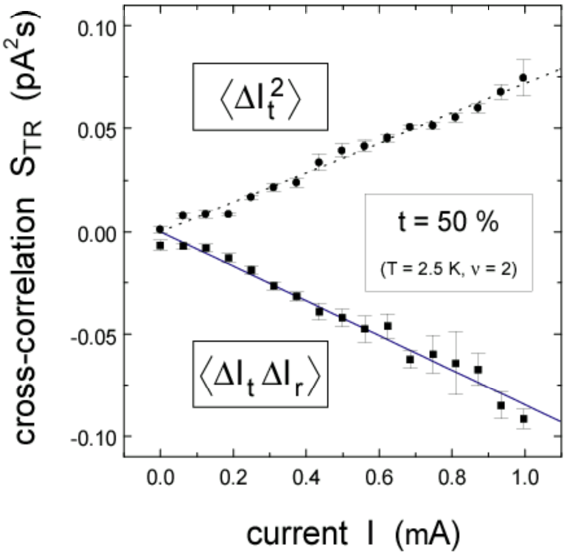
C=-1 (<0)

$$\left(if\,,\,p=\left|\langle a\,|\,1\rangle\right|^2=\left|\langle a\,|\,2\rangle\right|^2=\left|\langle b\,|\,1\rangle\right|^2=\left|\langle b\,|\,2\rangle\right|^2\right)$$

Antibunching of fermions

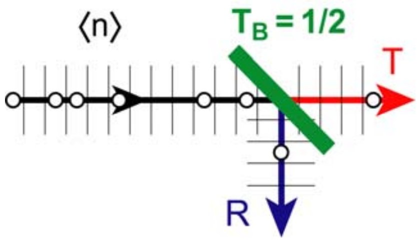






M. Henny et al. Science **284**, 296 (1999)

HBT: dependence on statistics

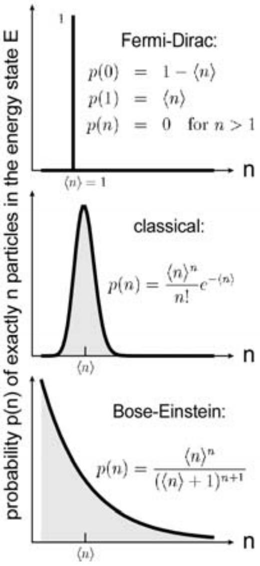


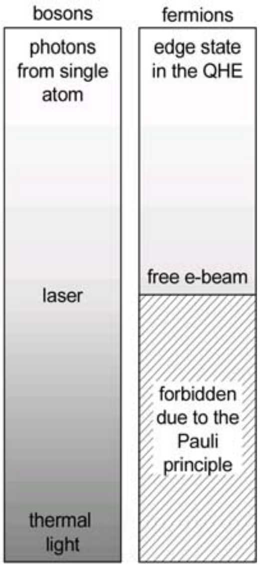
$$C = \left\langle \Delta n_T \Delta n_R \right\rangle \propto \left\langle \Delta n^2 \right\rangle - \left\langle n \right\rangle$$

fluctuations in the incoming beam

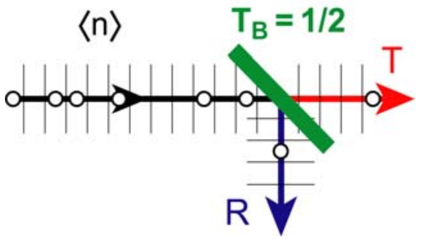
particle nature

sub-Poissonian	***C < 0***	$\left\langle \Delta n^2 \right\rangle < \left\langle n \right\rangle$
Poissonian	***C = 0***	$\left\langle \Delta n^2 \right\rangle = \left\langle n \right\rangle$
super-Poissonian	***C > 0***	$\left\langle \Delta n^2 \right\rangle > \left\langle n \right\rangle$





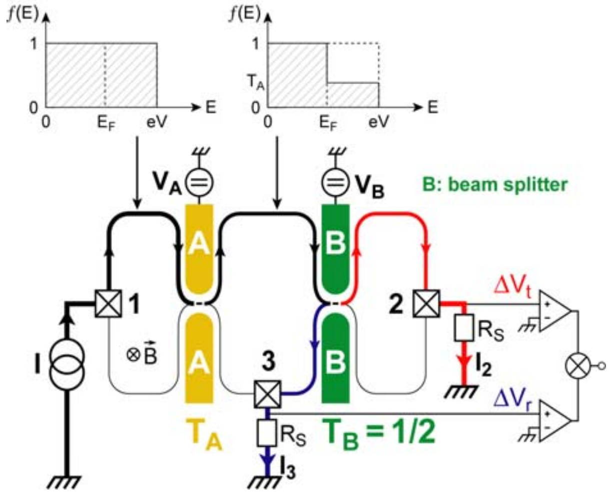
HBT: dependence on statistics

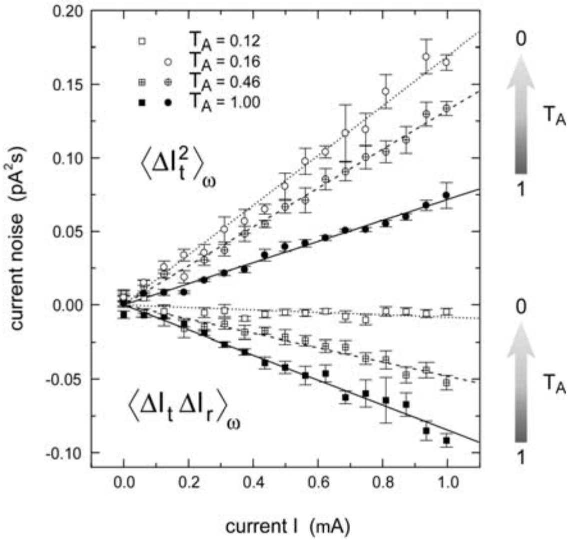


$$C = \left\langle \Delta n_T \Delta n_R \right\rangle \propto \left\langle \Delta n^2 \right\rangle - \left\langle n \right\rangle$$

fluctuations in the incoming beam

particle nature

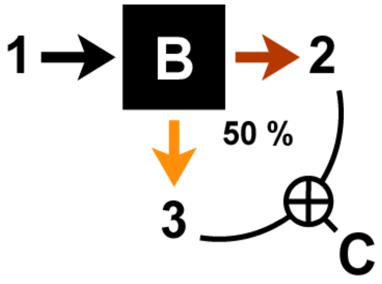




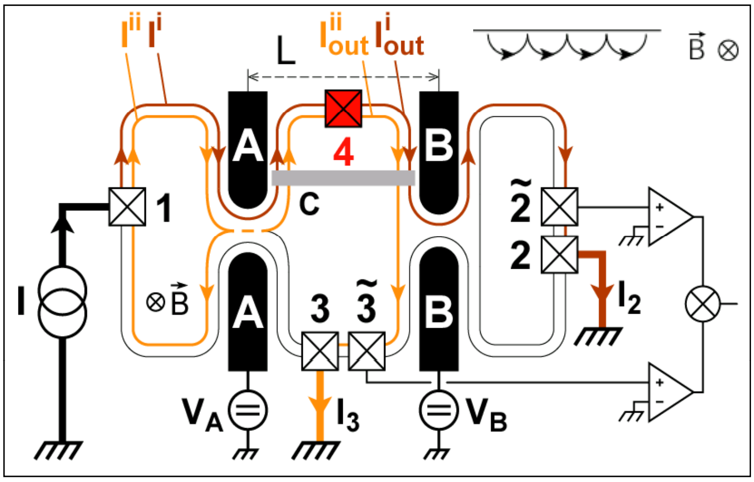
S. Oberholzer *et al.*, Physica E **6**, 314 (2000)

Note: pos. correlations may also be classical

"black box"



assume incoming noise current in lead 1. If "black box" equilibrates the fluctuations it will appear on both 2 and 3, and 2 and 3 are positive correlated!



- *partitioning at A creates noise in the second edge-state (ii):*

$$\left\langle \left(\Delta I^{ii}\right)^2 \right\rangle_\omega = 2G_0 T_A^{ii}\left(1-T_A^{ii}\right)\mu_1$$

- *equilibration in contact 4:*

$$\Delta I^i_{out} = \Delta I^{ii}_{out} = \Delta I^{ii} / 2$$

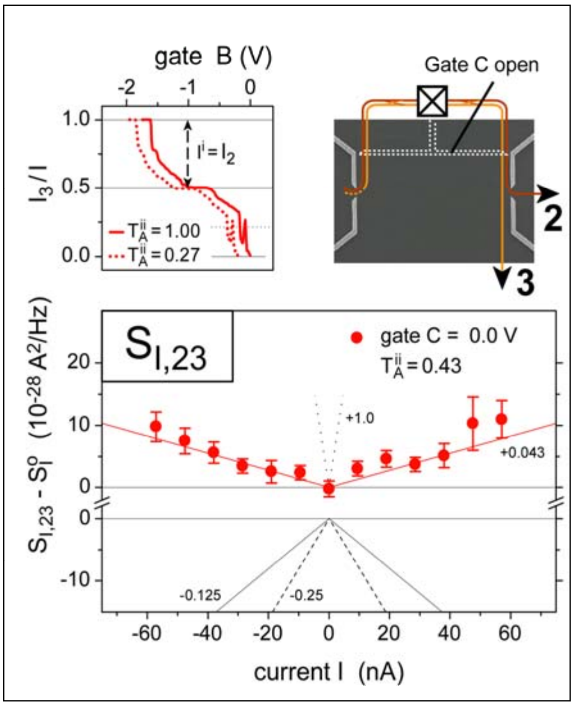
- *gate B separates the two edge-states into different contacts:*

$$\frac{\left\langle \Delta I_2 \Delta I_3 \right\rangle}{2e|I|} = +\frac{1}{4}\frac{T_A^{ii}(1-T_A^{ii})}{1+T_A^{ii}}$$

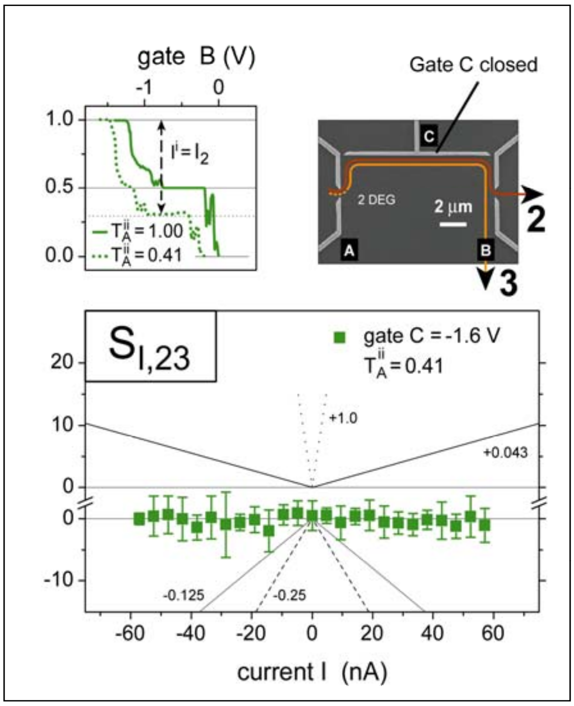
< 1 / 24 !!

Note: correlations may also be classical

equilibration:



no equilibration:



Positive Cross Correlations in a Normal-Conducting Fermionic Beam Splitter, S. Oberholzer, E. Bieri, C. Schönenberger, M. Giovannini, and J. Faist, Phys. Rev. Lett. **96**, 046804 (2006)

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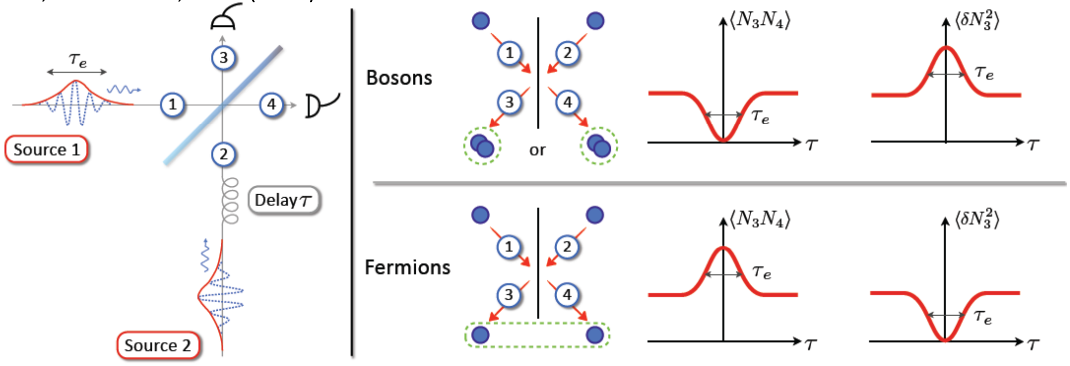
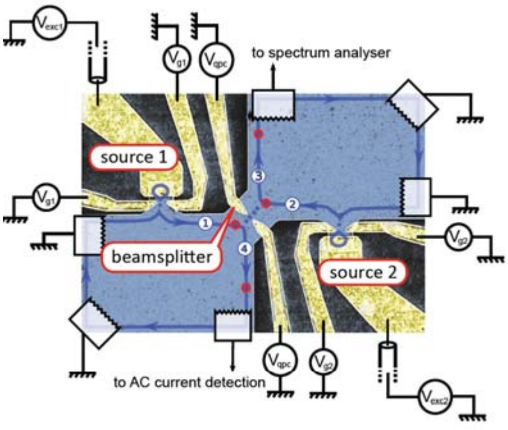
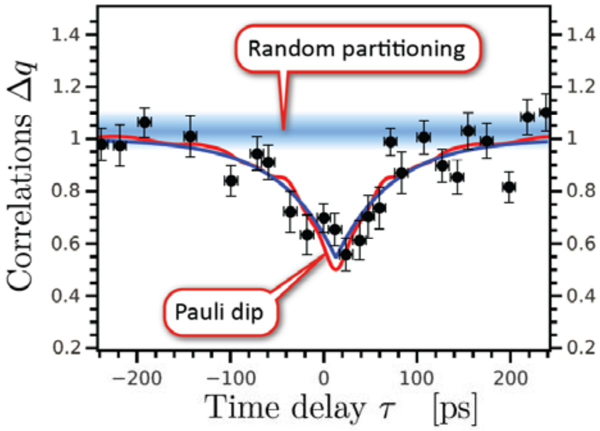
today: even two beam correlations

E. Bocquillon, Science 339, 6123 (2013)







other exp.: Liu, Odom, Yamamoto & Tarucha, Nature (98); Neder et al. Nature (2007); Glattli et al. (2014)

Extension to high frequencies

“Quantum” noise

classical: $kT >> \hbar\omega$ → $S_{\delta I}(-\omega) = S_{\delta I}(\omega) = 2kTG$

quantum: $kT << \hbar\omega$ → $S_{\delta I}(-\omega) \neq S_{\delta I}(\omega) = G\hbar\omega$ (corresponds to zero-point fluctuations)

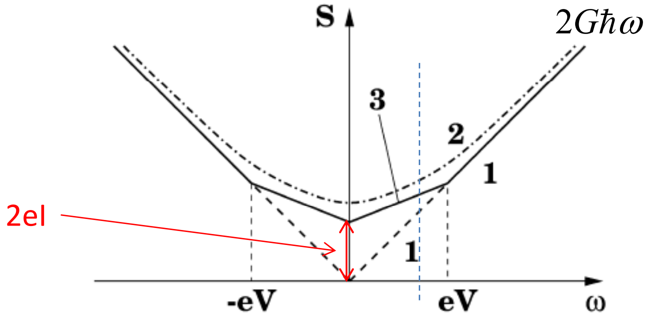
$$\widetilde{S}_{\delta I}(\omega) = \frac{e^2}{2\pi\hbar}\left\{2\hbar\omega\coth\left(\frac{\hbar\omega}{2k_B T}\right)\sum_n T_n^2 + \left[(\hbar\omega + eV)\coth\left(\frac{\hbar\omega + eV}{2k_B T}\right) + (\hbar\omega - eV)\coth\left(\frac{\hbar\omega - eV}{2k_B T}\right)\right]\sum_n T_n(1-T_n)\right\} .$$

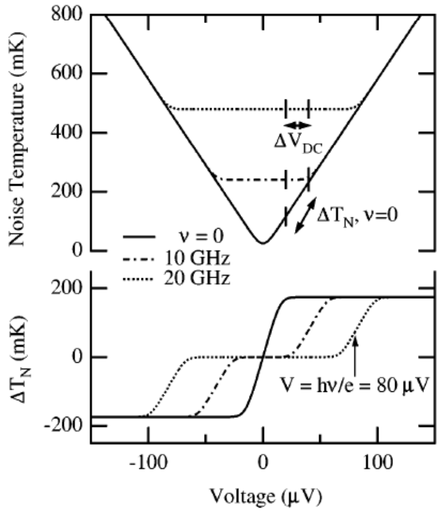
Büttiker 1992

(note: this is the symmetrized form)

equilibrium (V=0): $\widetilde{S}_{\delta I}(\omega) = 2G\hbar\omega\coth\left(\frac{\hbar\omega}{2kT}\right)$

non-equilibrium (T =0):





“Quantum” noise

VOLUME 78, NUMBER 17 PHYSICAL REVIEW LETTERS 28 APRIL 1997

Frequency Dependence of Shot Noise in a Diffusive Mesoscopic Conductor

R. J. Schoelkopf, P. J. Burke, A. A. Kozhevnikov, and D. E. Prober
Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06520 8284

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(Received 22 November 1996)

Detailed measurements of the voltage, temperature, and frequency dependence of the nonequilibrium current fluctuations for a diffusive mesoscopic conductor are reported. The data confirm predictions that a mesoscopic conductor shorter than the electron-electron inelastic length will display shot noise. Furthermore, the low temperatures (100 mK) and high frequencies (1–20 GHz) used for the measurements allow tests in the high-frequency regime (i.e., $h\nu \gg eV$ and kT) of the shot noise, which clearly show the influence of vacuum fluctuations. The quantum noise causes a high-frequency “cutoff” in the shot noise, *i.e.*, the noise is independent of bias voltage for frequencies $\nu >$
[S0031-9007(97)02944-X]

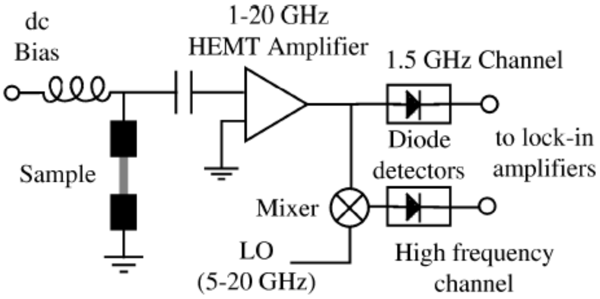
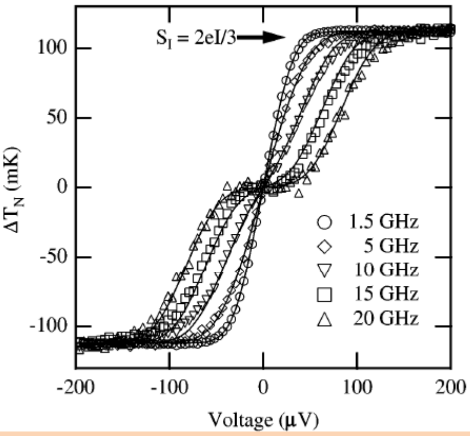


FIG. 2. Schematic of noise measurement apparatus.

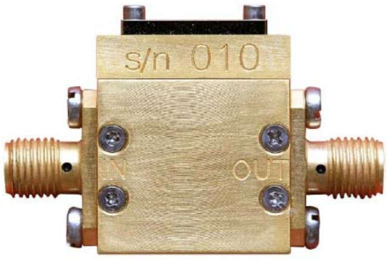
excess noise



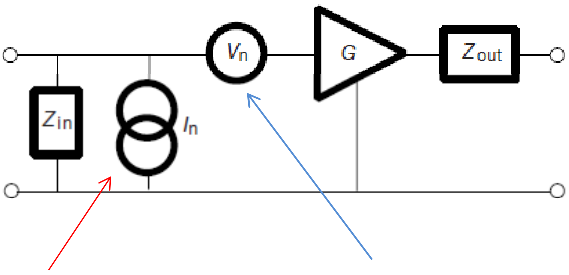
it may sound great to be „quantum“, but it is nothing but equilibrium noise yielding not more information than conductance. Hence, we need to work in a regime where ω is not too large

Measurement techniques

linear amplifiers



taken from Devoret & Schoelkopf, Nature 406, 1043 (2000)



„backaction“ noise

noise added by the amplifier referred to the input

usually one would assume that the two noise source are uncorrelated, but in front-end devices they are not

noise temperature T_N=6 K

$$A = kT_N / \hbar\omega \approx 25$$ at 5 GHz

noise impedance: $Z_N = \sqrt{S_V / S_I}$

best performance in terms of signal to noise if effective source impedance equals Z_N, called **noise matching**

noise energy: $E_N = \sqrt{S_V S_I}$

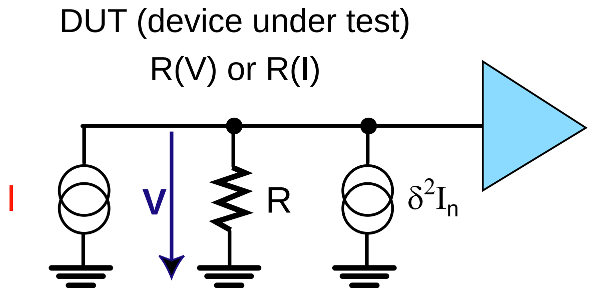
„Heisenberg limit“: $E_N \geq \dfrac{\hbar\omega}{2}$

today‘s SET (rf-SET) and SQUIDs (rf-SQUID) operating ~ 1Ghz reach $< 5\hbar\omega$

parametric amplifiers can reach $< 1\hbar\omega$ (at 5GHz or above)

Measurement techniques: **current source**

a) use a (DC) current bias ➔ I is fixed and known



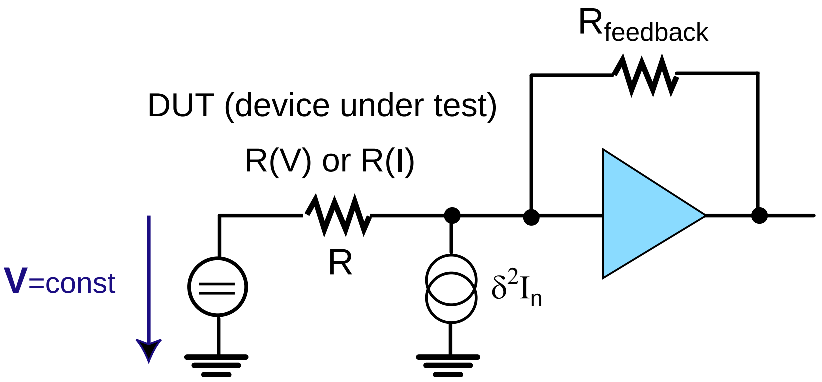
b) amplifiers yields V(I) and therefore also dV/dI (I)

c) in terms of noise, we measure: $\delta^2 V_n = \delta^2 I_n \left(\frac{\partial V}{\partial I}\right)^2$

d) to get the result, need to **divide** by **differential resistance.**

Measurement techniques: **voltage source**

a) use a (DC) voltage bias ➔ V is now fixed and known



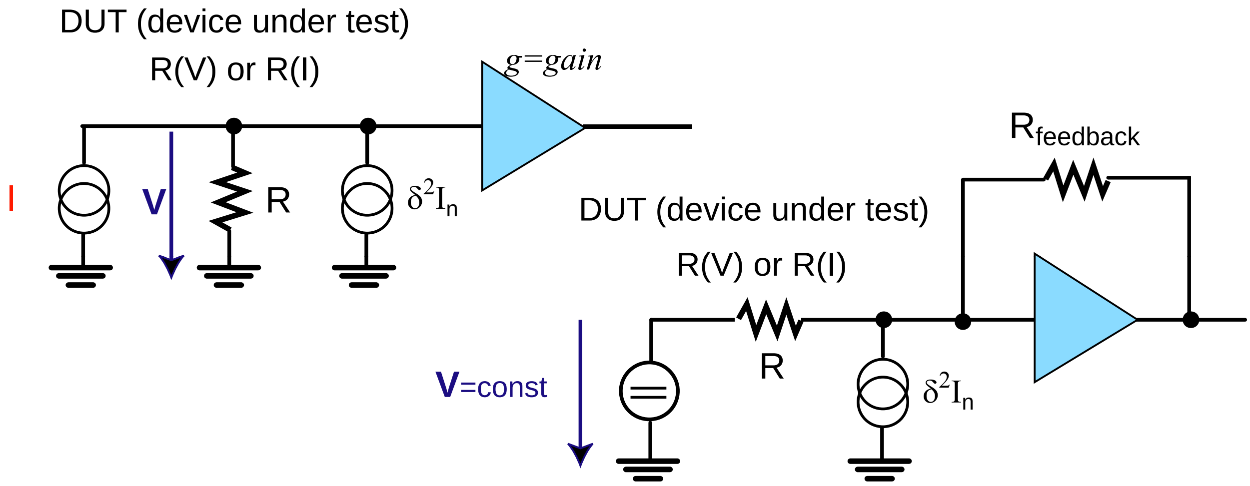
b) amplifiers yields an output proportional to I and therefore also dI/dV

c) in terms of noise, we measure: $\delta^2 V_{output} = \delta^2 I_n R_{feedback}$

d) ... hence, we actually get S_I(**V**) and need to convert to S_I(**I**)

note, in both cases do we need to consider the non-linearities.

voltage or current amplifier ?



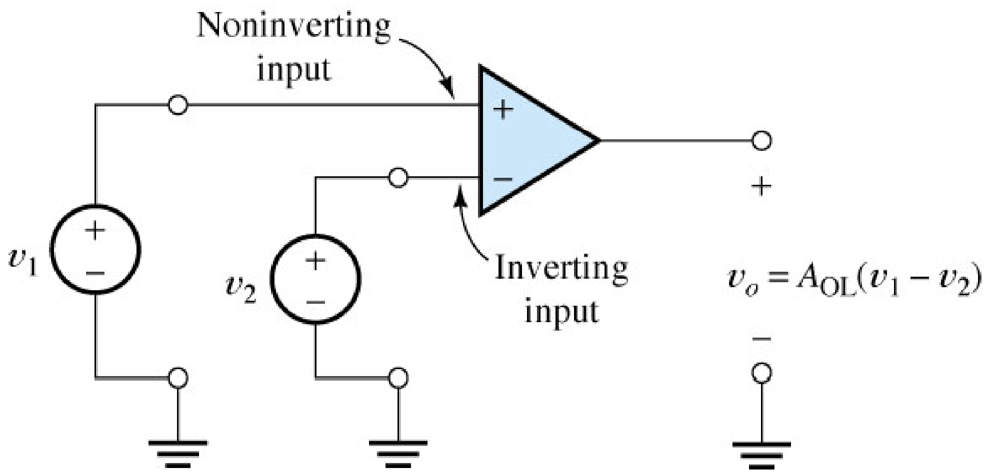
A) ➔ voltage (pre-) amplifier

B) ➔ current voltage converter

what is better ?

Analog Electronics is based on Op-Amp‘s

Operational Amplifiers take small voltages and make them MUCH larger.

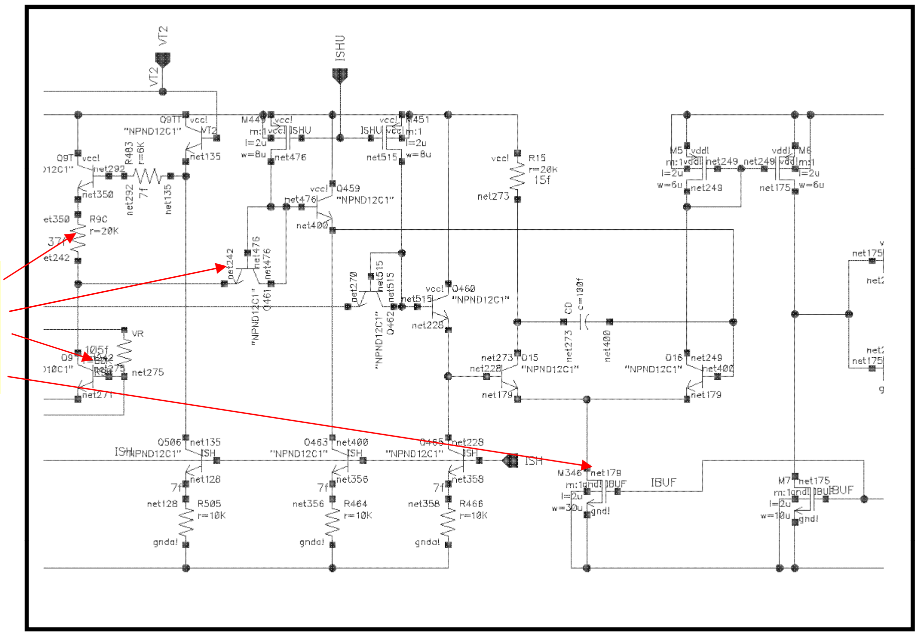


Golden Rules (Op amp with negative feedback):

(1) No-current flows into either (+) or (-) inputs.

(2) The (+) and (-) inputs are at the same voltage.

Noise of the Op-Amp ?



Each component is a (multiple) noise source

Note, that (pure) capacitors or inductors do not produce noise

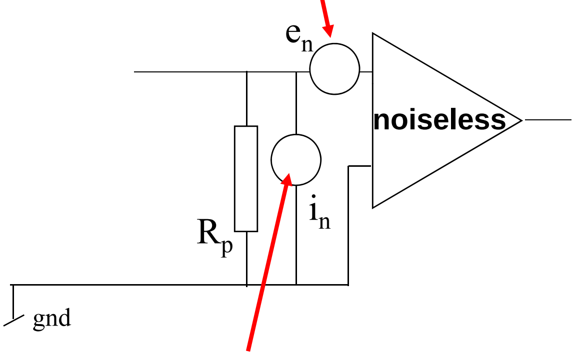
---

Noise of the Op-Amp ?

The noise properties of any amplifier can be described fully in terms of a **voltage noise source** and **current noise source** at the **amplifier input**. Typical magnitudes are nV/ sqrt Hz and pA/ sqrt Hz .

Circuit equivalent voltage noise source

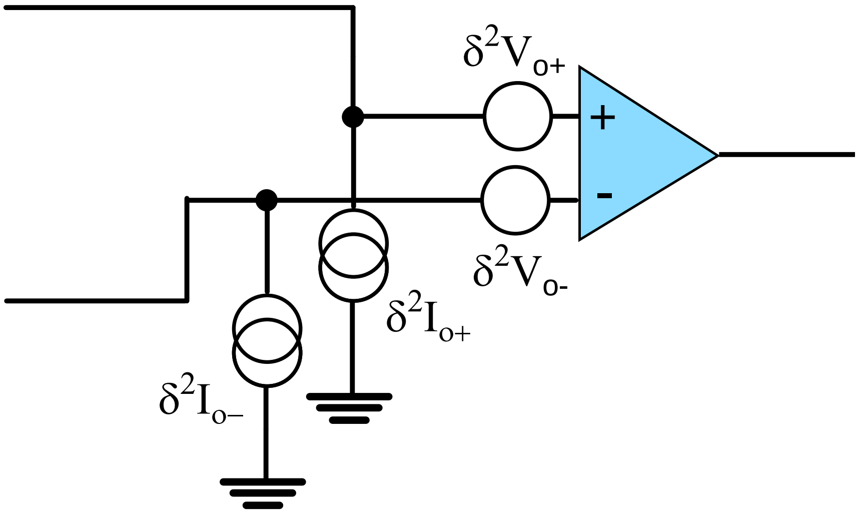
$$\delta^2 V_o := \left\langle (\delta V_o(t))^2 \right\rangle_{\Delta f} = \delta_{\Delta f}{}^2 V_o$$



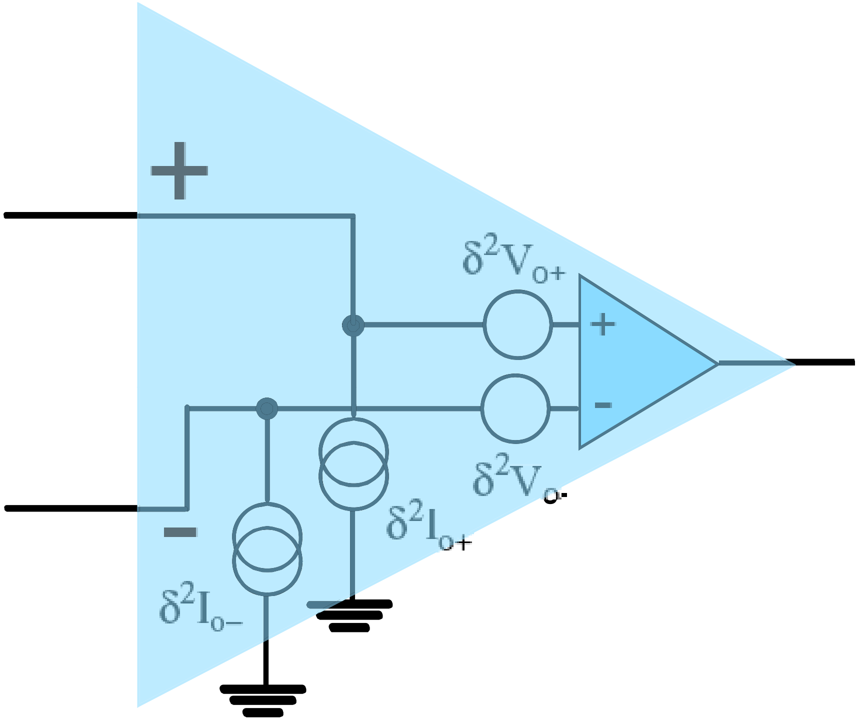
Circuit equivalent current noise source

$$\delta^2 I_o := \left\langle (\delta I_o(t))^2 \right\rangle_{\Delta f} = \delta_{\Delta f}{}^2 I_o$$

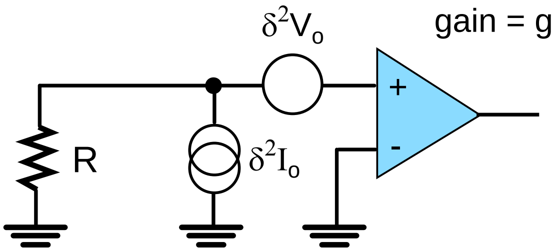
Noise of the Op-Amp ?



Noise of the Op-Amp ?



voltage or current amplifier ?



$$output\_noise = gain^2 \cdot \left(\delta^2 I_o \cdot R^2 + \delta^2 V_o\right)$$

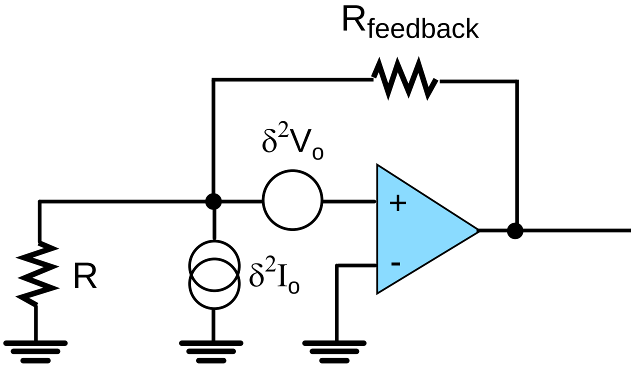
$R \to \infty$ obviously, gets too large

$R \to 0$ obviously, is much better

therefore, use a V-amplifier if $R \le R_n := \dfrac{\delta V_o}{\delta I_o}$

---

voltage or current amplifier ?



$$\delta^2 V_{output} = R^2_{feddback} \cdot \left(\delta^2 I_o + \delta^2 V_o / R^2\right)$$

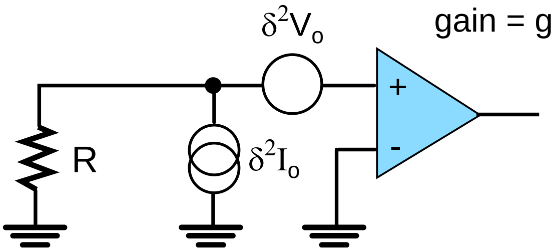
$R \to 0$ obviously, gets too large

$R \to \infty$ obviously, is much better

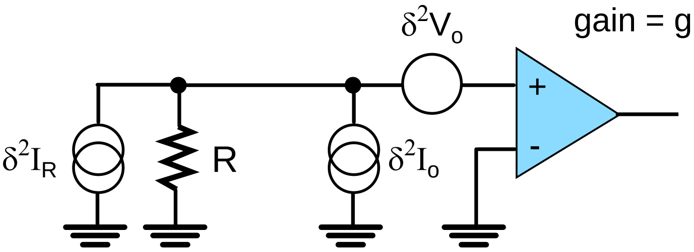
therefore, use an I-amplifier if $R \ge R_n := \dfrac{\delta V_o}{\delta I_o}$

Equivalent Amplifier Noise Resistance

	Uos	Ios	**Rn**
	nV/sqrt Hz	pA/sqrt Hz	**k Ohm**
LI 75	1.2	0.012	**100.00**
Brookdeal 5003	2	0.014	**142.86**
SR560	4	0.02	**200.00**
220FS	0.45	0.13	**3.46**
Brookdeal 5004	0.8	0.1	**8.00**
OpAmp LT 1028	1	1.6	**0.63**
OpAmp AD797	0.9	2	**0.45**



Noise Factor and Noise Figure



The noise characteristics of an amplifier is often expressed by the *noise figure*

First, the *noise factor F* is defined as the ratio of the total noise signal (device resistance and amplifier) to the thermal noise of the device resistance R.

$$F = \left(4kTR + R^2\delta^2 I_0 + \delta^2 V_0\right) / \, 4kTR$$

$$F = 1 + \frac{R\delta^2 I_0 + \delta^2 V / R}{4kT}$$

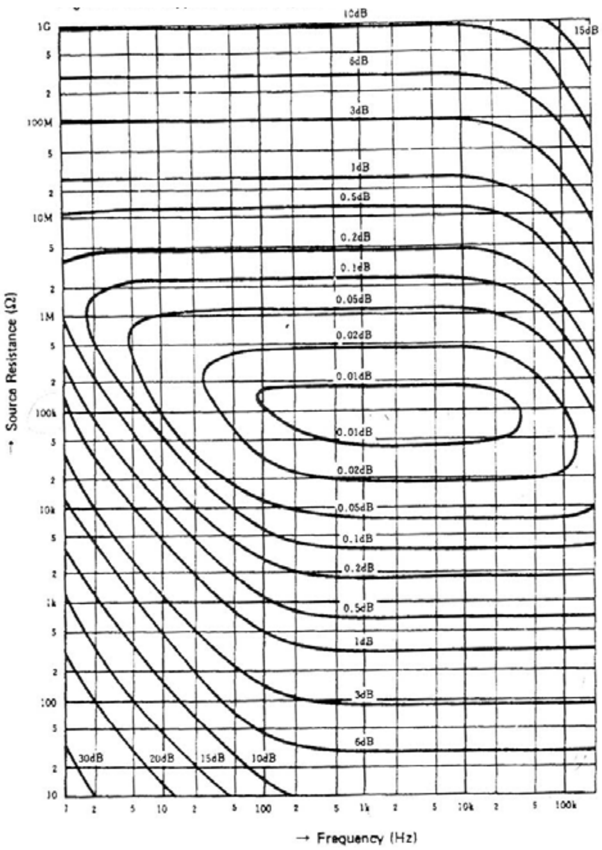
(note, there is an optimum resistance, i.e. noise matching)

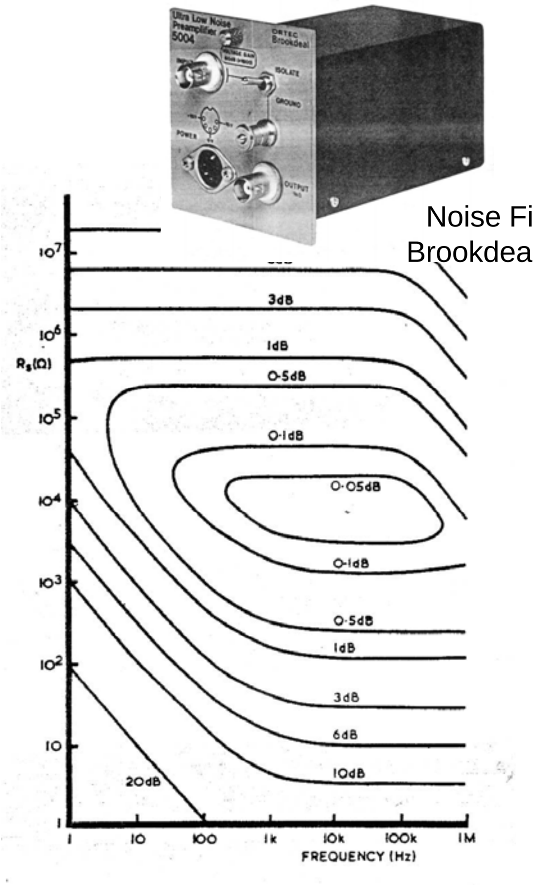
The *noise figure* is equal to $10\log(F)$ in dB

The noise temperature of the amplifier is then: $T_N = \frac{\sqrt{\delta^2 V_0 \cdot \delta^2 I_0}}{2k_B}$

Noise Factor and Noise Figure

Noise Figure of LI-75A





Noise Figure
Brookdeal 5004

Noise Matching

principle of noise matching must be applied with caution, because power is not always the relevant measure

noise energy:

- commercial (FET) amplifiers operating at room temperature: **0.5 K**
- with correlation technique (also FETs at ambient): **< 50 mK**
- SQUIDs (at 4K) : < **1 mK** possible

another factor, which should not be underestimated, is **bandwith**!