Single Electron Tunneling

tutorial

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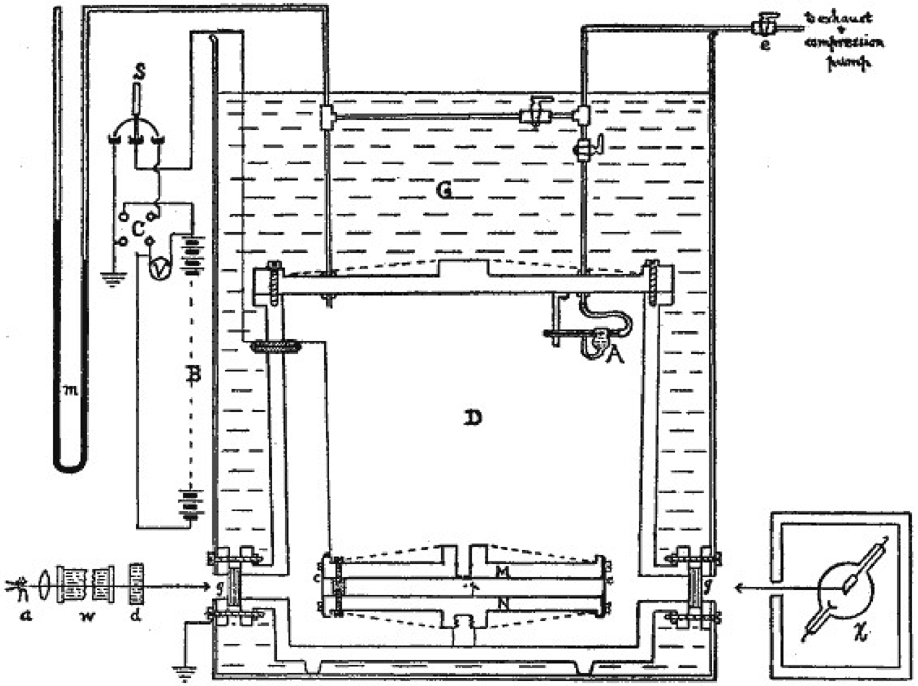
Content

1. Introduction: Quantized Charge

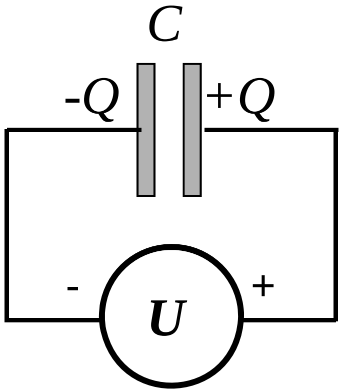
R. A. Millikan (1911): Oil droplet experiment
charge is quantized in
units of *e*



granularity of charge does not usually show up in macroscopic quantities such as current and voltage.

Reason: charge flow is the **continuous** movement of an **electron fluid**

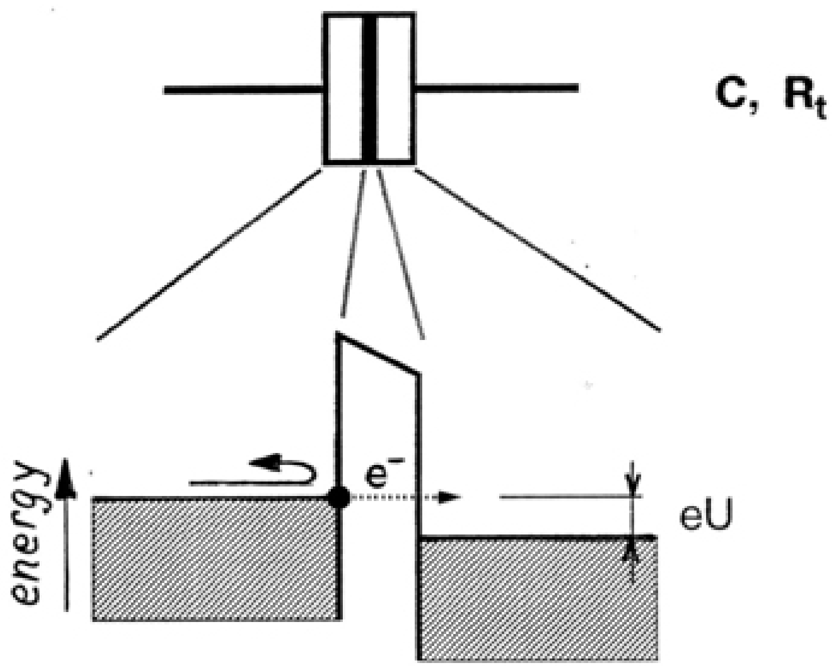
An example is the charging of a capacitor *C*, where the charge on the plate can take up any value, i.e. is not quantized!



correct statement is $\langle Q(t) \rangle = \overline{Q}$ is not quantized

but there are fluctuations around the average $\delta Q = e$

Tunnel Contact (Tunnel Capacitor)



in a tunnel junction charge flow is no longer continuous, since electrons are transferred **one by one** → we expect full shot noise

it is build from two metals with an oxide barrier (e.g. Al) or in an STM with a vacuum tunneling barrier

“golden rule” perturbation result from QM, i.e.

$$\Gamma_{i->f} = \frac{2\pi}{\hbar} \left| \langle f | H_t | i \rangle \right|^2 \delta\left(E_f - E_i - \Delta G\right)$$

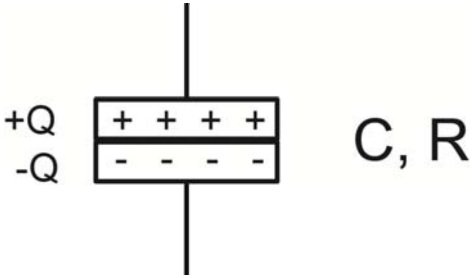
ΔG is the free energy gain, H_t the tunneling “matrix element”, *i,f* the initial and final states, respectively

by integrating over initial and final states, one obtains:

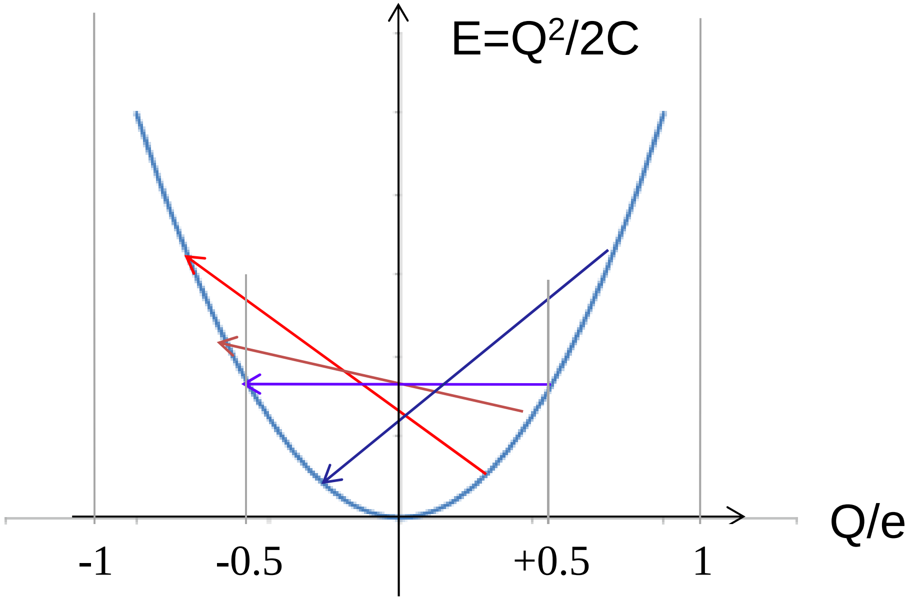
sketch derivation on blackboard
and draw curve Γ(ΔE), I(eV)
discuss role of environment

$$\Gamma(\Delta G) = \frac{1}{\mathrm{Re}^2} \frac{\Delta E}{1 - \exp(-\Delta G / k_B T)}$$

Tunnel Contact (Tunnel Capacitor)



let *Q=CU* be the charge before an electron tunnels



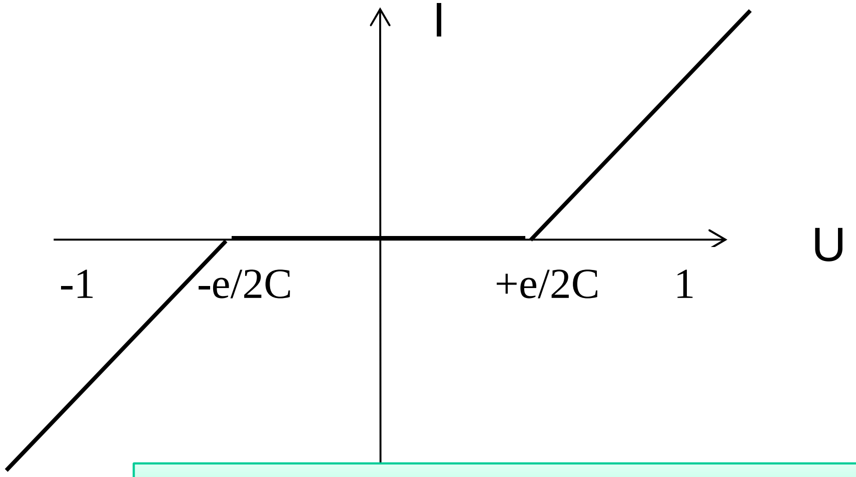
$$\Delta E(Q->Q\pm e)=\frac{(Q\pm e)^2}{2C}-\frac{Q^2}{2C}=\frac{e^2}{2C}\pm eU$$

change in energy (energy after tunneling event minus energy before tunneling event)

process is only allowed if there is an energy gain, i.e. provided: $|U|>e/2C$

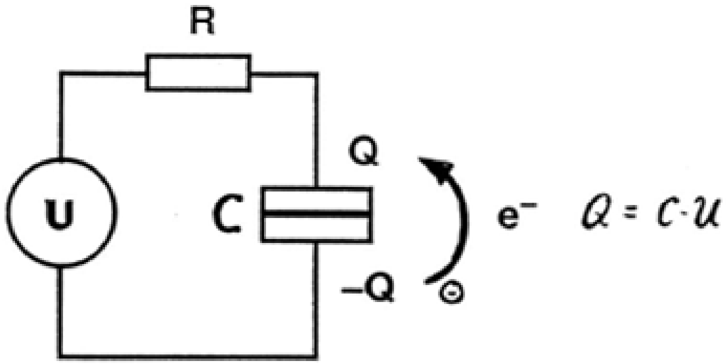
this is known as **Coulomb blockade requirement:** $E_C=\frac{e^2}{2C}>>kT$

this treatment is not fully correct
(as we have neglected the environment)
and adopted a local picture!



Exercice 1: estimate Ec for metallic spheres

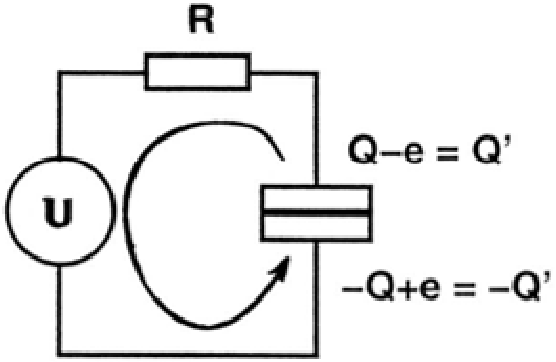
Tunnel Capacitor with Environment



we consider a tunnel capacitor attached to a voltage source, but this time through an impedance

initial state: C is charged to Q=CU

1. step: an electron tunnels as shown on the left. Note, this is not the final step!



2. step: there is a voltage difference over the resistance, so that a current should flow.
→ the **voltage source** is now pushing the electron back to the other side, **doing the work eU** in this process

if process 2 is **too fast**, there is **no Coulomb blockade**, since the final state has the same energy than the initial state, so that there is indeed no effective barrier to overcome

from energy time uncertainty relation: $\frac{\hbar}{\tau} << \frac{e^2}{2C}$

with τ=*RC* one obtains: $R >> R_Q = \frac{h}{2e^2}$

CB occurs is E_C>>*kT* and if *R*>>R_Q
but precursor already visible even for
small R values

Tunnel Capacitor with Environment

M.H. Devoret et al. Phys. Rev. Lett. 64, 1824 (1990)

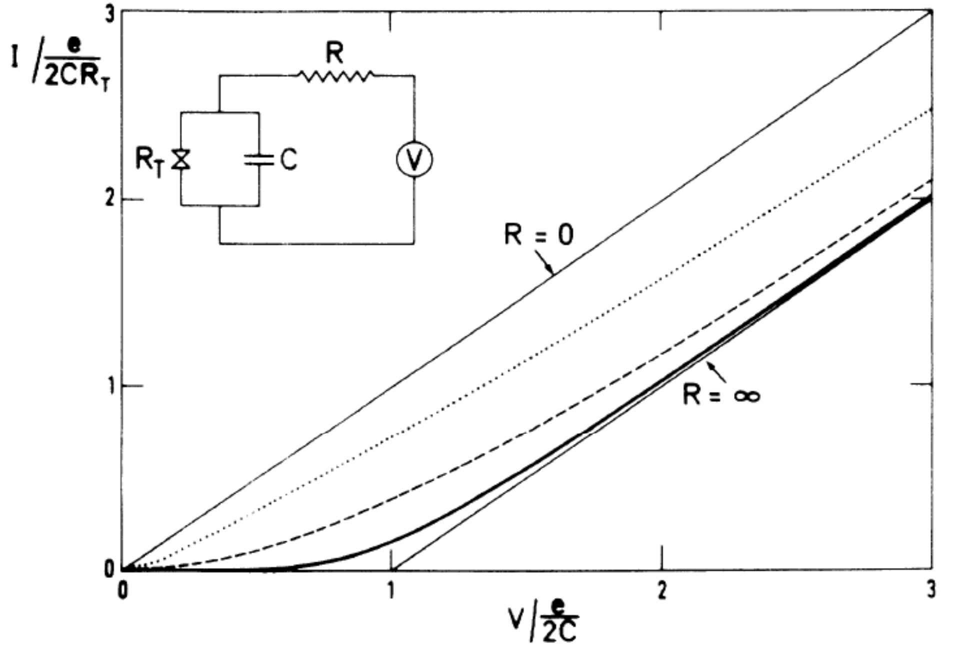
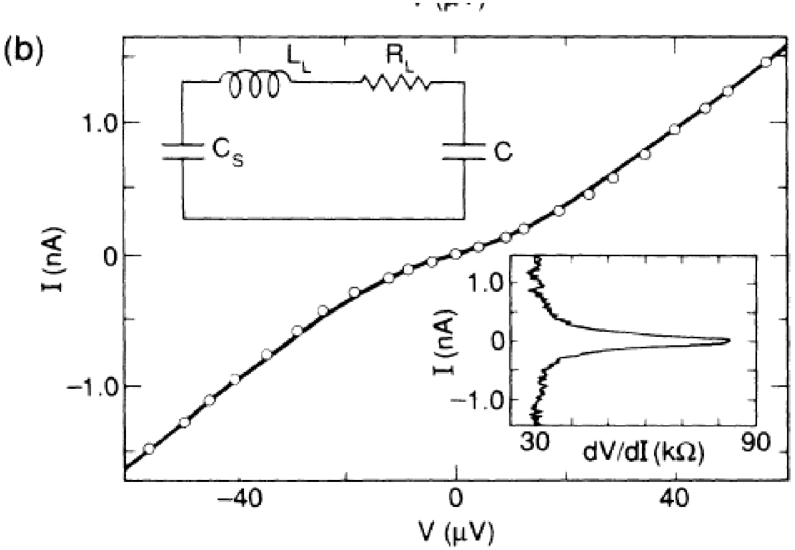


FIG. 2. The *I-V* characteristic of a tunnel junction coupled to an environment characterized by a resistance *R* (see inset) for $R/R_Q = 0, 0.1, 1, 10$, and ∞.

A.N. Cleland et al. Phys. Rev. Lett. 64, 1565 (1990)



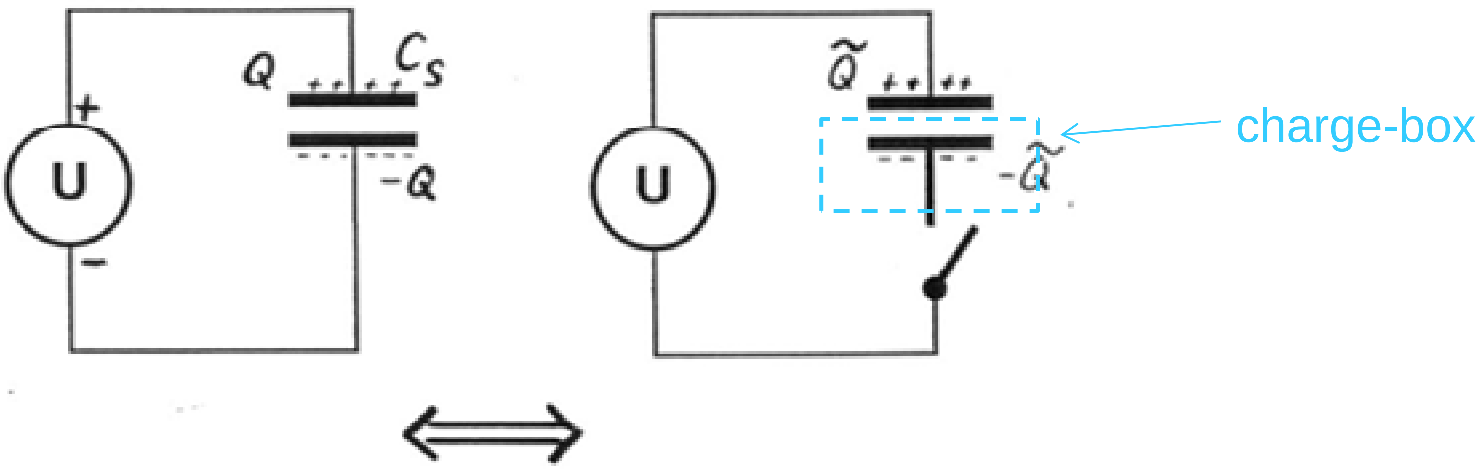
relation to noise: it is the noise in the environment that “drives” the transition through the tunnel junction. Langevin formalism. Limit kT large:

$$\frac{\left\langle \delta Q^2 \right\rangle}{2C} = \frac{kT}{2}$$

$$I = \frac{1}{eR_T} \int_{-\infty}^{+\infty} dE \int_{-\infty}^{+\infty} dE' \{ f(E)[1 - f(E')] P(E + eV - E') - [1 - f(E)] f(E') P(E' - E - eV) \} ,$$

$$P(E) = (2\pi\hbar)^{-1} \int_{-\infty}^{+\infty} dt \exp[J(t) + iEt/\hbar] \qquad J(t) = \langle [\phi(t) - \phi(0)] \phi(0) \rangle$$

The single electron box



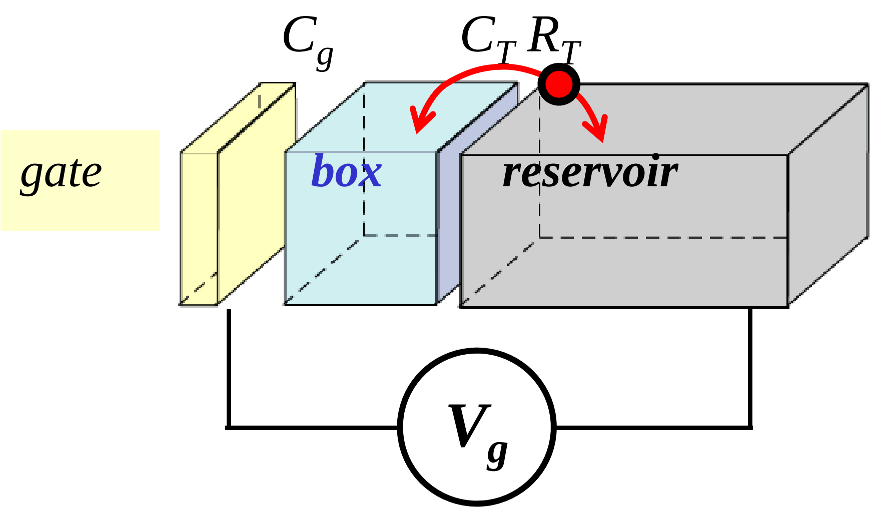
Gedankenexperiment:

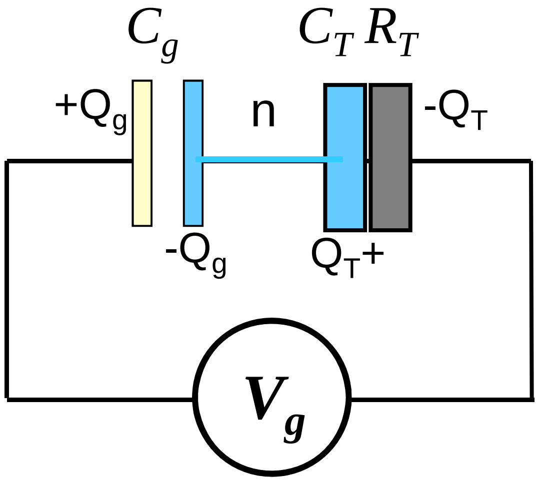
1. we charge up capacitor left by connecting it to source U shown on the left side. What is $<\delta Q^2>$ in the classical limit?

2. next, we open the switch. Same question: what is $<\delta Q^2>$?

left we have:	right we have:
$\langle Q\rangle = CU$	$\tilde{Q} = ne = const$
$\langle \delta Q^2\rangle = k_B TC$	$\langle \delta\tilde{Q}^2\rangle = 0$

The single electron box





instead of switch, here a tunnel junction one can view like a “leaky” switch

we assume $\frac{e^2}{2C} \gg k_B T$ and $R_t \gg R_Q$

1. if V_g is increased from zero up, charge will be transferred to the island and n increases in steps
2. there is electrostatic energy $E(Q_t,Q_g)$ stored in the capacitors, but there is also work done by the voltage source V_g .
3. The Gibbs free energy G(V,n) is the electrostatic energy $E(Q_t,Q_g)$ minus the work done by the voltage source V_g given by Q_gV_g

result:

$$G(V_g,n) = E_C\left(n - C_g V_g / e\right)^2 - C_g V_g^2 / 2$$

$$E_C = \frac{e^2}{2\left(C_g + C_t\right)^2}$$

The single electron box

electrostatics:

$$Q = Q_t - Q_g \qquad \frac{Q_t}{C_t} + \frac{Q_g}{C_g} = V_g$$

use last equations to express Q_t and Q_g as a function of V_g and Q

$$Q_g = \frac{C_g\left(C_t V_g - Q\right)}{C_t + C_g} \qquad Q_t = \frac{C_t\left(C_g V_g + Q\right)}{C_t + C_g}$$

total electrostatic energy

$$E(V_g, Q) = \frac{Q_t^2}{2C_t} + \frac{Q_g^2}{2C_g}$$

$$E(V_g, Q) = \frac{C_t C_g V_g^2}{2\left(C_t + C_g\right)} + \frac{Q^2}{2\left(C_t + C_g\right)}$$

Exercice 2: calculate the free energy for this problem using another ansatz.

free energy $G(V_g,Q)$

$$G(V_g, Q) = E(V_g, Q) - Q_g V_g$$

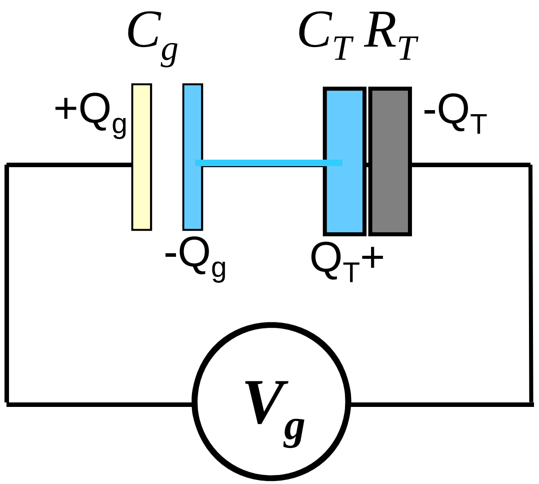
$$\boxed{G(V_g, n) = E_C\left(n - C_g V_g / e\right)^2 - C_g V_g^2 / 2}$$

$$E_C = \frac{e^2}{2C_\Sigma}$$

and we assumed that Q=ne, i.e. that CB holds

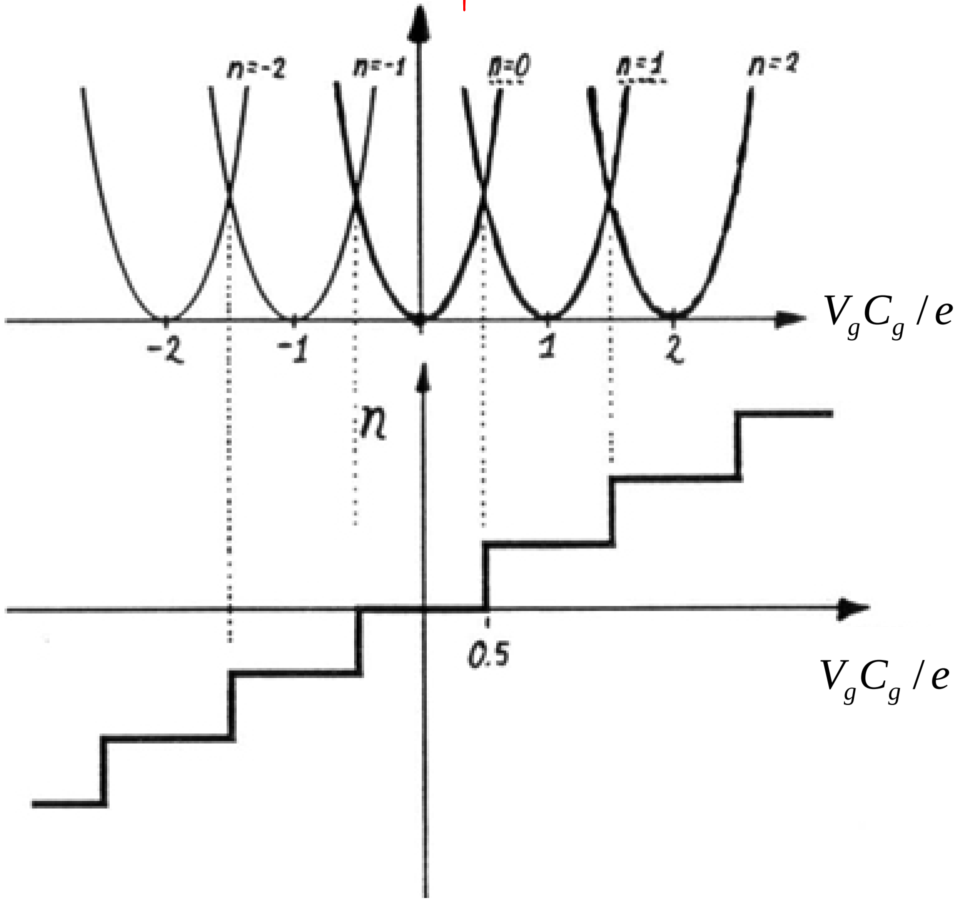
note, the relevant **single-electron charging energy** has the two **capacitors in parallel**, i.e. it is the capacitance seen from the island!

The single electron box



free energy for the single electron box

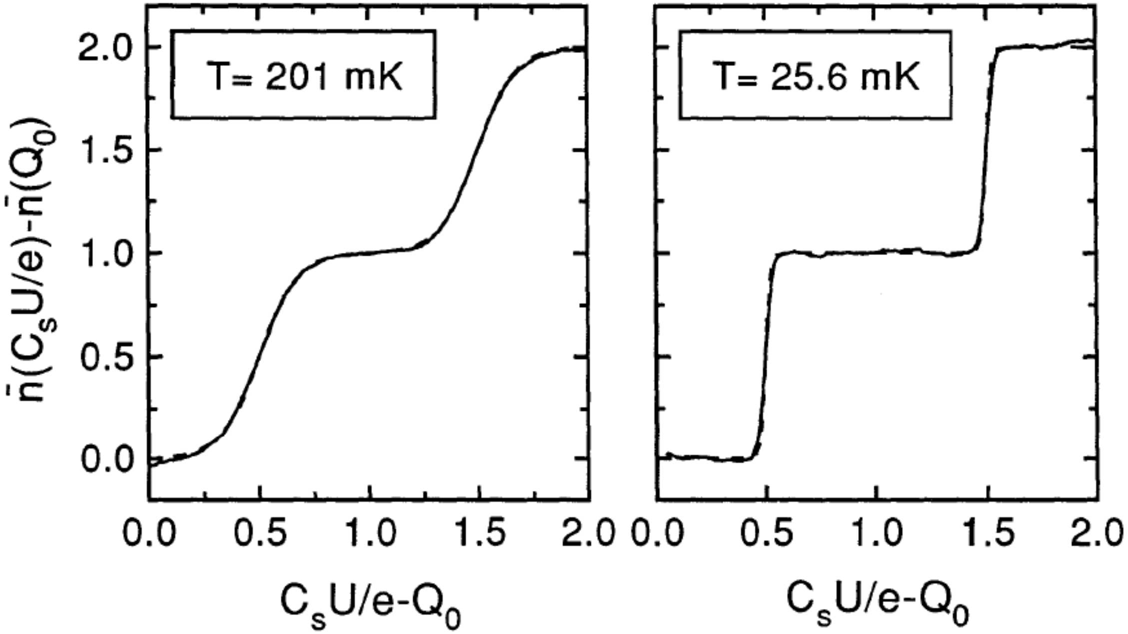
$$G(V_g,n)=\underbrace{E_C\left(n-C_gV_g/e\right)^2}-C_gV_g^2/2$$

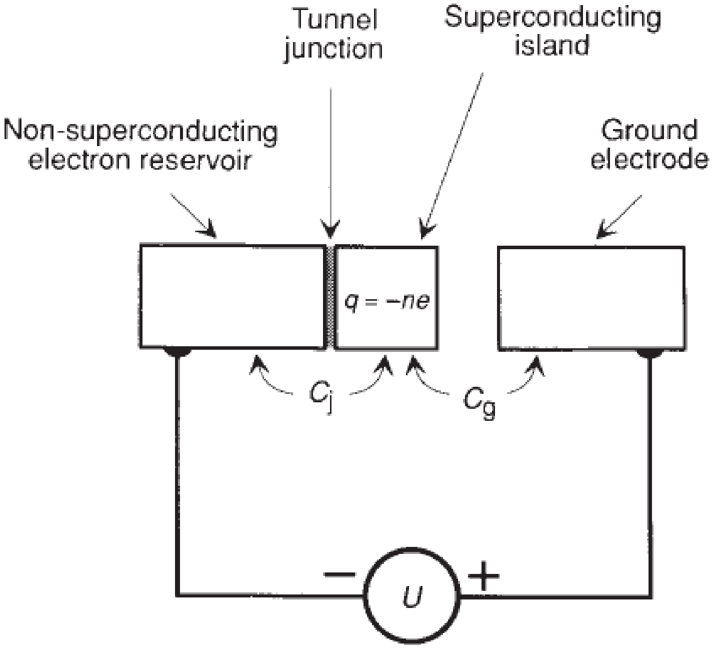


one can view this as a solid state Millikan experiment, the charge is quantized and controlled by the gate voltage

The single electron box

P. Lafarge, thesis Université de Paris 1993





The single electron box

Two-electron quantization of the charge on a superconductor

Nature 365, 422 (1993)

P. Lafarge, P. Joyez, D. Esteve, C. Urbina & M. H. Devoret

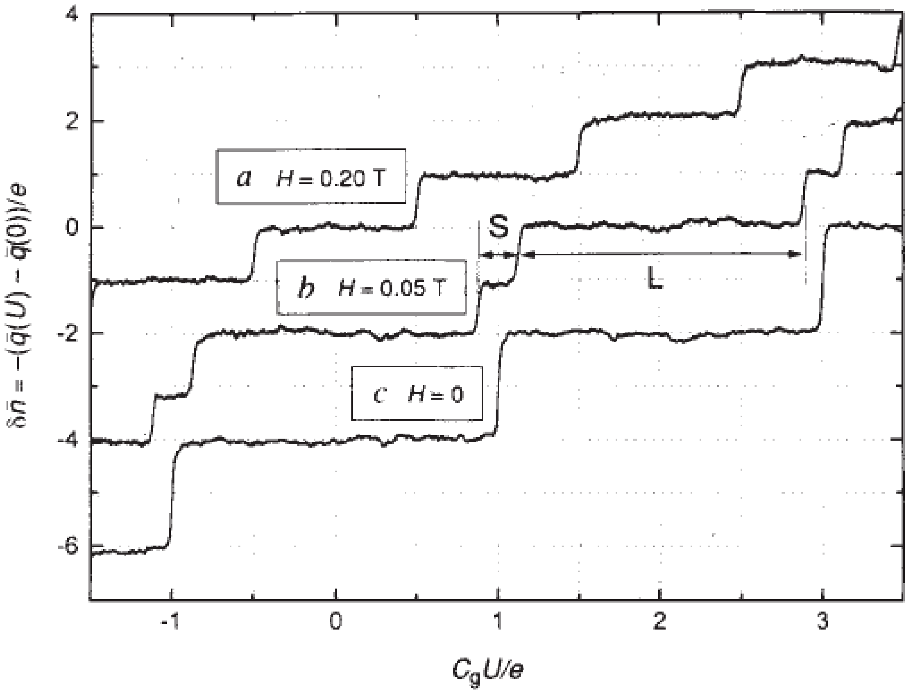
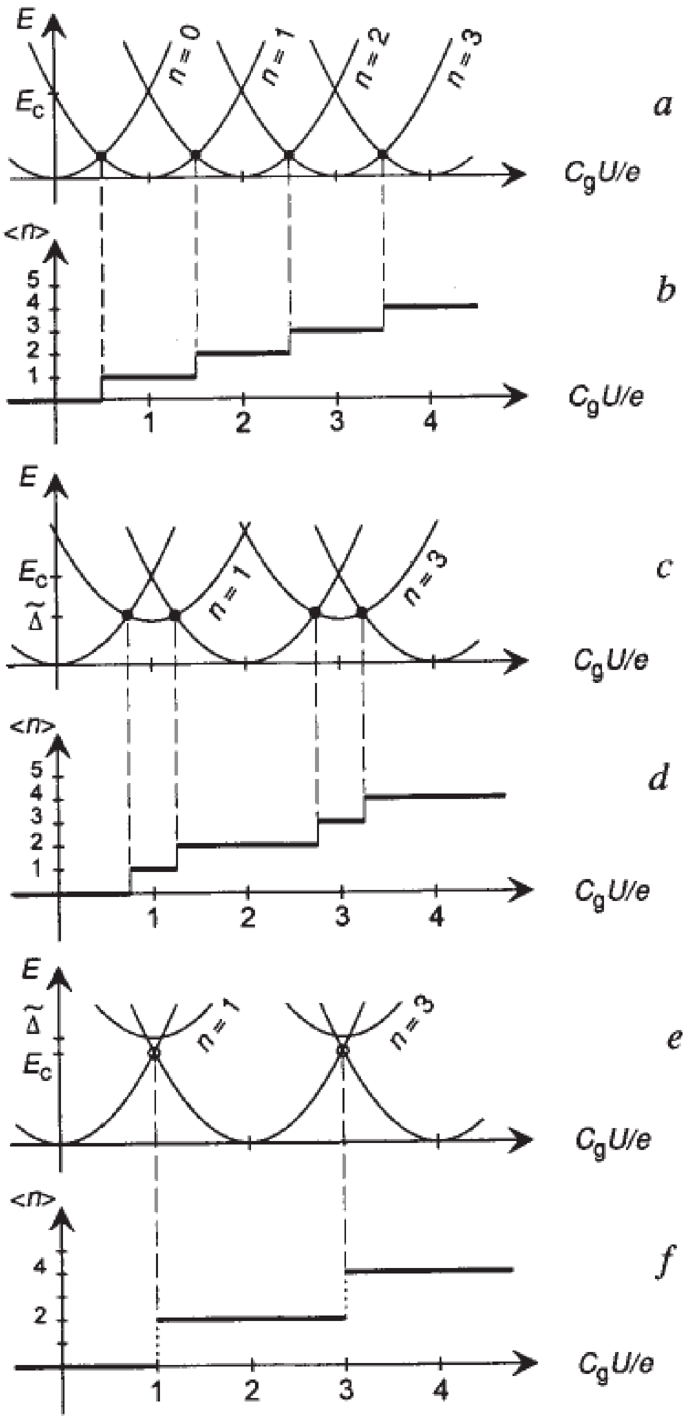
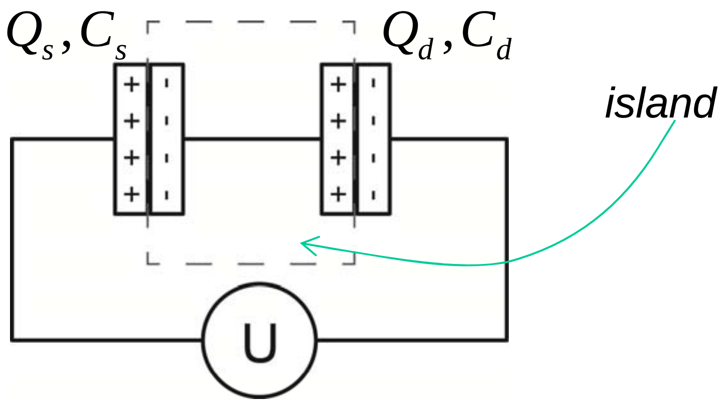


FIG. 2 Variations of the average value $\bar{q}$, in units of e, with the polarization C_gU/e, at $T = 28$ mK, for three values of the magnetic field H applied to the sample. *a*, Non-superconducting island. *b* and *c*, Superconducting island. For clarity, *b* and *c* have been offset vertically by 2 and 4 units, respectively. The letters L and S refer to the long and short steps, respectively.



The double-barrier tunneling junction



$$Q = Q_d - Q_s$$

we assume that $R_t \gg R_Q$ so that charge Q on the island is discrete

$$Q = ne$$

express Q_s and Q_d in *n* and *U:*

$$Q_s = \frac{C_s C_d}{C_\Sigma}\left(U - \frac{ne}{C_d}\right) \qquad Q_d = \frac{C_s C_d}{C_\Sigma}\left(U + \frac{ne}{C_d}\right)$$

the electrostatic energy is then given by $E = \frac{Q_s^2}{2C_s} + \frac{Q_d^2}{2C_d} = \frac{(ne)^2}{2C_\Sigma} + \frac{C_s C_d}{2C_\Sigma} U^2$

let us look what happens at instances, when an electron tunnels changing *n*→*n+1*

for the first electron to tunneling there is a "on-site" charging energy of $E_C = \frac{e^2}{2C_\Sigma}$

hence, as before, if $kT \ll E_C$, no current can flow: **Coulomb blockade** = CB

but if an electron tunnels, the source is doing some work on the system, leading to an effective free energy change ΔG

$$\Delta G_l^+(0) = E_c - e\frac{C_d}{C_\Sigma}U$$

free energy for adding an electron through the left (source) junction, considering the charge transition n=0→1

$$\Delta G_r^-(1) = -E_c - e\frac{C_s}{C_\Sigma}U$$

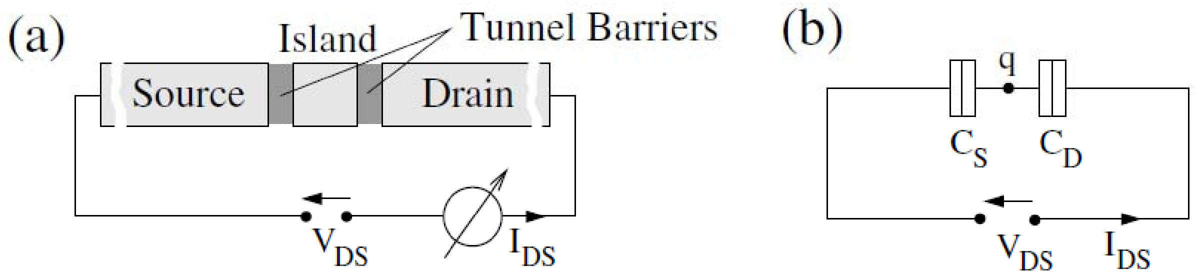
free energy for removing an electron through the right (drain) junction, considering the charge transition 1→0.

Exercice 3: determine the "relaxation" charge δQ that is moved through the source when an electron tunnels through either junction

The double-barrier tunneling junction

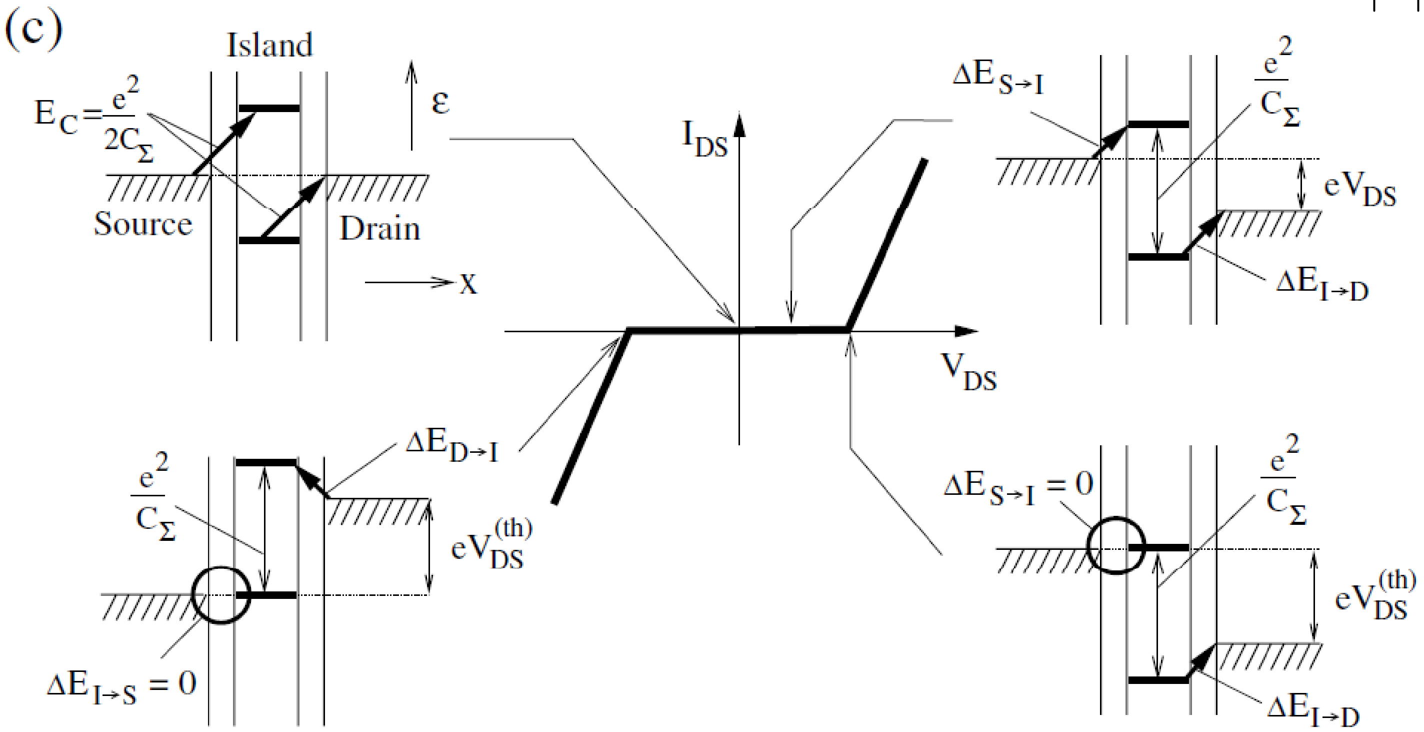
$\Delta G_l^{+}(0) = E_c - e\frac{C_d}{C_\Sigma}U$ free energy for adding an electron through the left (source) junction, i.e. considering the charge transition n=0→1

$\Delta G^{-}(0) = E_c - e\frac{C_d}{C_\Sigma}U$ free energy for removing an electron through the right (drain) junction, considering the charge transition n=0→-1



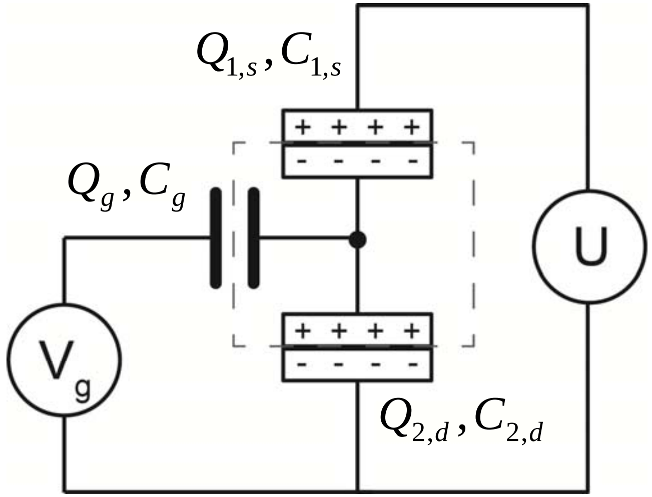
In order for a current to flow, there is a threshold voltage U_{th}

$$|U| \geq U_{th} = \min\left(\frac{e}{2C_s}, \frac{e}{2C_d}\right)$$



taken from Jürgen Weis, CFN Lectures on Functional Nanostructures Vol. 1., Springer Lecture Notes in Physics Volume 658, 2005, pp 87-121 (2004)

The single-electron transistor



let us first assume that *U=0*. Then, C_1 and C_2 are in parallel and we are back to the SET box.

The effective barrier that is seen for an electron to tunnel onto the island 0→1 is:

$$\Delta G^+(0) = E_c - e\frac{C_g}{C_\Sigma}V_g \qquad C_\Sigma = C_1 + C_2 + C_g$$

$$\Delta G^+(0) \le 0 \quad \text{if} \quad V_g \ge \frac{e}{2C_g}$$

Can one also tunnel out, i.e. what is the effective barrier for the transition 1→ 0 ?

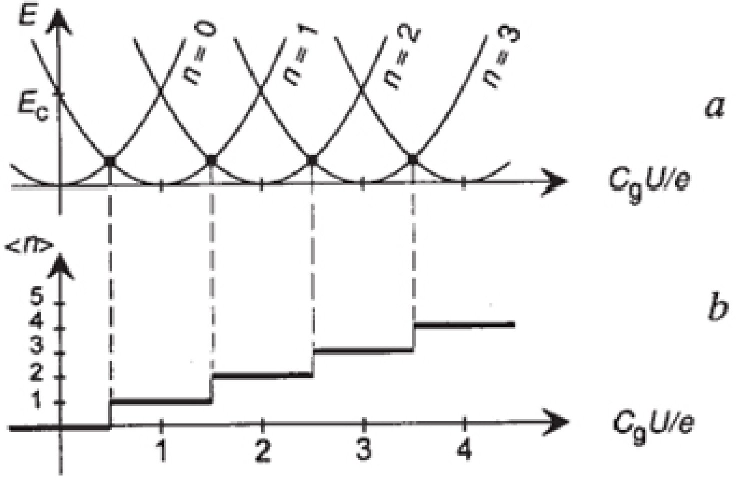
$$\Delta G^-(1) = -E_c + e\frac{C_g}{C_\Sigma}V_g \qquad \Delta G^-(1) \le 0 \quad \text{if} \quad V_g \le \frac{e}{2C_g}$$

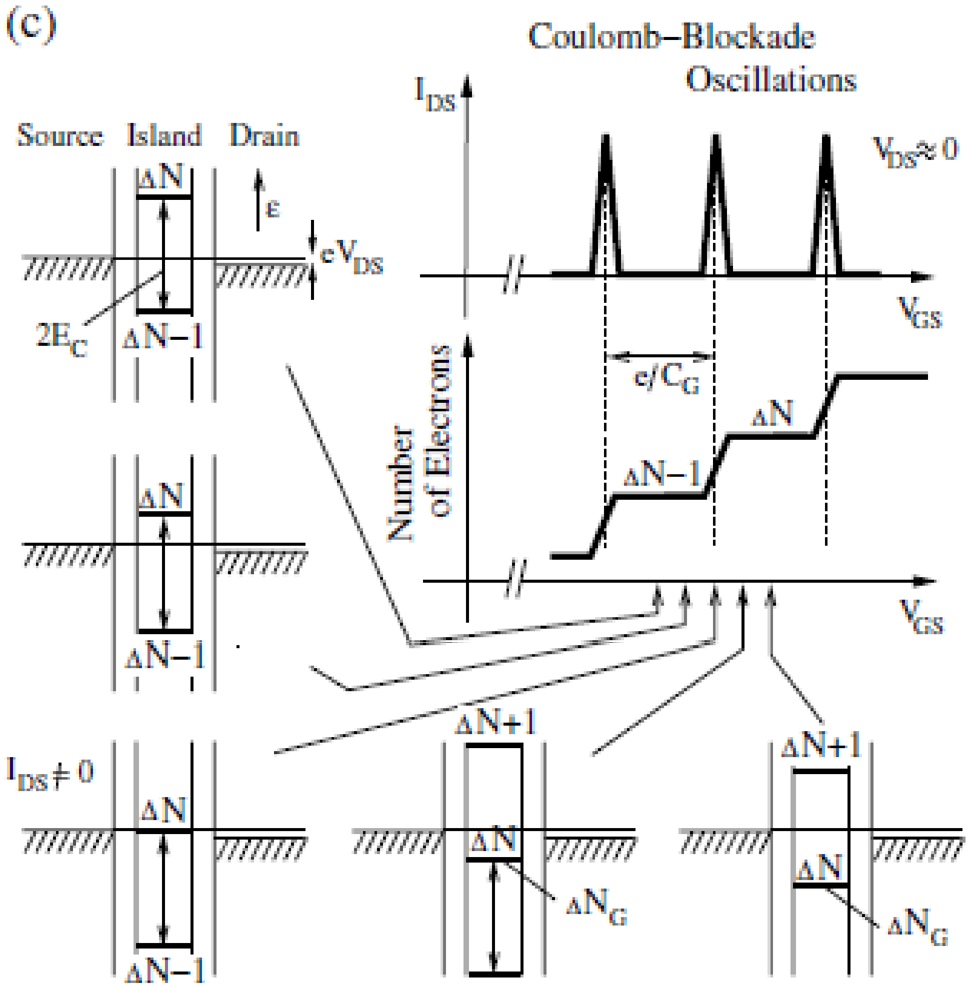
Hence, at exactly $V_g=e/2C_g$ is it possible to tunnel **in and out** (transition 0←→1). But this means that a current can flow, or more precisely that the conductance is low at that gate voltage value.

if one looks at different transitions, i.e. n←→n+1. one obtains the condition $V_g = \frac{e}{C_g}\left(n + \frac{1}{2}\right)$

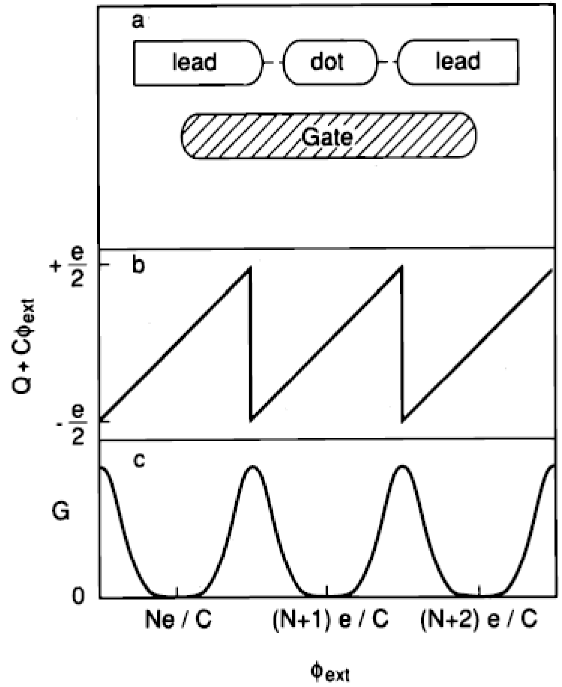
effect is known as Coulomb oscillations!
SET transistor is a periodic in the gate charge

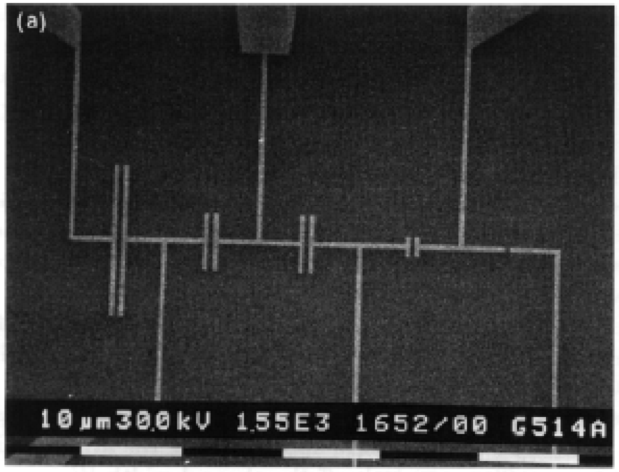
The single-electron transistor

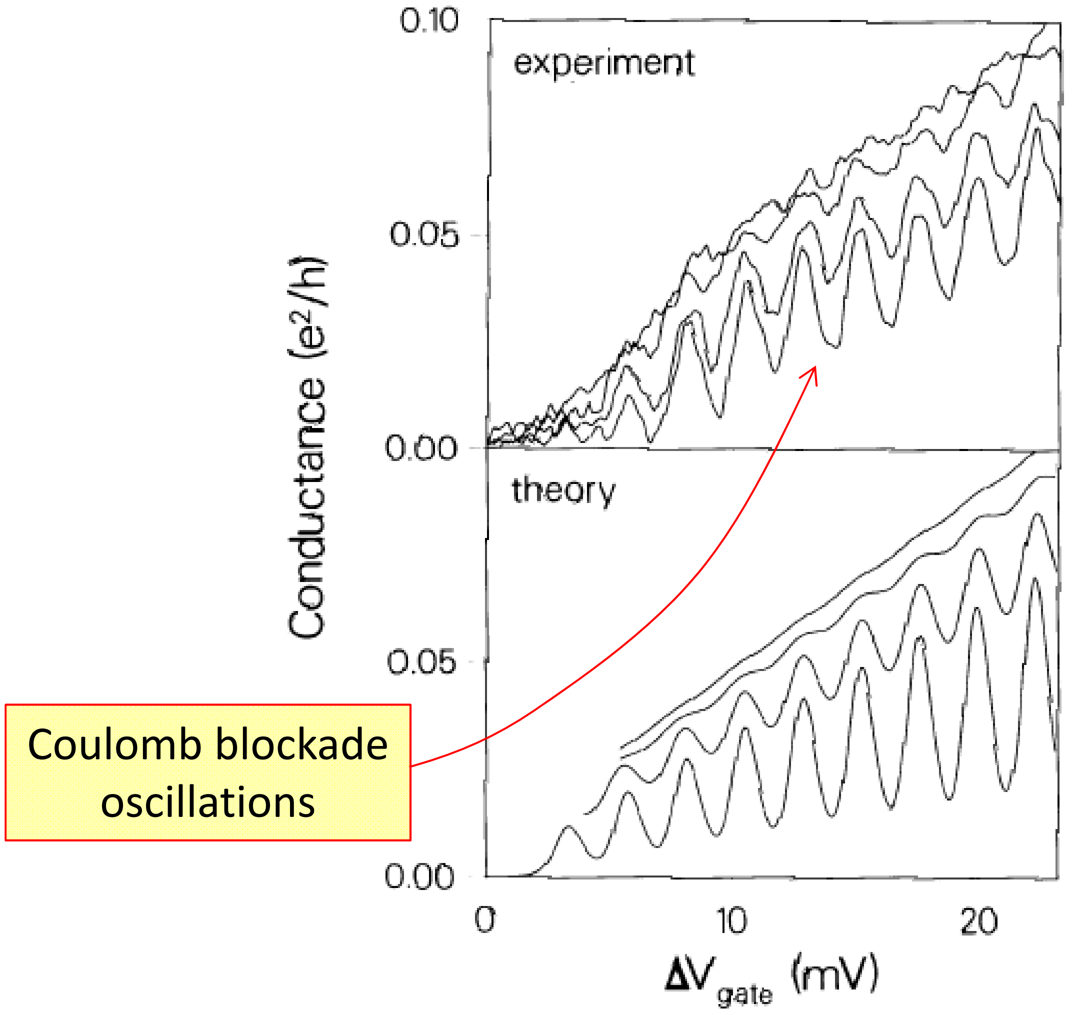




The single-electron transistor



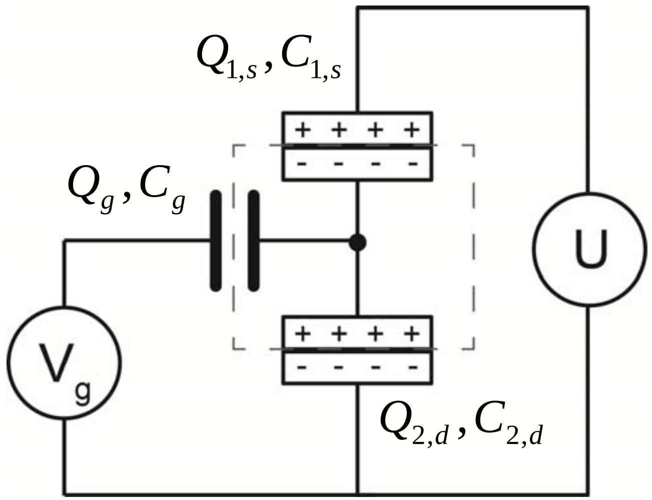




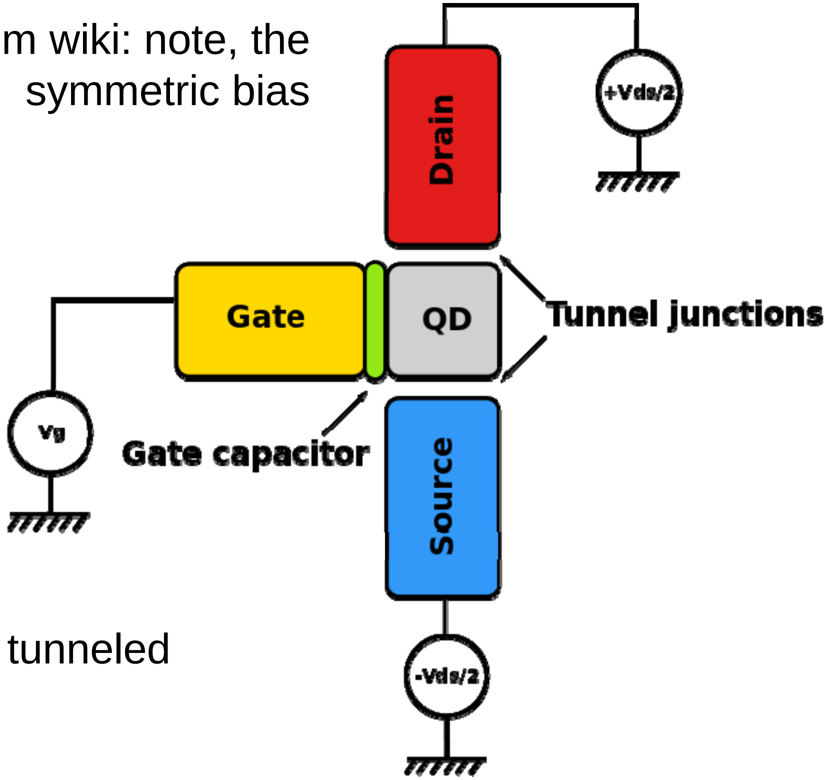
Coulomb blockade oscillations

early results measured in a (disordered) semicond. wire by A.A.M. Staring

The single-electron transistor



from wiki: note, the
symmetric bias



generalization:

we denote by n_1 (n_2) the number of electrons that have tunneled over junction 1 (2). Then, $n=n_1-n_2$

$$G(n_1,n_2,U,V_g)=\frac{(ne-C_gV_g)^2}{2C_\Sigma}-\frac{n_1(C_2+C_g)+n_2C_1}{C_\Sigma}eU$$

$$\Delta G_1^\pm(n)=\frac{e}{C_\Sigma}\left[\frac{e}{2}\pm\left(ne-C_gV_g-(C_2+C_g)U\right)\right]$$

free energy change for transition n→n±1
induced by tunneling over junction 1

$$\Delta G_2^\pm(n)=\frac{e}{C_\Sigma}\left[\frac{e}{2}\pm\left(ne-C_gV_g+C_1U\right)\right]$$

free energy change for transition n→n±1
induced by tunneling over junction 2

Exercice 4: use above equation $G(n_1,n_2)$ to check the final two results

The single-electron transistor

$\Delta G_1^+(n) \le 0$ transition n→n+1 over junction 1

$$C_g V_g + \left(C_2 + C_g\right)\cdot U \ge \left(n+\frac{1}{2}\right)e$$

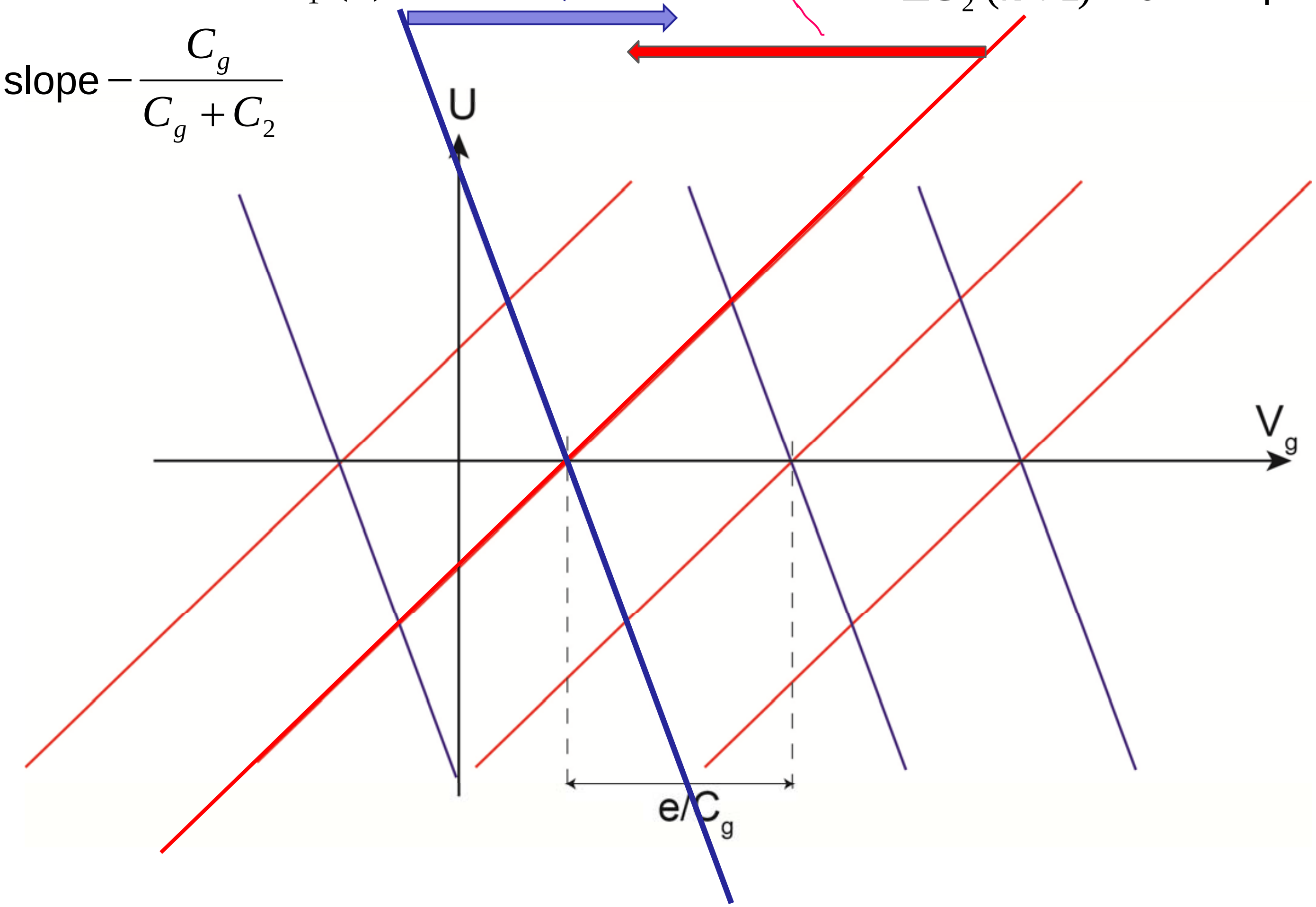
$\Delta G_2^-(n+1) \le 0$ transition n+1→n over junction 2

$$C_g V_g - C_1 \cdot U \ge \left(n+\frac{1}{2}\right)e$$

$\Delta G_1^+(n) = 0$

$\Delta G_2^-(n+1) = 0$ slope $\frac{C_g}{C_1}$

slope $-\frac{C_g}{C_g + C_2}$



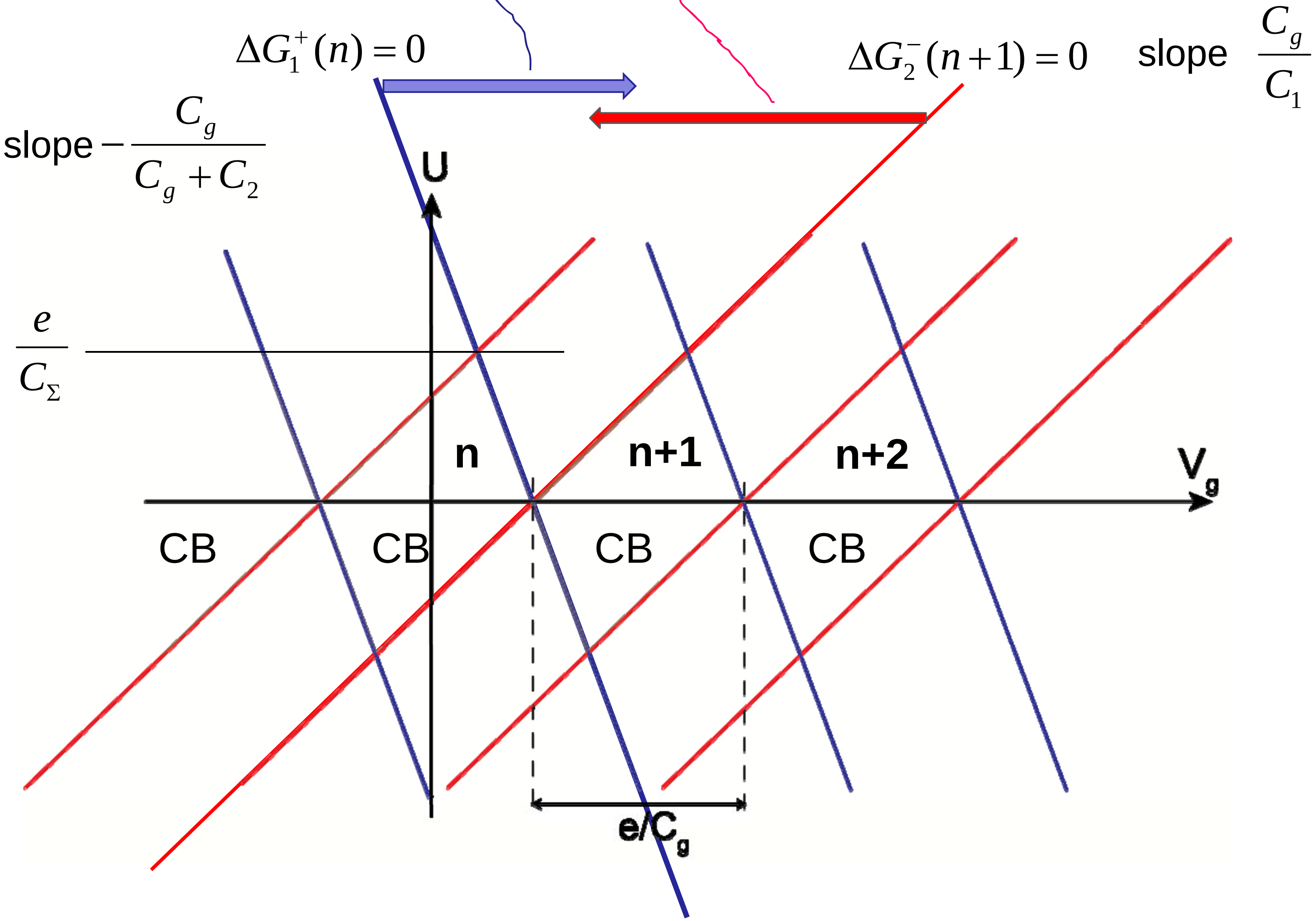
The single-electron transistor

$\Delta G_1^+(n) \le 0$ transition n→n+1 over junction 1

$$C_g V_g + (C_2 + C_g) \cdot U \ge \left(n + \frac{1}{2}\right) e$$

$\Delta G_2^-(n+1) \le 0$ transition n+1→n over junction 2

$$C_g V_g - C_1 \cdot U \ge \left(n + \frac{1}{2}\right) e$$



$\Delta G_1^+(n) = 0$

$\Delta G_2^-(n+1) = 0$ slope $\frac{C_g}{C_1}$

slope $-\frac{C_g}{C_g + C_2}$

The single-electron transistor

$\Delta G_1^+(n) \le 0$ transition n→n+1 over junction 1

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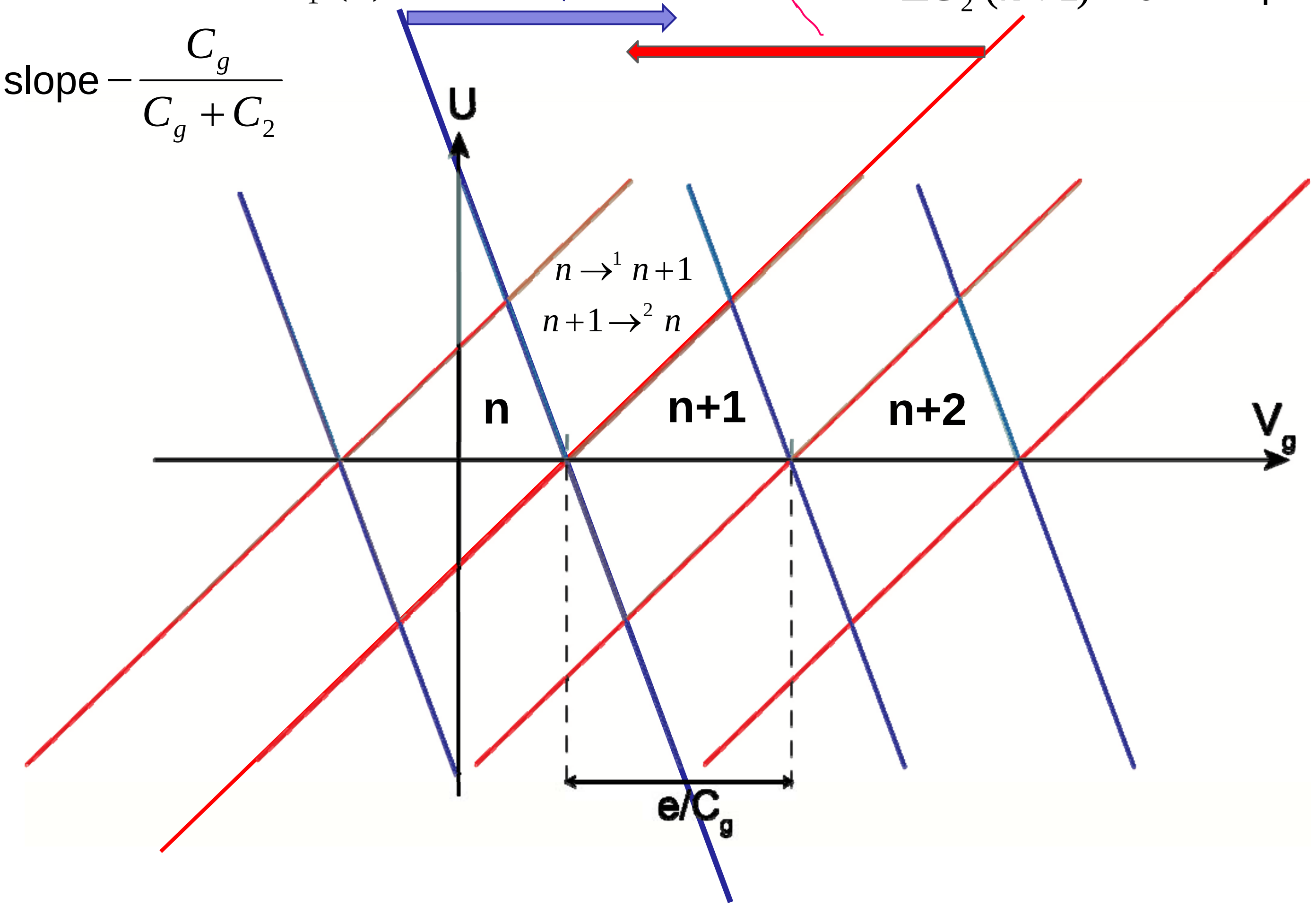


The single-electron transistor

$\Delta G_1^+(n) \le 0$ transition n→n+1 over junction 1

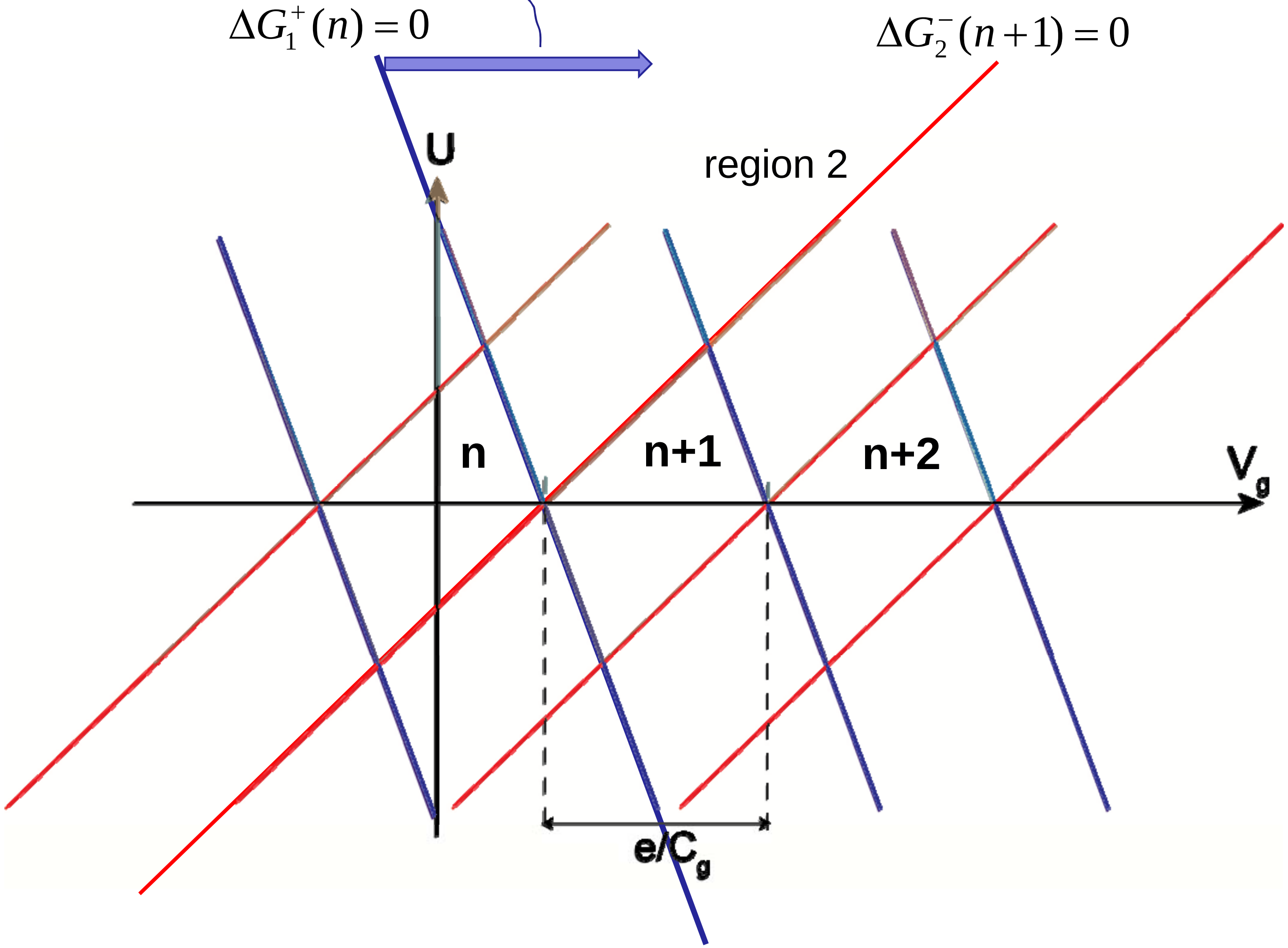
$$C_g V_g + (C_2 + C_g)\cdot U \ge \left(n + \frac{1}{2}\right)e$$

$\Delta G_2^-(n+1) \le 0$ transition n+1→n over junction 2

$$C_g V_g - C_1 \cdot U \ge \left(n + \frac{1}{2}\right)e$$

$\Delta G_1^+(n) = 0$

$\Delta G_2^-(n+1) = 0$



region 2

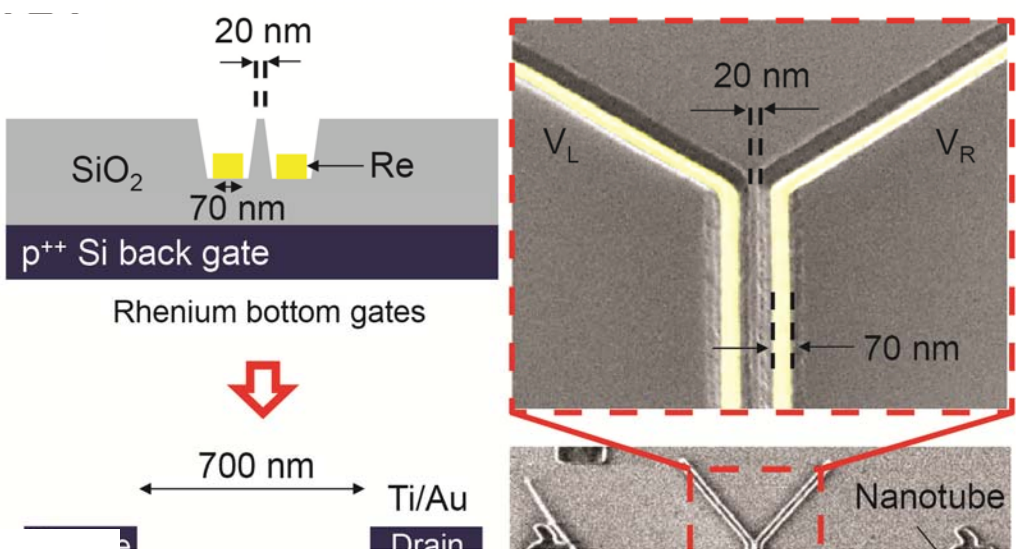
$n \rightarrow^{1} n+1$

$n+1 \rightarrow^{1} n+2$

$n+1 \rightarrow^{2} n$

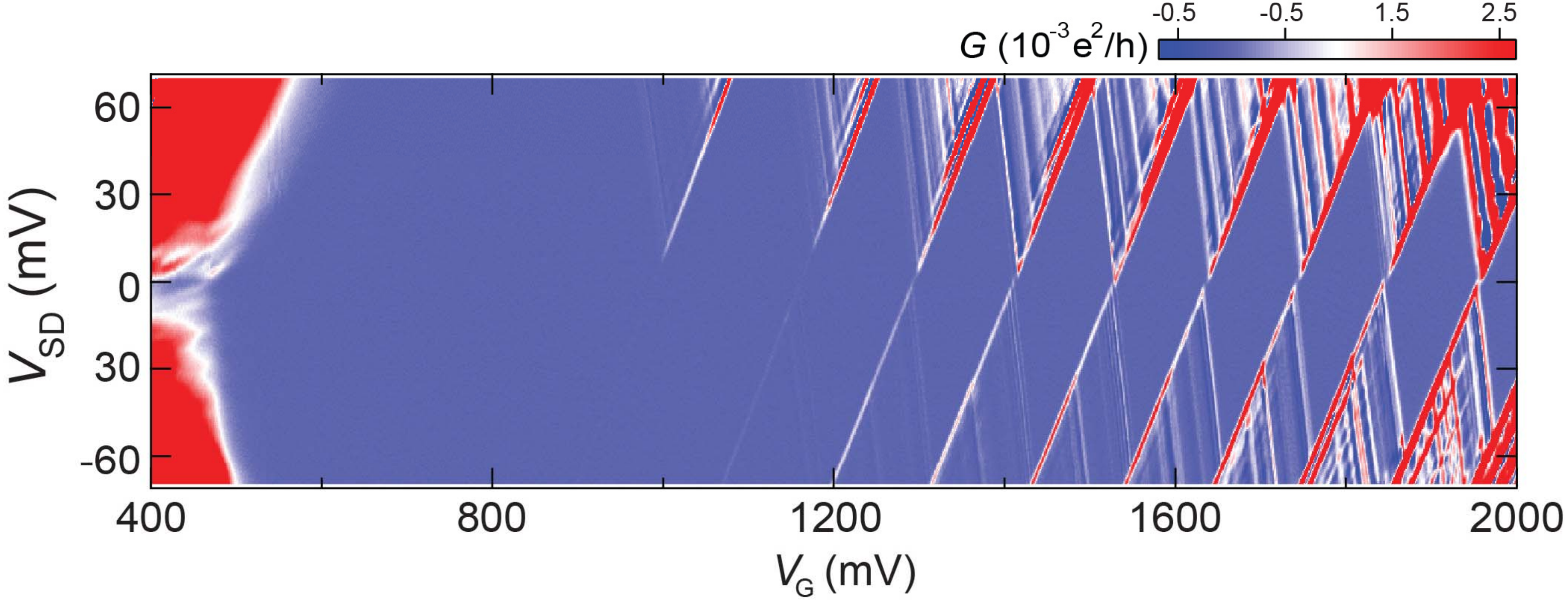
$n+2 \rightarrow^{2} n+1$

The single-electron transistor

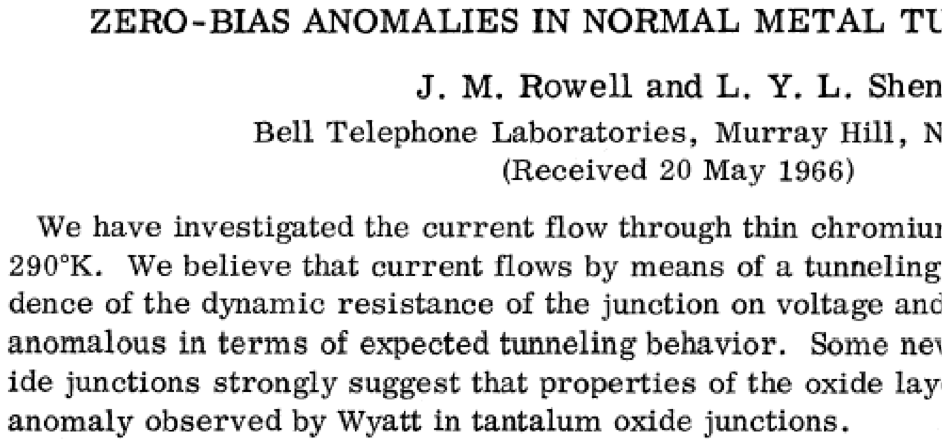


a carbon-nanotube quantum dot

Minkyung Jung et al. CS group



Zero-bias anonamly



ZERO-BIAS ANOMALIES IN NORMAL METAL TU

J. M. Rowell and L. Y. L. Shen
Bell Telephone Laboratories, Murray Hill, N

(Received 20 May 1966)

We have investigated the current flow through thin chromiu
290°K. We believe that current flows by means of a tunneling
dence of the dynamic resistance of the junction on voltage and
anomalous in terms of expected tunneling behavior. Some ne
ide junctions strongly suggest that properties of the oxide lay
anomaly observed by Wyatt in tantalum oxide junctions.

Phys. Rev. Lett. 17, p15 (1966)

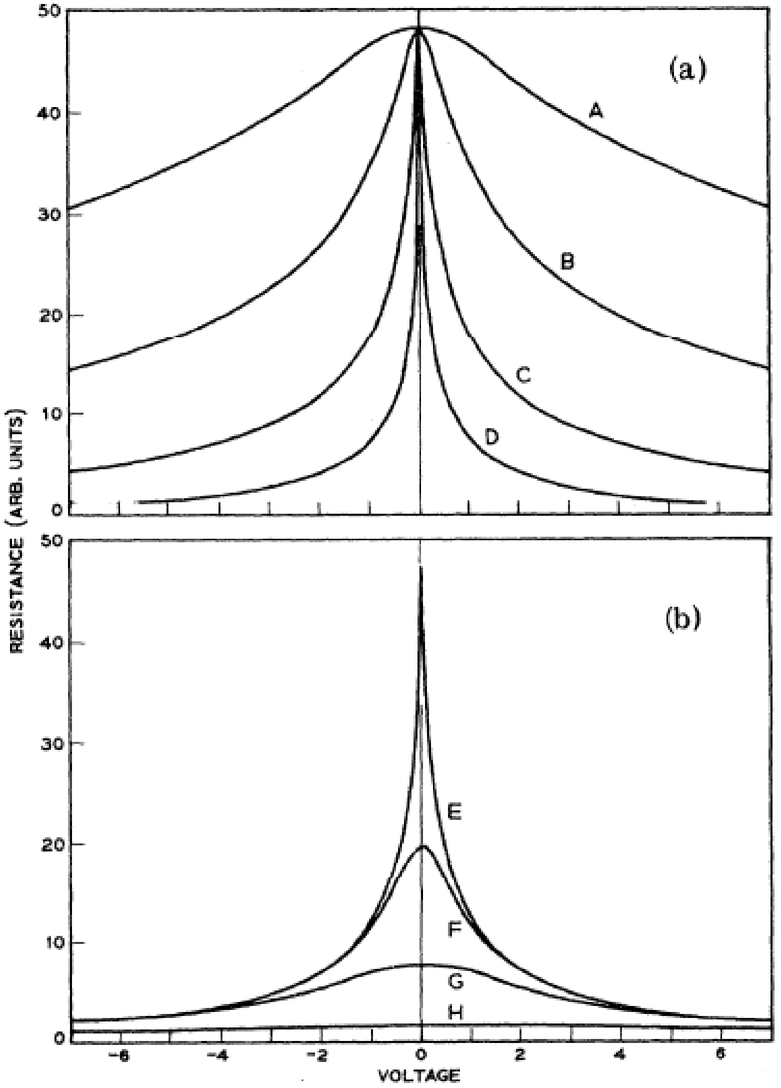
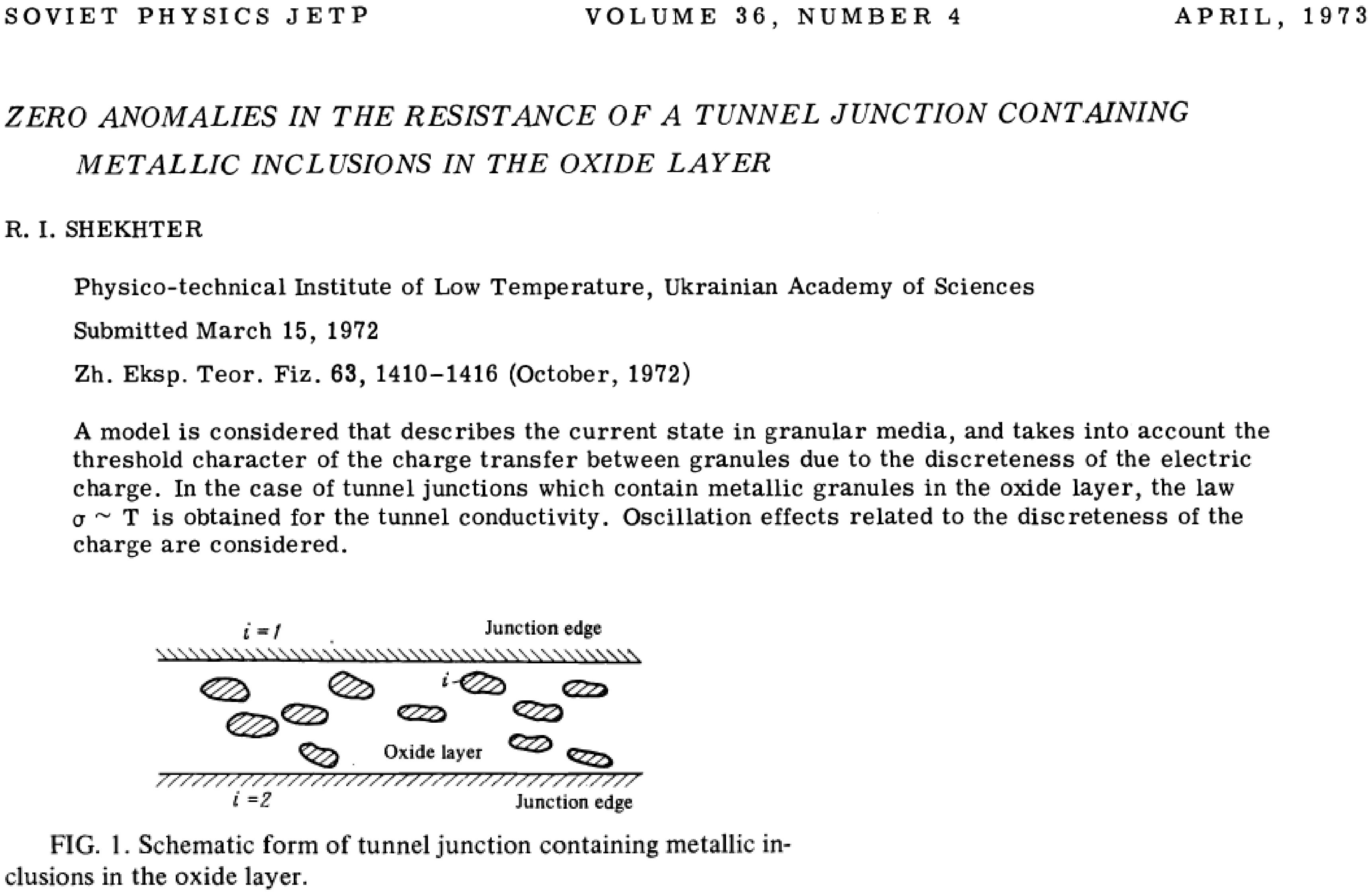


FIG. 1. (a) The dynamic resistance versus voltage for a Cr-I-Ag junction at 0.9°K. The voltage scales are $A = 0.2$ mV/division, $B = 1.0$ mV/division, $C = 5$ mV/division, $D = 20$ mV/division. (b) The dynamic resistance versus voltage for a Cr-I-Ag junction at various temperatures. $E = 0.9$, $F = 20.4$, $G = 77$, and $H = 290$°K. The voltage scale is 10 mV/division.

Zero-bias anomaly



SOVIET PHYSICS JETP VOLUME 36, NUMBER 4 APRIL, 1973

ZERO ANOMALIES IN THE RESISTANCE OF A TUNNEL JUNCTION CONTAINING METALLIC INCLUSIONS IN THE OXIDE LAYER

R. I. SHEKHTER

Physico-technical Institute of Low Temperature, Ukrainian Academy of Sciences

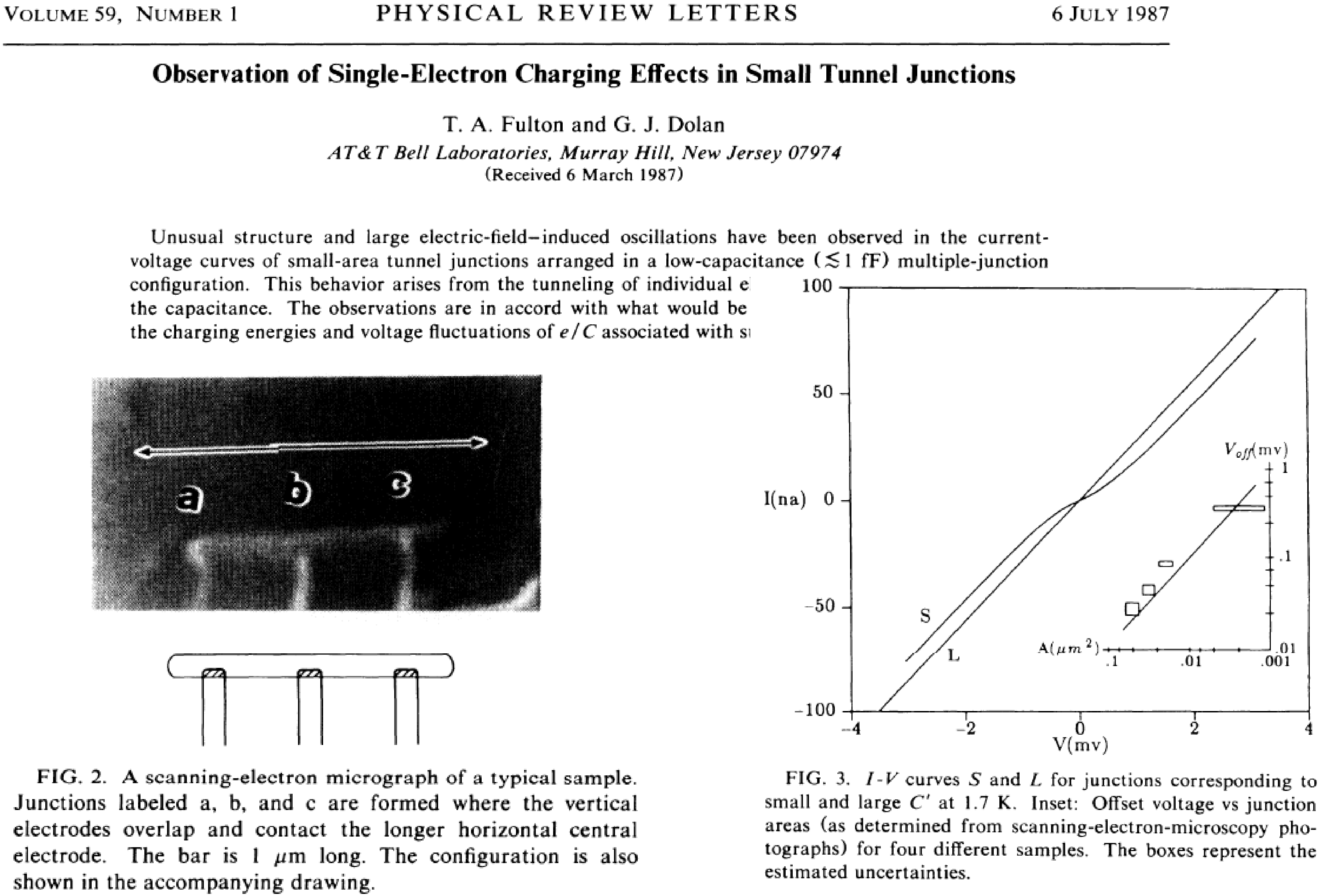
Submitted March 15, 1972

Zh. Eksp. Teor. Fiz. **63**, 1410–1416 (October, 1972)

A model is considered that describes the current state in granular media, and takes into account the threshold character of the charge transfer between granules due to the discreteness of the electric charge. In the case of tunnel junctions which contain metallic granules in the oxide layer, the law $\sigma \sim T$ is obtained for the tunnel conductivity. Oscillation effects related to the discreteness of the charge are considered.

FIG. 1. Schematic form of tunnel junction containing metallic inclusions in the oxide layer.

Fulton-Dolan



VOLUME 59, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1987

Observation of Single-Electron Charging Effects in Small Tunnel Junctions

T. A. Fulton and G. J. Dolan
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

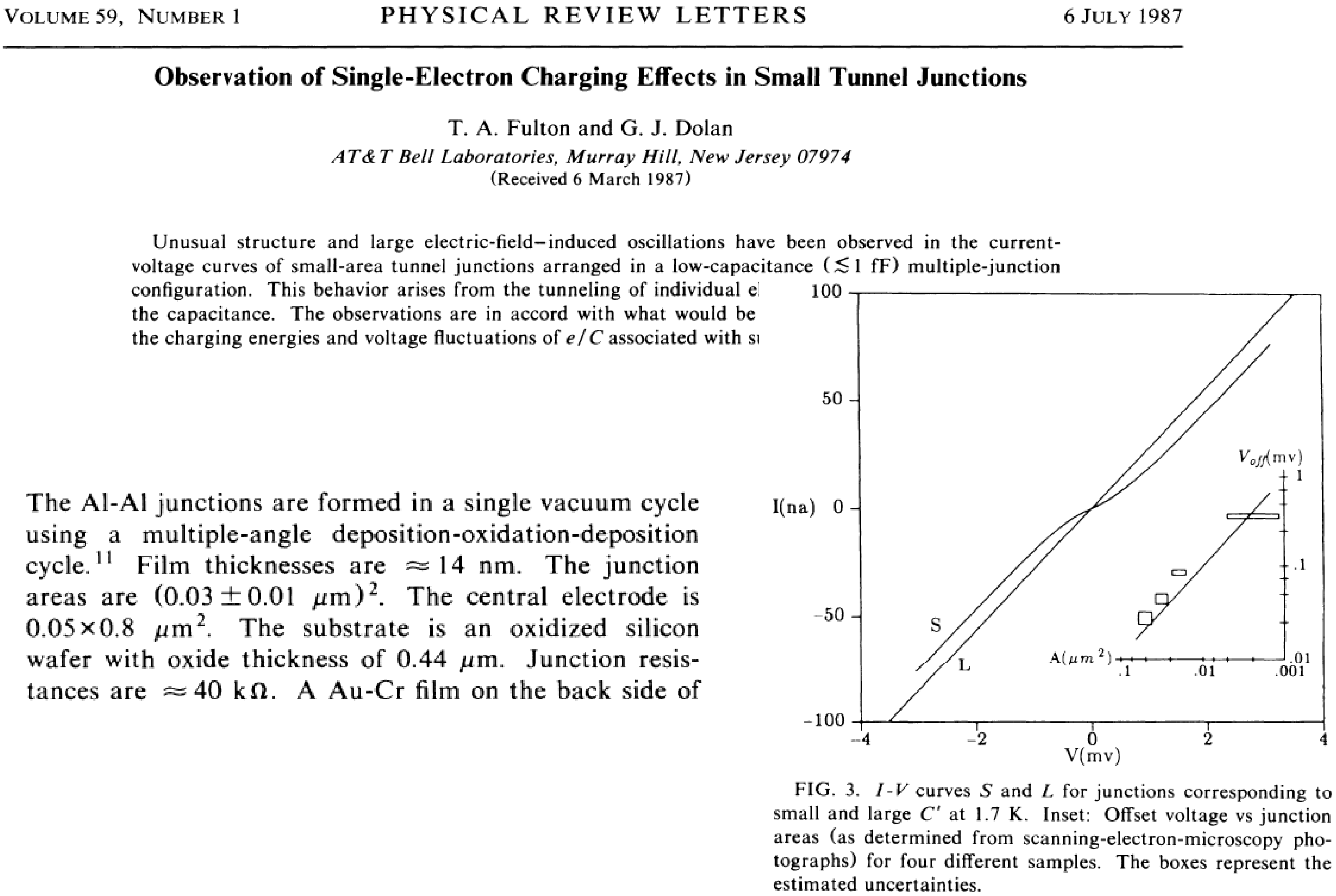
(Received 6 March 1987)

Unusual structure and large electric-field–induced oscillations have been observed in the current-voltage curves of small-area tunnel junctions arranged in a low-capacitance ($\lesssim 1$ fF) multiple-junction configuration. This behavior arises from the tunneling of individual e
the capacitance. The observations are in accord with what would be
the charging energies and voltage fluctuations of e/C associated with sı

FIG. 2. A scanning-electron micrograph of a typical sample. Junctions labeled a, b, and c are formed where the vertical electrodes overlap and contact the longer horizontal central electrode. The bar is 1 μm long. The configuration is also shown in the accompanying drawing.

FIG. 3. I-V curves S and L for junctions corresponding to small and large C' at 1.7 K. Inset: Offset voltage vs junction areas (as determined from scanning-electron-microscopy photographs) for four different samples. The boxes represent the estimated uncertainties.

Fulton-Dolan



VOLUME 59, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1987

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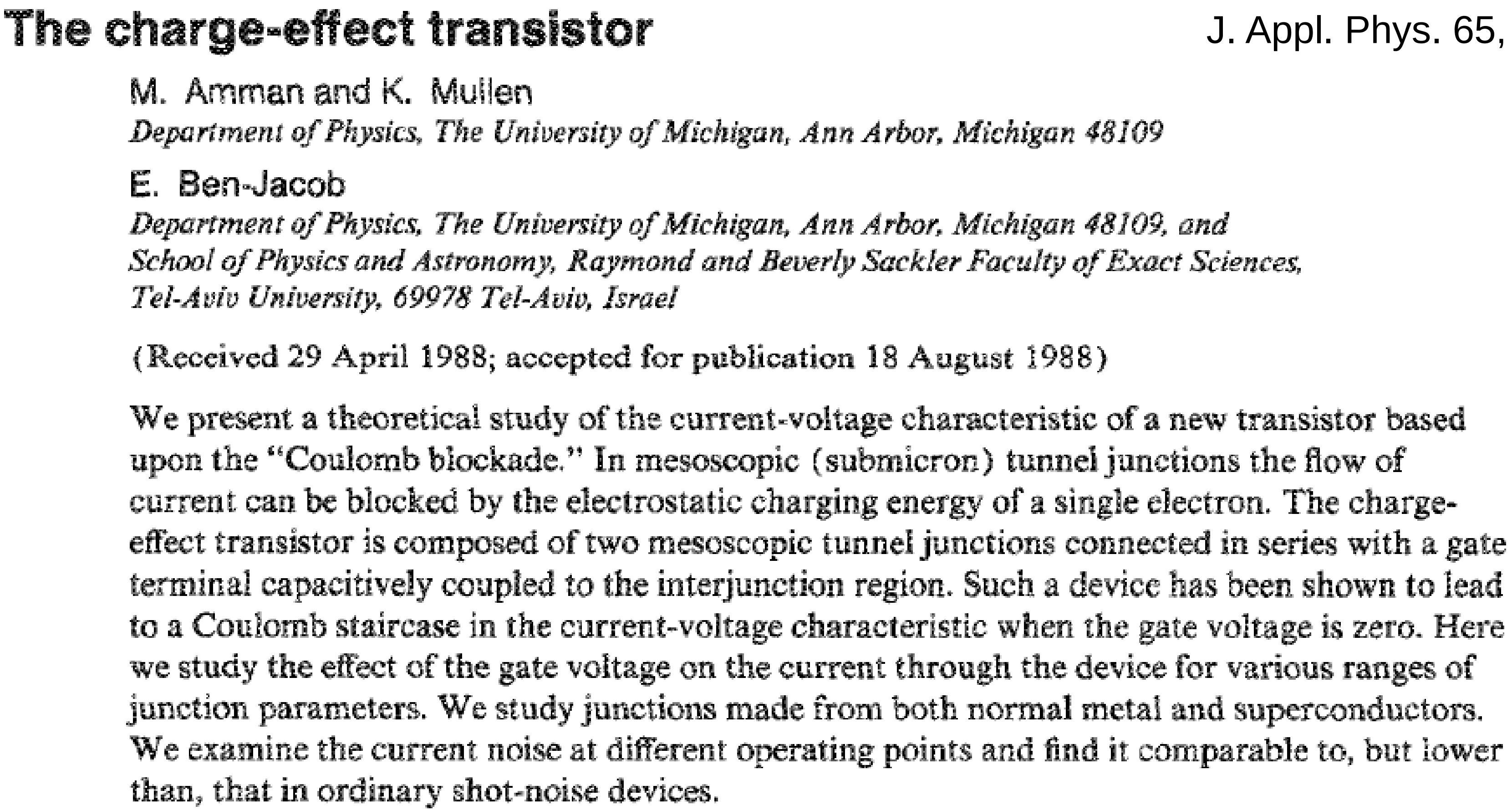
The Al-Al junctions are formed in a single vacuum cycle using a multiple-angle deposition-oxidation-deposition cycle.[11] Film thicknesses are ≈ 14 nm. The junction areas are $(0.03 \pm 0.01\ \mu\text{m})^2$. The central electrode is $0.05 \times 0.8\ \mu\text{m}^2$. The substrate is an oxidized silicon wafer with oxide thickness of 0.44 μm. Junction resistances are ≈ 40 kΩ. A Au-Cr film on the back side of

FIG. 3. *I-V* curves *S* and *L* for junctions corresponding to small and large *C'* at 1.7 K. Inset: Offset voltage vs junction areas (as determined from scanning-electron-microscopy photographs) for four different samples. The boxes represent the estimated uncertainties.

The single-electron transistor

J. Appl. Phys. 65, 339 (1989)



The charge-effect transistor

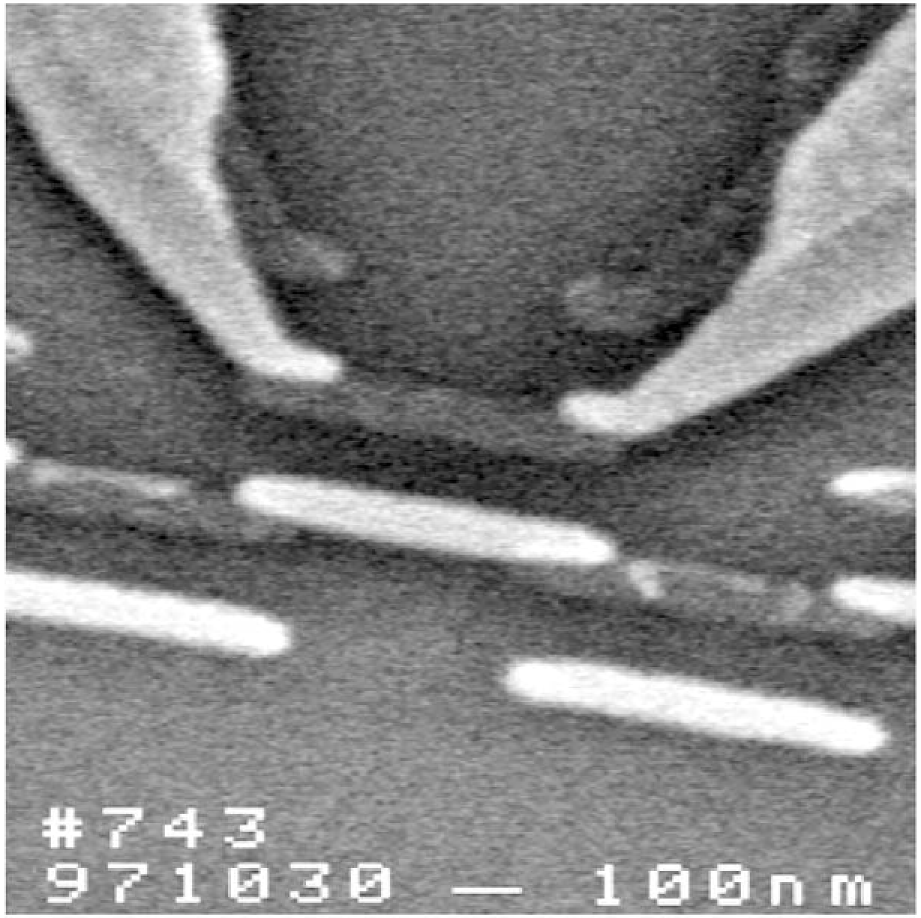
M. Amman and K. Mullen
Department of Physics, The University of Michigan, Ann Arbor, Michigan 48109
E. Ben-Jacob
Department of Physics, The University of Michigan, Ann Arbor, Michigan 48109, and School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, 69978 Tel-Aviv, Israel

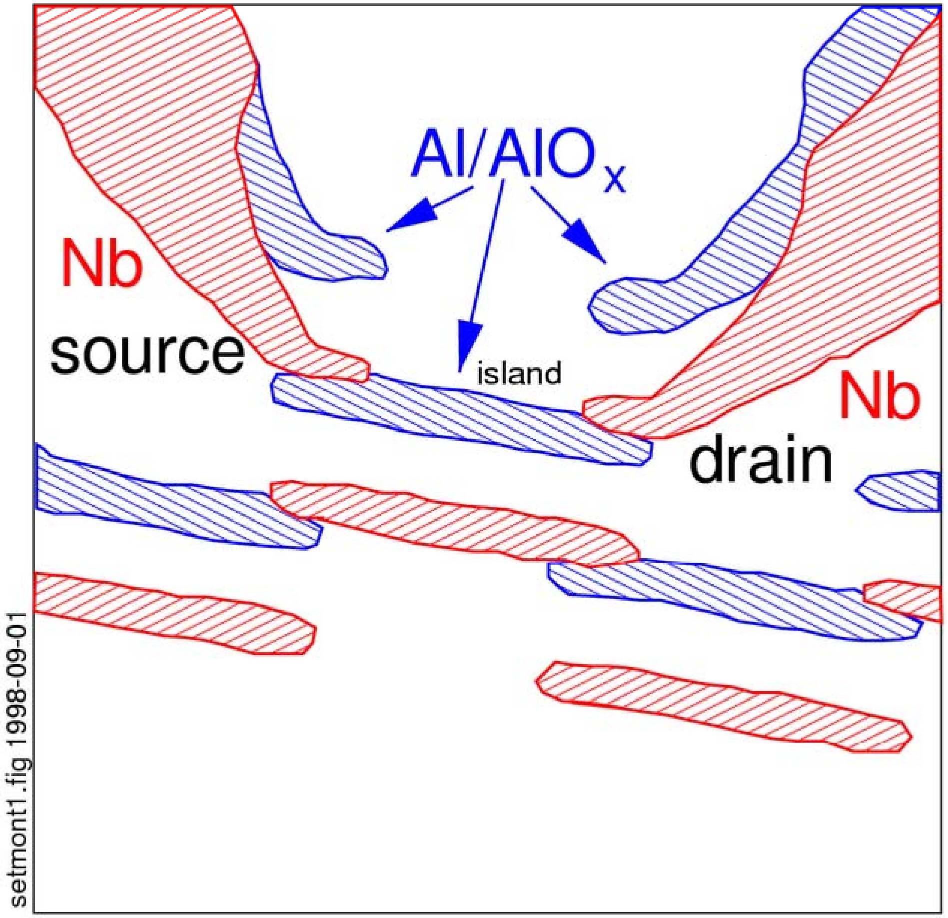
(Received 29 April 1988; accepted for publication 18 August 1988)

We present a theoretical study of the current-voltage characteristic of a new transistor based upon the "Coulomb blockade." In mesoscopic (submicron) tunnel junctions the flow of current can be blocked by the electrostatic charging energy of a single electron. The charge-effect transistor is composed of two mesoscopic tunnel junctions connected in series with a gate terminal capacitively coupled to the interjunction region. Such a device has been shown to lead to a Coulomb staircase in the current-voltage characteristic when the gate voltage is zero. Here we study the effect of the gate voltage on the current through the device for various ranges of junction parameters. We study junctions made from both normal metal and superconductors. We examine the current noise at different operating points and find it comparable to, but lower than, that in ordinary shot-noise devices.

introduces, **Coulomb blockade**, **Coulomb staircase**, Coulomb oscillation
also examine shot-noise

SET in different kind of devices





fabricated by angle-dependent evaporation
through a shadow mask in close proximity
typically made with Al, Al_2O_3, Al

SET in metallic devices

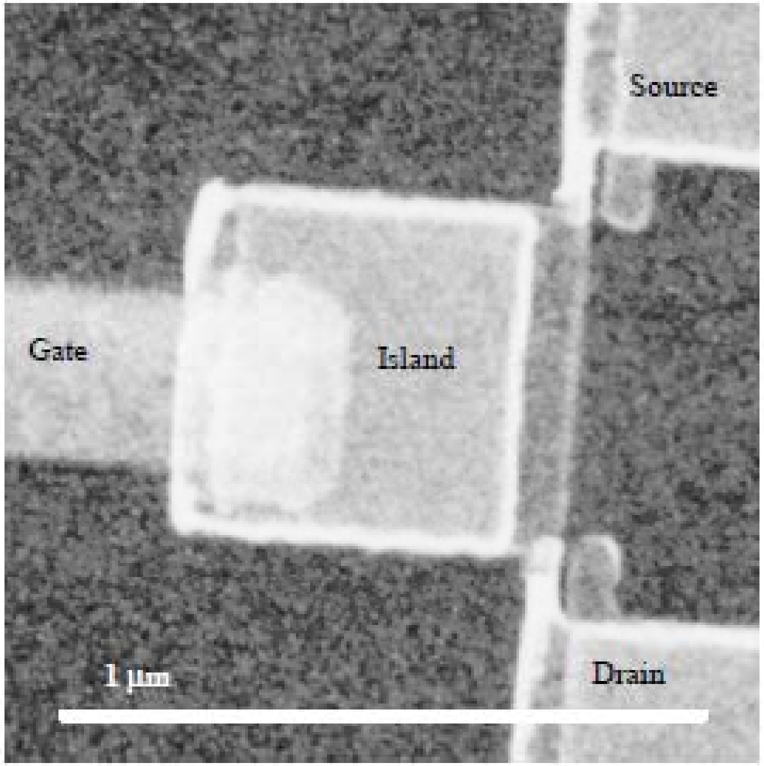
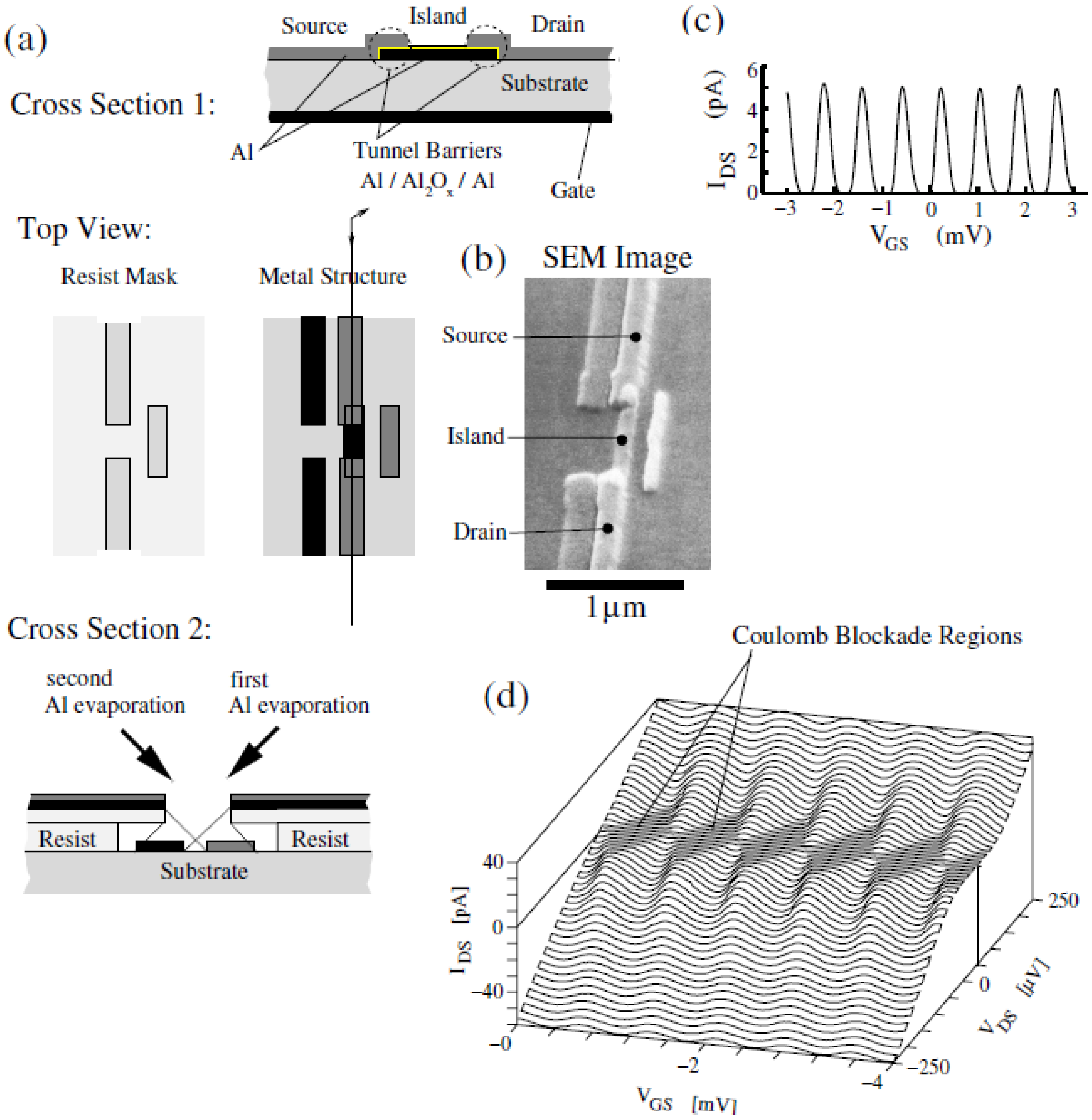
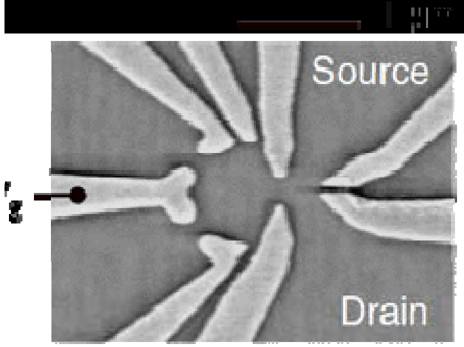


FIGURE 1 A SEM photo of a capacitively coupled SET transistor. First gold layer was deposited on an oxidized Si substrate and this was patterned by liftoff to form the gate. Next SiO was deposited to electrically isolate the gate and the island. Finally the aluminum source, drain, and island were defined by liftoff. The two tunnel junctions at the corners where the island meets the source and the drain were defined by shadow evaporation. (*Courtesy of Erik Visscher*)

SET in metallic devices

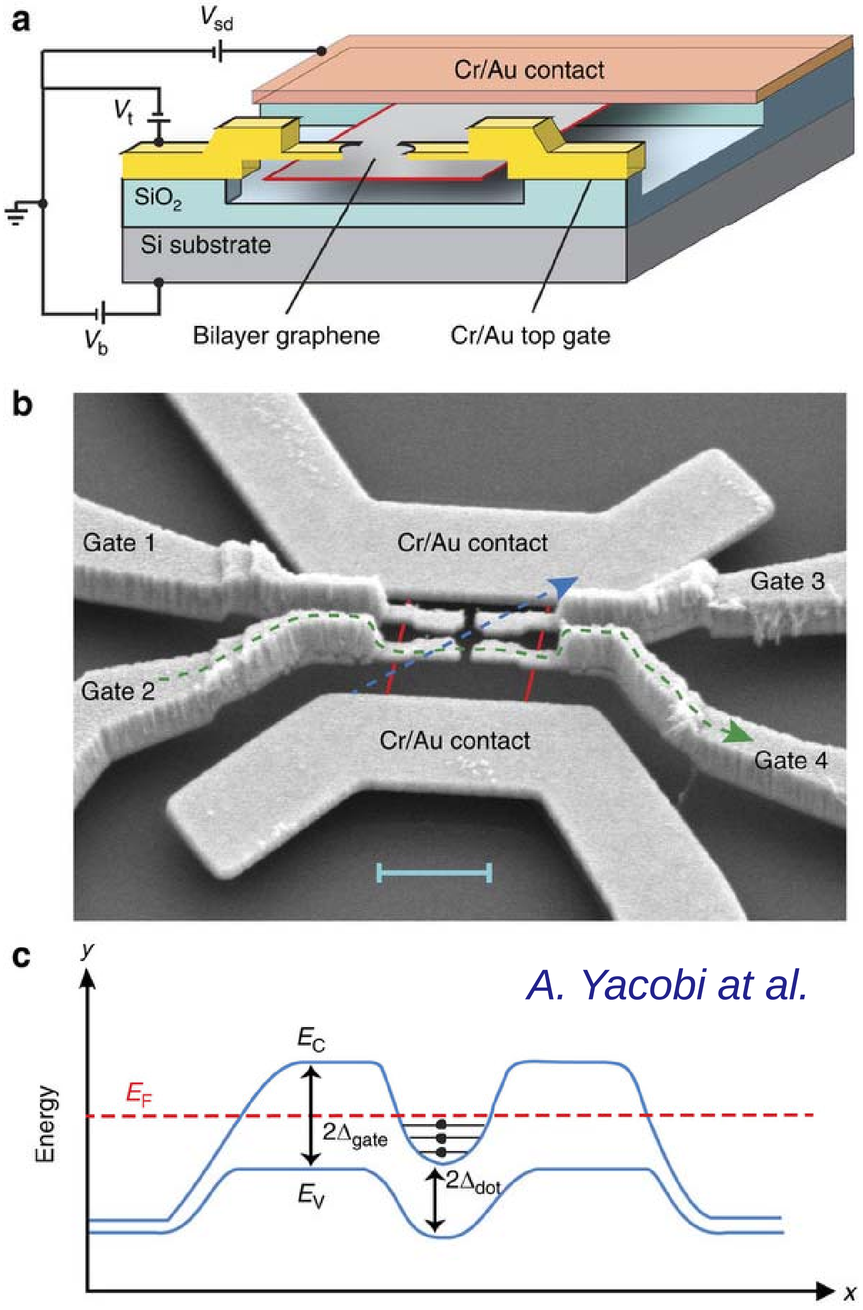


SET in semiconductors



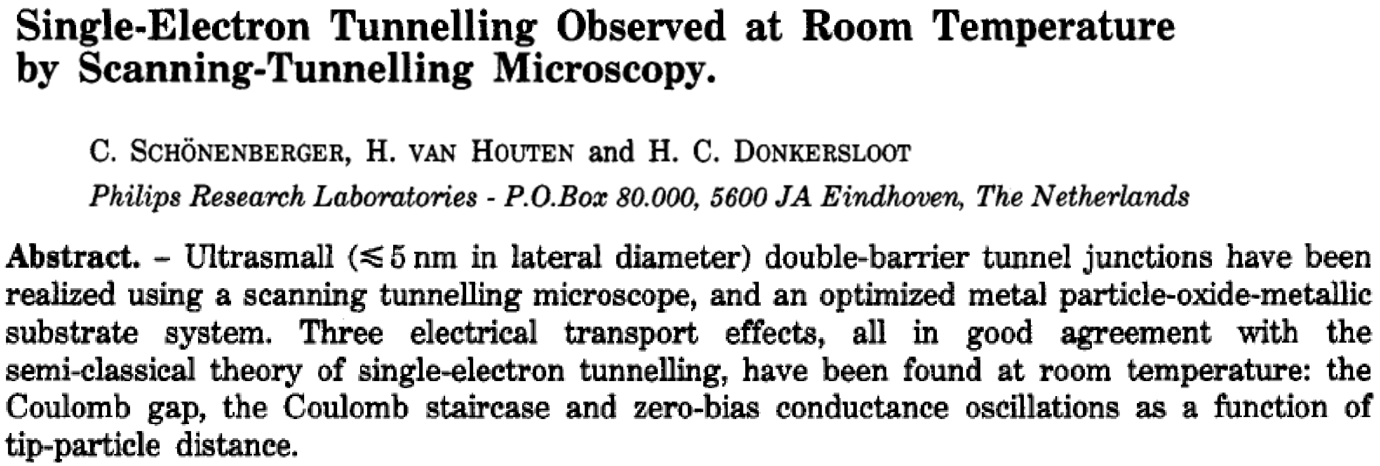
defined in semiconductors by gates, mostly GaAs, but now also in Si/Ge....graphene, CNTs

→ next chapters on quantum dots by D. Zumbühl



A. Yacobi at al.

Scanning-tunneling microscope



Single-Electron Tunnelling Observed at Room Temperature by Scanning-Tunnelling Microscopy.

C. Schönenberger, H. van Houten and H. C. Donkersloot

Philips Research Laboratories - P.O.Box 80.000, 5600 JA Eindhoven, The Netherlands

Abstract. – Ultrasmall ($\lesssim 5$ nm in lateral diameter) double-barrier tunnel junctions have been realized using a scanning tunnelling microscope, and an optimized metal particle-oxide-metallic substrate system. Three electrical transport effects, all in good agreement with the semi-classical theory of single-electron tunnelling, have been found at room temperature: the Coulomb gap, the Coulomb staircase and zero-bias conductance oscillations as a function of tip-particle distance.

EPL 1992

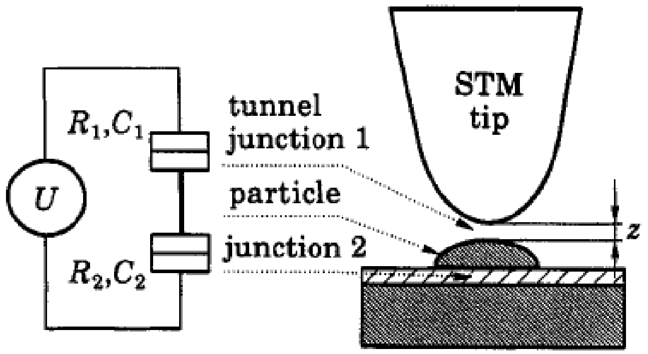


Fig. 1.

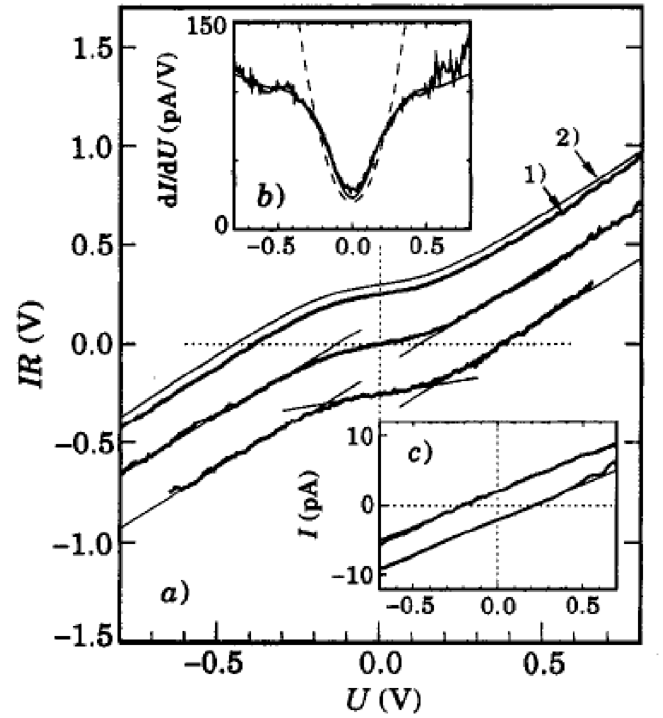
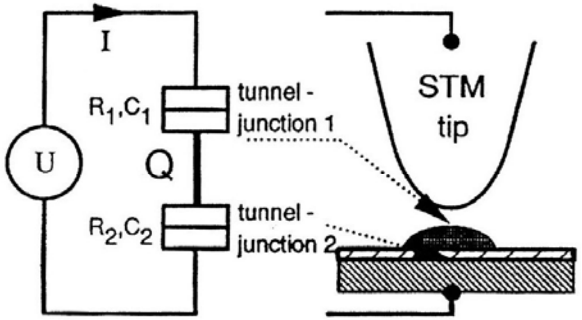
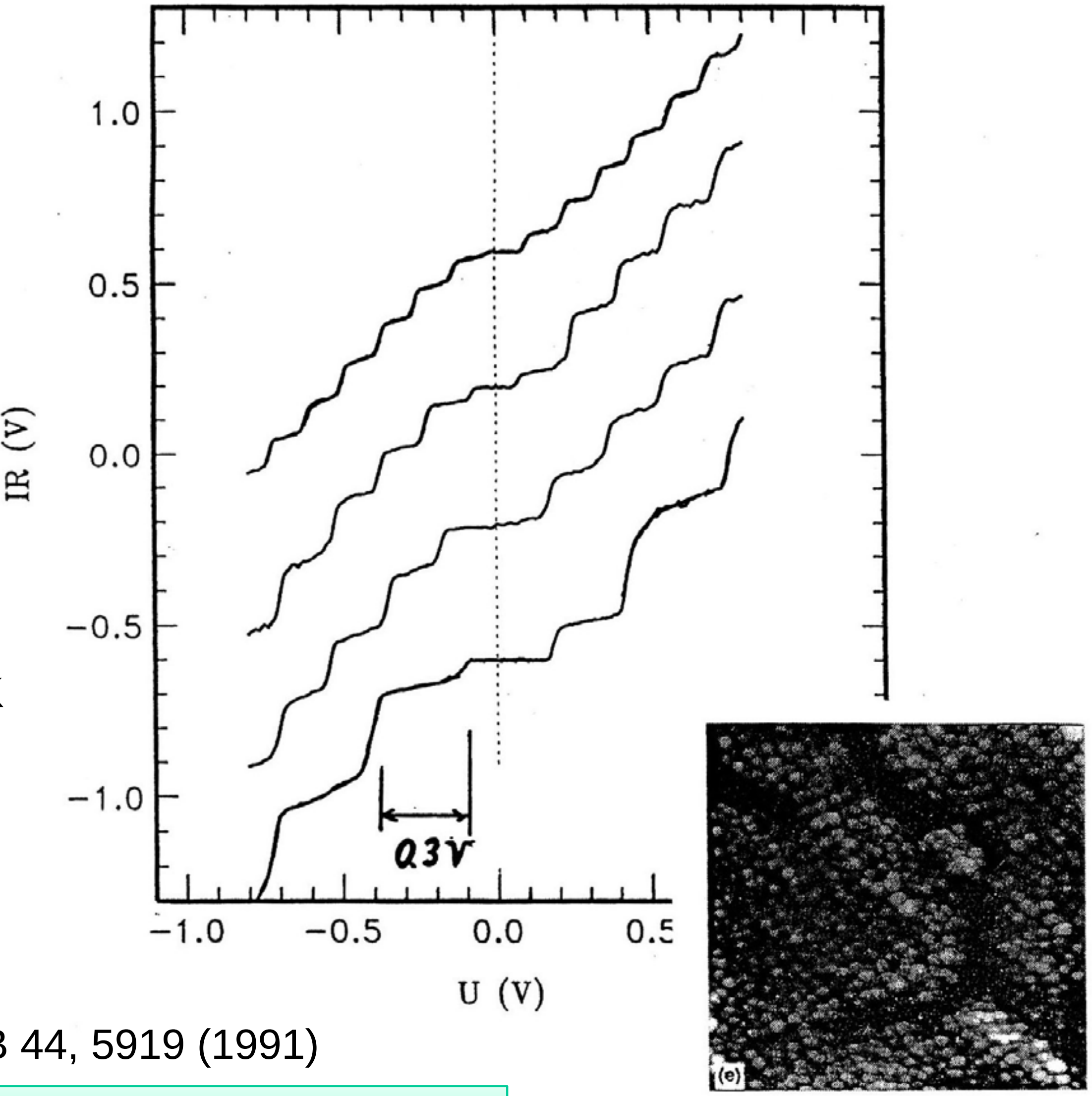


Fig. 2.

Scanning-tunneling microscope



measurements in
a low-T STM at 4K
showing Coulomb
staircases

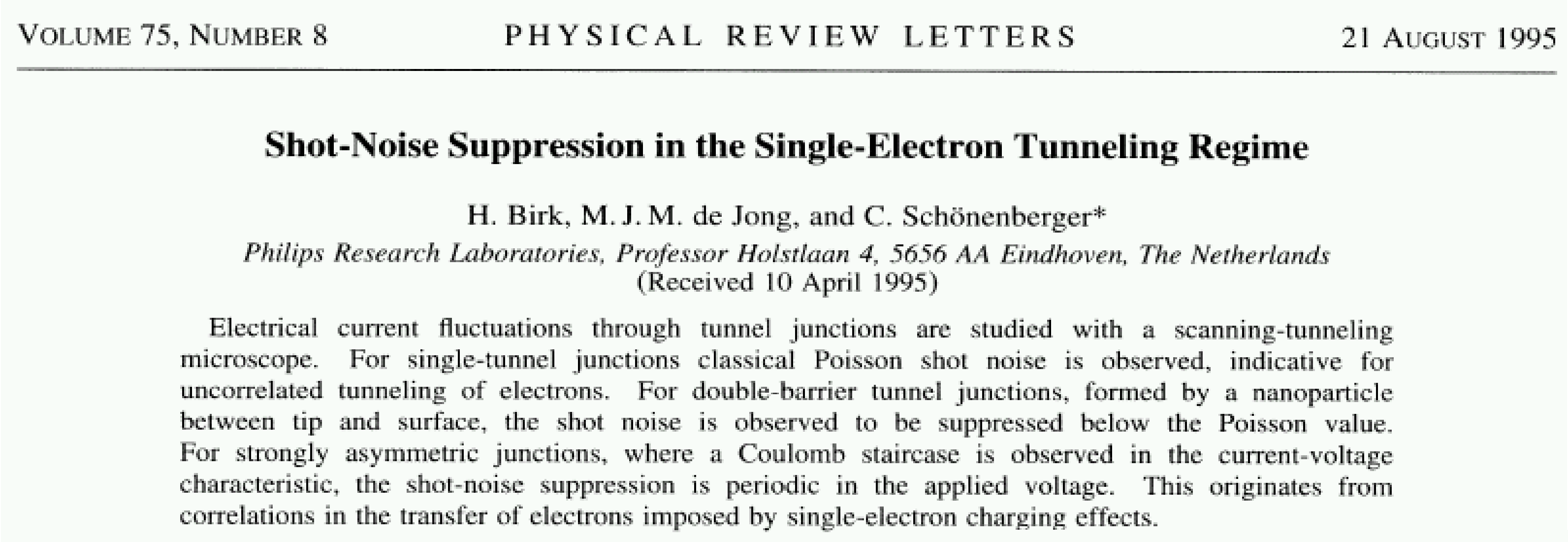


shape of staircase is nicely explains
in A.E. Hanna + M. Tinkham Phys. Rev. B 44, 5919 (1991)

Exercise 5: read above paper → discuss different shapes in exercise class.

Exercise 6: if you have fun (and time), write a program that generates I-V curves

SET with the STM



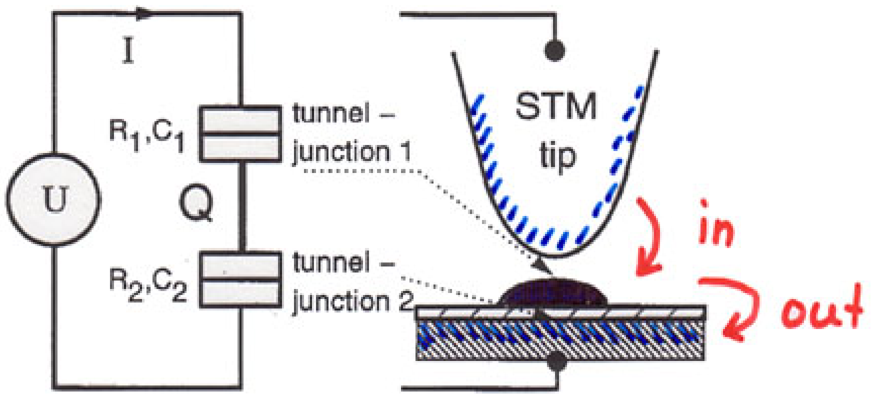
VOLUME 75, NUMBER 8 PHYSICAL REVIEW LETTERS 21 AUGUST 1995

Shot-Noise Suppression in the Single-Electron Tunneling Regime

H. Birk, M. J. M. de Jong, and C. Schönenberger*
Philips Research Laboratories, Professor Holstlaan 4, 5656 AA Eindhoven, The Netherlands

(Received 10 April 1995)

Electrical current fluctuations through tunnel junctions are studied with a scanning-tunneling microscope. For single-tunnel junctions classical Poisson shot noise is observed, indicative for uncorrelated tunneling of electrons. For double-barrier tunnel junctions, formed by a nanoparticle between tip and surface, the shot noise is observed to be suppressed below the Poisson value. For strongly asymmetric junctions, where a Coulomb staircase is observed in the current-voltage characteristic, the shot-noise suppression is periodic in the applied voltage. This originates from correlations in the transfer of electrons imposed by single-electron charging effects.



single-electron tunneling regime
with single-electron charging energies
U > 0.1eV

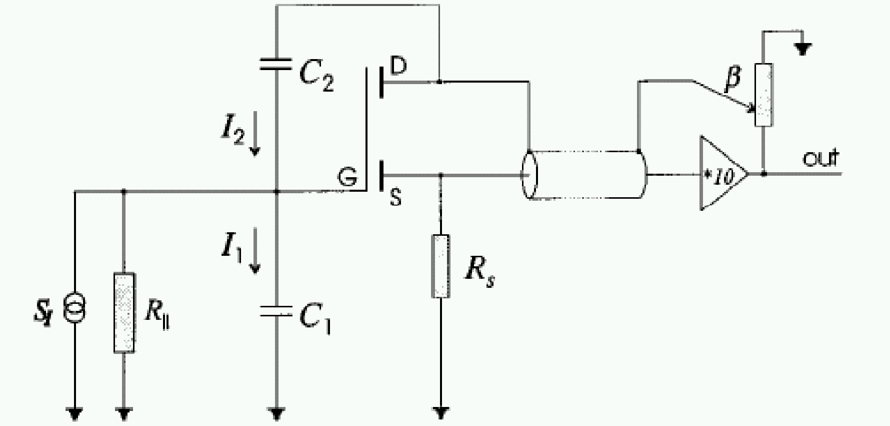
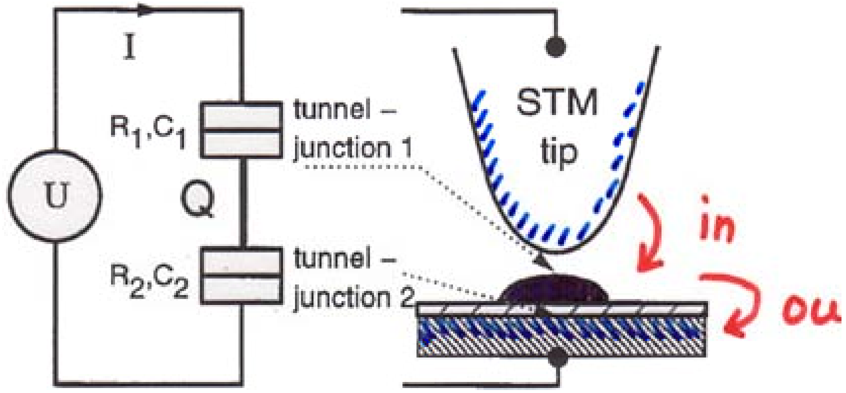
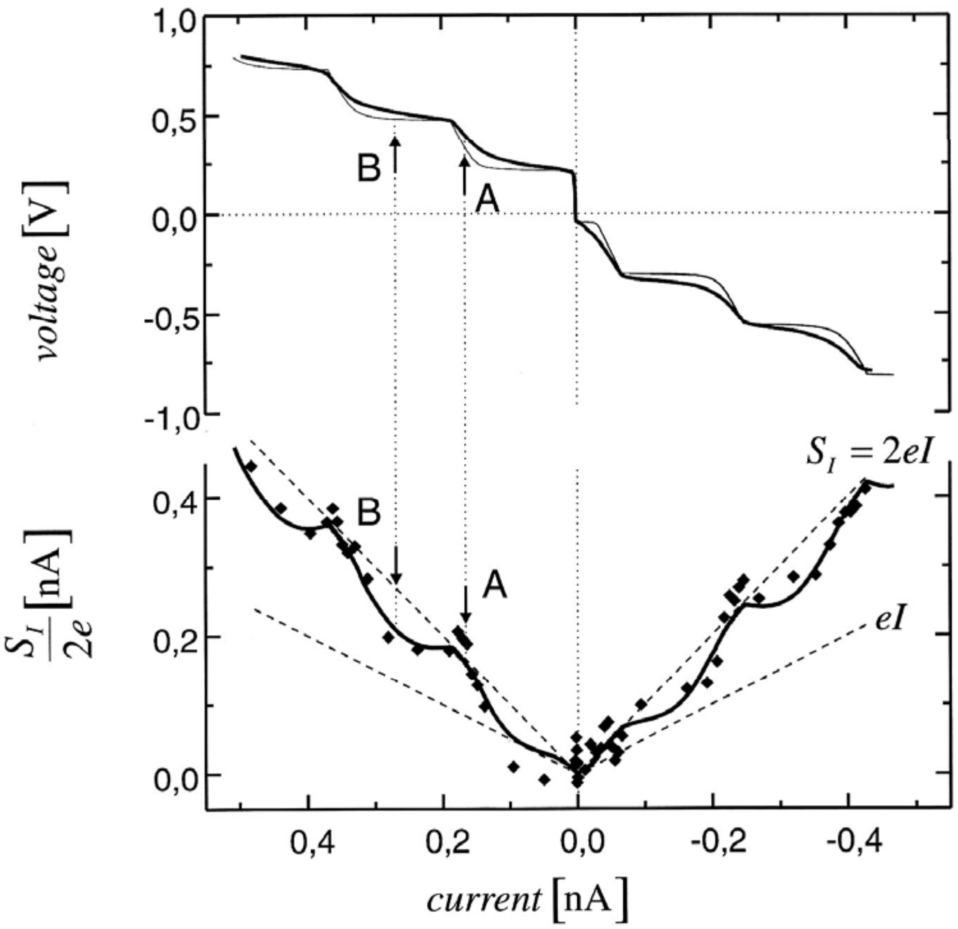


FIG. 2. Equivalent circuit for the noise measurements: The current noise S_I is converted into measurable voltage fluctuations by the resistor $R_{\|}=R_m \| R_t$. Also shown is the coaxial cable which connects the low temperature part (left-hand side) to an additional amplifier at room temperature (shown on the right-hand side). The feedback gain $\beta = U_D/U_S$ can be adjusted and is coupled back to the drain and substrate contact of the FET via the shield of the coaxial cable. This allows us to compensate the input capacitance and increases the bandwidth considerably.

Shot-noise suppression in SET

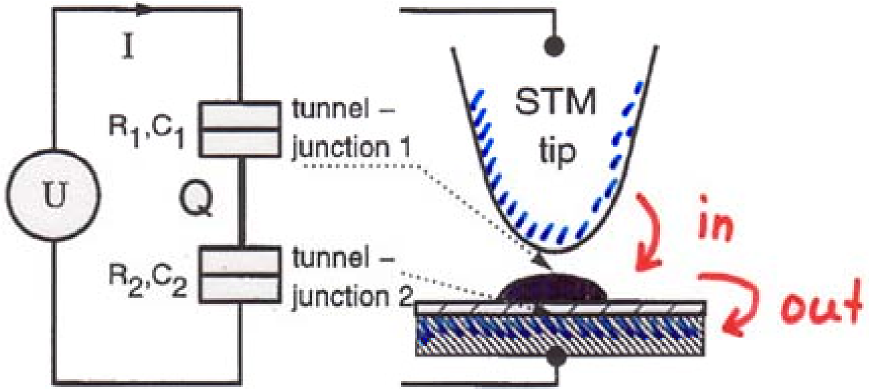




single-electron tunneling regime
with single-electron charging energies
U > 0.1eV

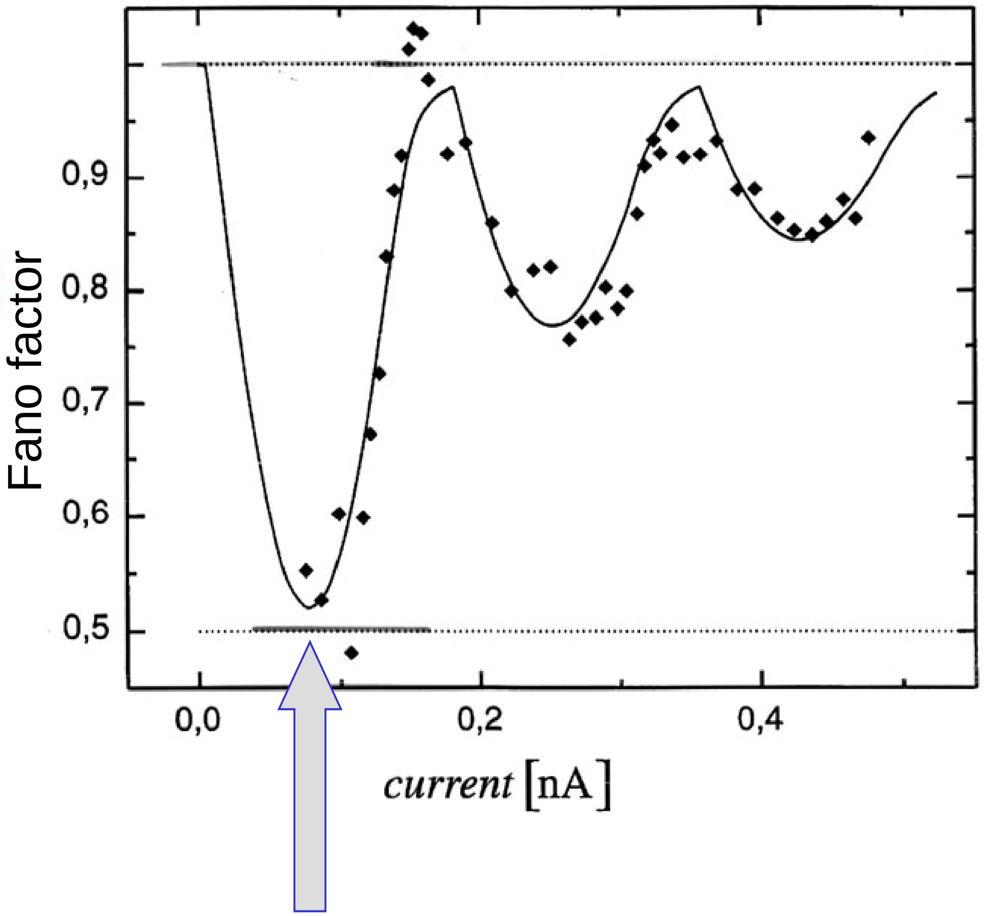
Birk et al. Phys. Rev. Lett. 75, 1610 (1995)

Shot-noise suppression in SET



single-electron tunneling regime
with single-electron charging energies
U > 0.1eV

$$S_I = 2eI \frac{\Gamma_1^{-2} + \Gamma_2^{-2}}{\left(\Gamma_1^{-1} + \Gamma_2^{-1}\right)^2} \geq eI$$



<u>Origin of suppression:</u>
CB regulates the sequence of tunneling events between the two junctions:
„an electron can only tunnel into the island, only if the previous one has left it“

Birk et al. Phys. Rev. Lett. 75, 1610 (1995)

SET with “single molecules”

Single-electron transistor of a single organic molecule with access to several redox states

Nature 2003

**Sergey Kubatkin[1], Andrey Danilov[1], Mattias Hjort[2], Jérôme Cornil[2,3],
Jean-Luc Brédas[2,3]*, Nicolai Stuhr-Hansen[4], Per Hedegård[4]
& Thomas Bjørnholm[4]**

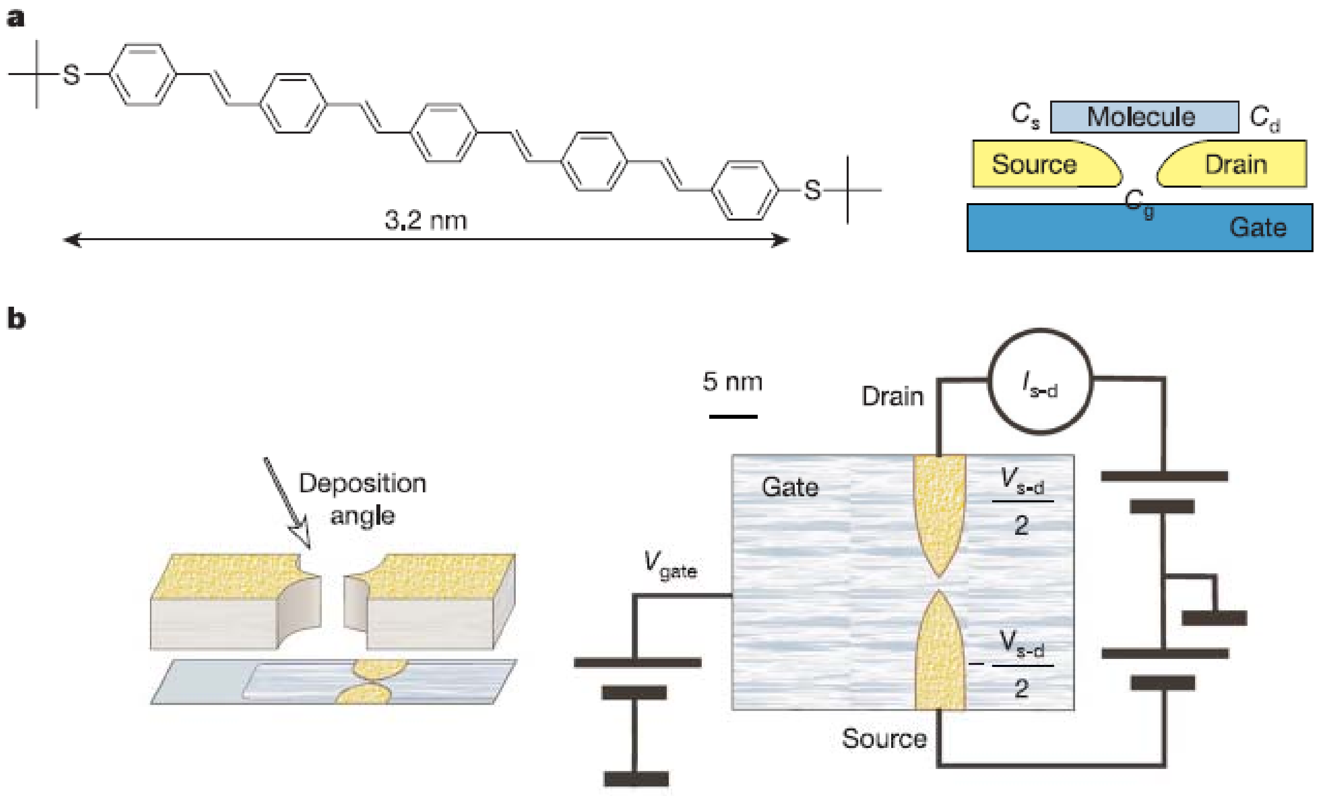


Figure 1 Device and experiment. **a**, Molecular structure of OPV5 and schematic experimental set-up. **b**, Schematic representation of the device preparation procedure.

SET with “single molecules”

Single-electron transistor of a single organic molecule with access to several redox states

Nature 2003

Sergey Kubatkin[1], **Andrey Danilov**[1], **Mattias Hjort**[2], **Jérôme Cornil**[2,3],
Jean-Luc Brédas[2,3]*, **Nicolai Stuhr-Hansen**[4], **Per Hedegård**[4]
& Thomas Bjørnholm[4]

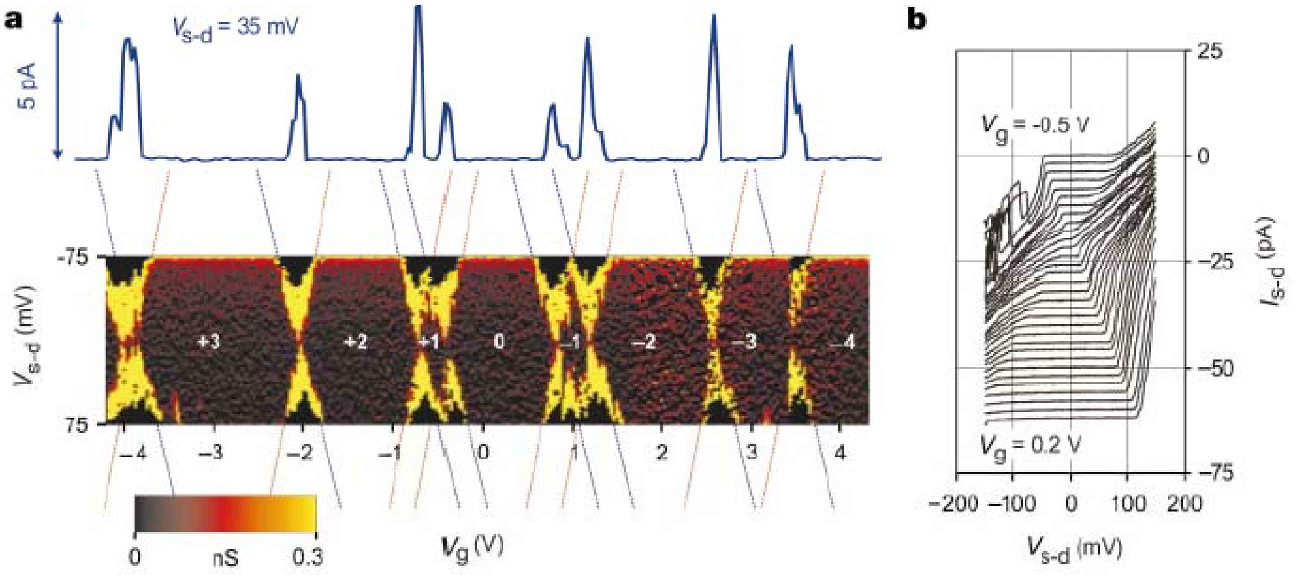


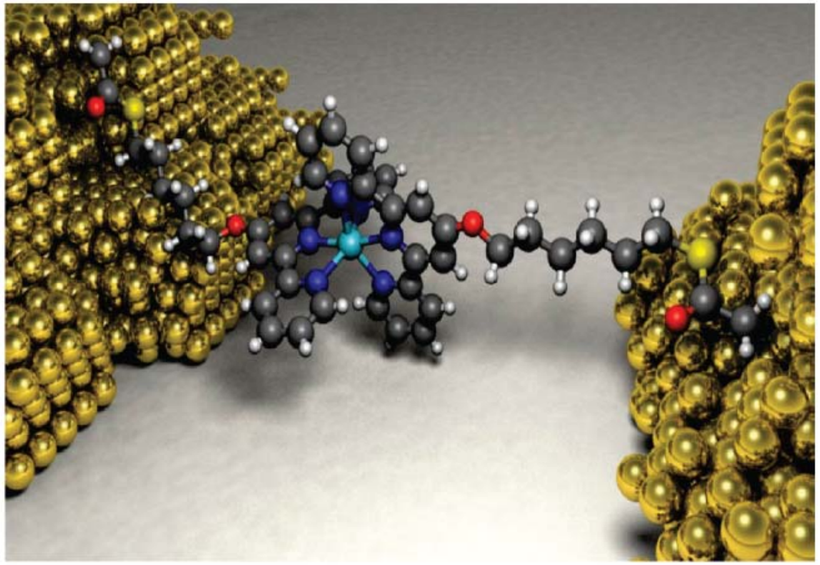
Figure 2 Experimental results. **a**, Measurements of the differential conductance ($dI_{s\text{-}d}/dV_{s\text{-}d}$) as function of $V_{s\text{-}d}$ and V_g. All red lines, and all blue lines, have identical slopes, as discussed in the text. The full solid line at the top of the figure shows a representative $I_{s\text{-}d}$–V_g trace. **b**, Examples of current–voltage curves $I_{s\text{-}d}$–($V_{s\text{-}d}$) for a single OPV5 molecule obtained at different gate potentials V_g (temperature $T = 4.2$ K). Curves are shifted vertically for clarity.

SET with “single molecules”

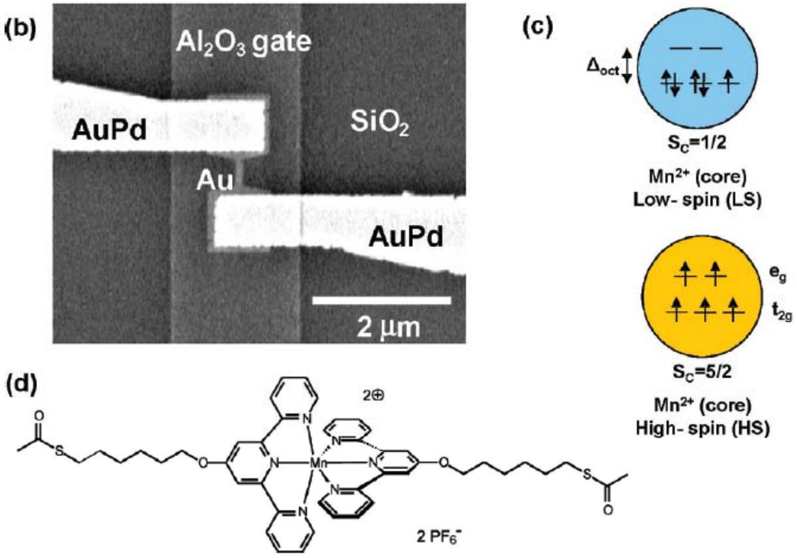
Electrical Manipulation of Spin States in a Single Electrostatically Gated Transition-Metal Complex

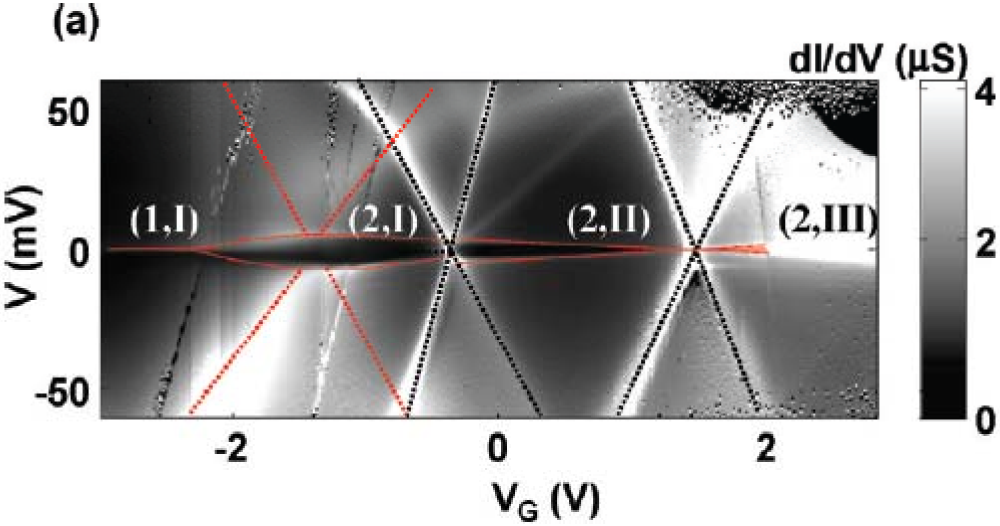
Nano Lett. 2010

Edgar A. Osorio,† Kasper Moth-Poulsen,† Herre S. J. van der Zant,*,† Jens Paaske,† Per Hedegård,† Karsten Flensberg,† Jesper Bendix,† and Thomas Bjørnholm†



junction fabricated by electromigration



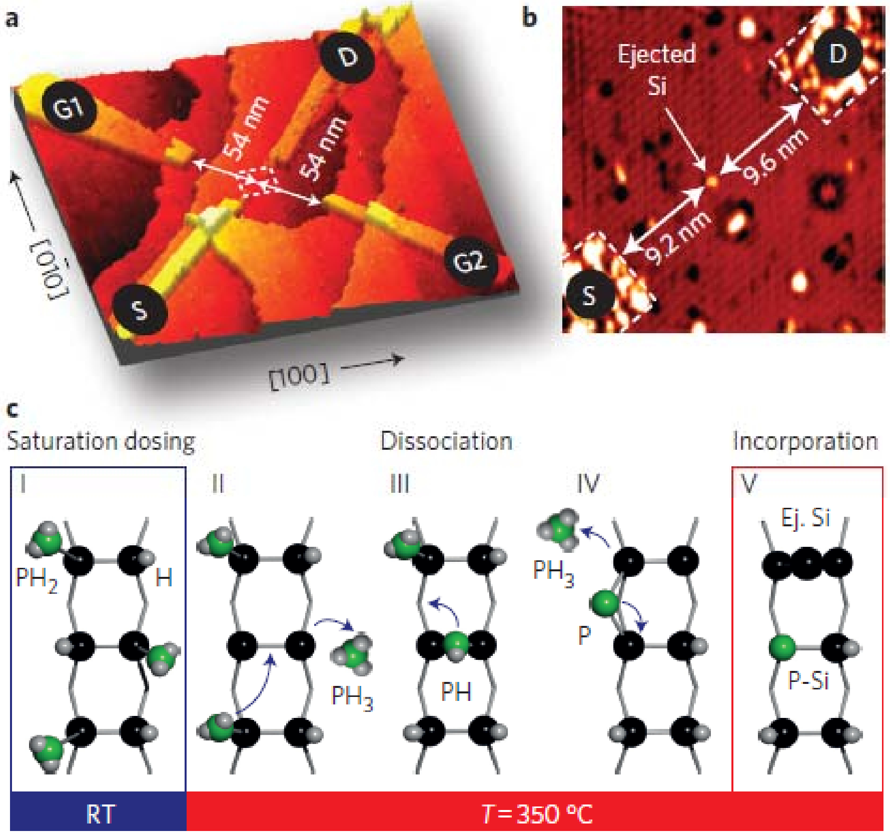


SET of a single dopant

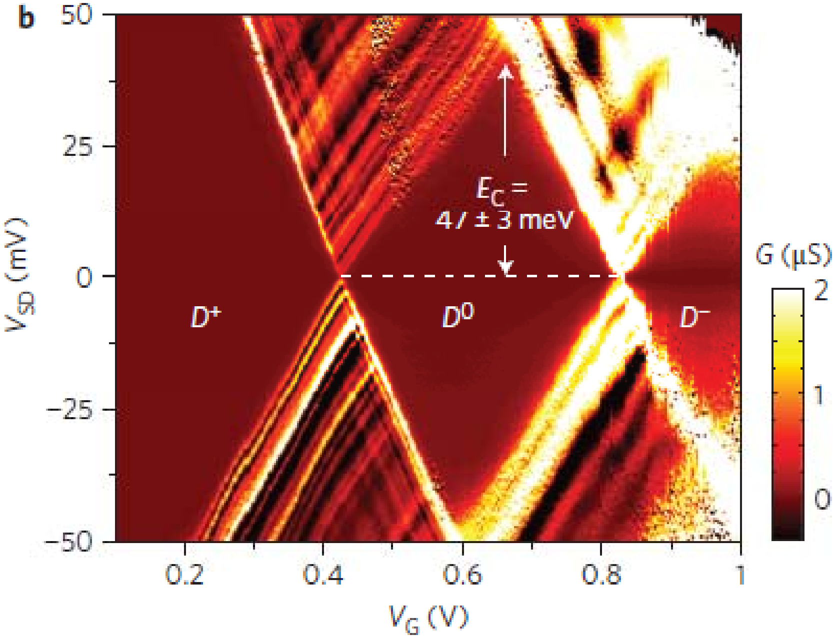
LETTERS
PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NNANO.2012.21
nature nanotechnology

A single-atom transistor

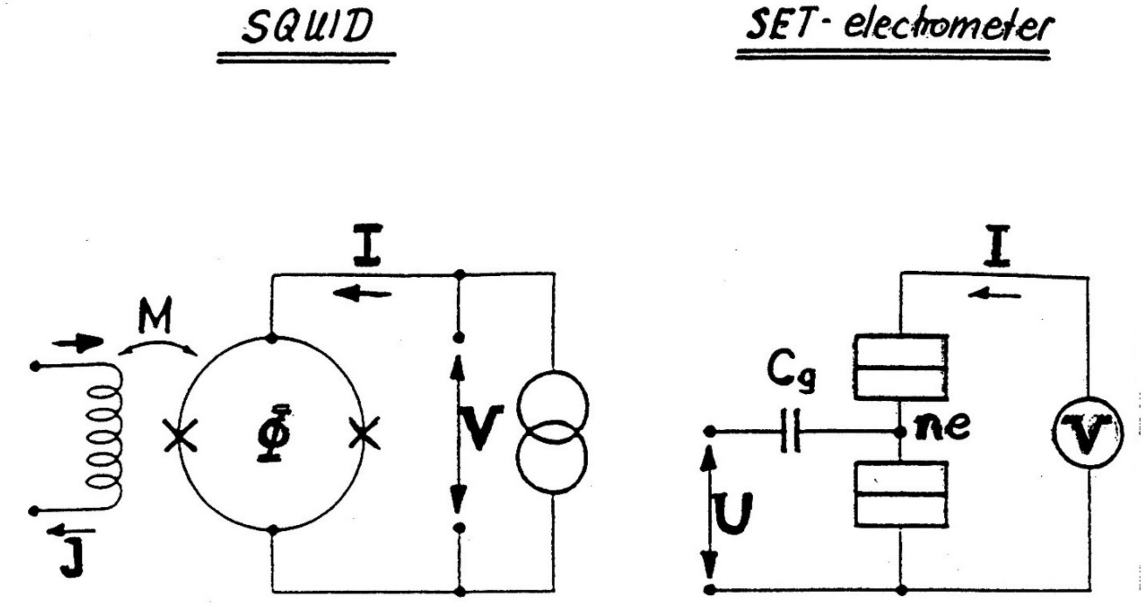
Martin Fuechsle[1], Jill A. Miwa[1], Suddhasatta Mahapatra[1], Hoon Ryu[2], Sunhee Lee[3], Oliver Warschkow[4], Lloyd C. L. Hollenberg[5], Gerhard Klimeck[3] and Michelle Y. Simmons[1]*



P-atom in/on Si



SET as electrometer



Loop		⟷	Island	
Flux	Φ	⟷	Charge	ne
Bias	I	⟷	Bias	V
Signal	J	⟷	Signal	U
Output	V	⟷	Output	I
Mutual inductance	M	⟷	Capacitance	C_g

SET as electrometer

Scanning Single-Electron Transistor Microscopy: Imaging Individual Charges

M. J. Yoo,* T. A. Fulton, H. F. Hess, R. L. Willett,
L. N. Dunkleberger, R. J. Chichester, L. N. Pfeiffer, K. W. West

Science 276, 579 (1999)

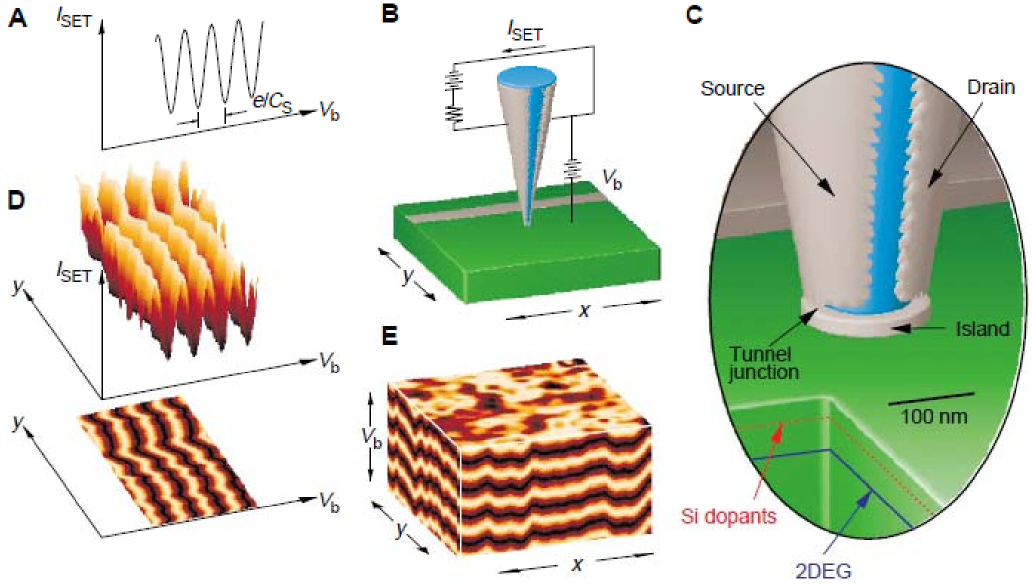
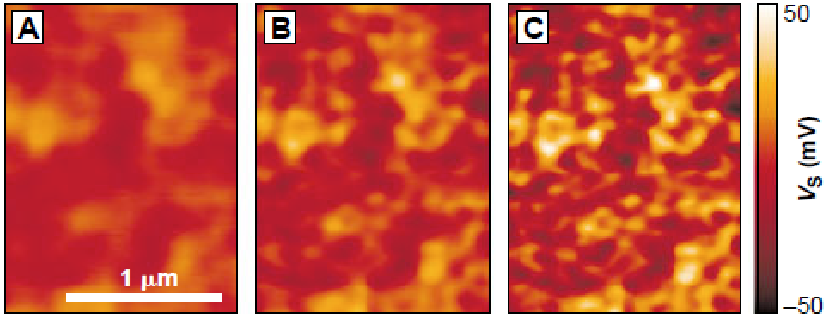


Fig. 1. (**A**) Typical current oscillations $I_{SET}(V_b)$ of a SET. (The zero of I_{SET} is offset; the amplitude is actually ~20% of the average). (**B**) Schematic depiction of the SET probe tip suspended above the GaAs/Al_xGa_{1-x}As heterostructure near a gate. (**C**) Magnified view of the tip and a cutaway view of the sample. (**D**) $I_{SET}(V_b)$ versus y for a 2-μm line scan: (top) 3D representations after (A); (bottom) 2D color representation of the same data. The wiggling of the stripes is caused by variations in the electrical charge density of the surface. (**E**) Color representation of a complete data set of $I_{SET}(V_b)$ versus x and y for a 2 μm by 2 μm raster scan over a nongated region. The top of the data block maps the electric field of the sample surface as detected by the SETSE. The side of the block is taken from (D).

Fig. 3. Surface potential V_s in the xy plane (relative to the average value of each image) showing fluctuations from dopants and surface charges as seen at scan heights of (**A**) 95 nm, (**B**) 50 nm, and (**C**) 25 nm. The spatial resolution and sensitivity improve with reduced height.



Charge-diagram of two islands

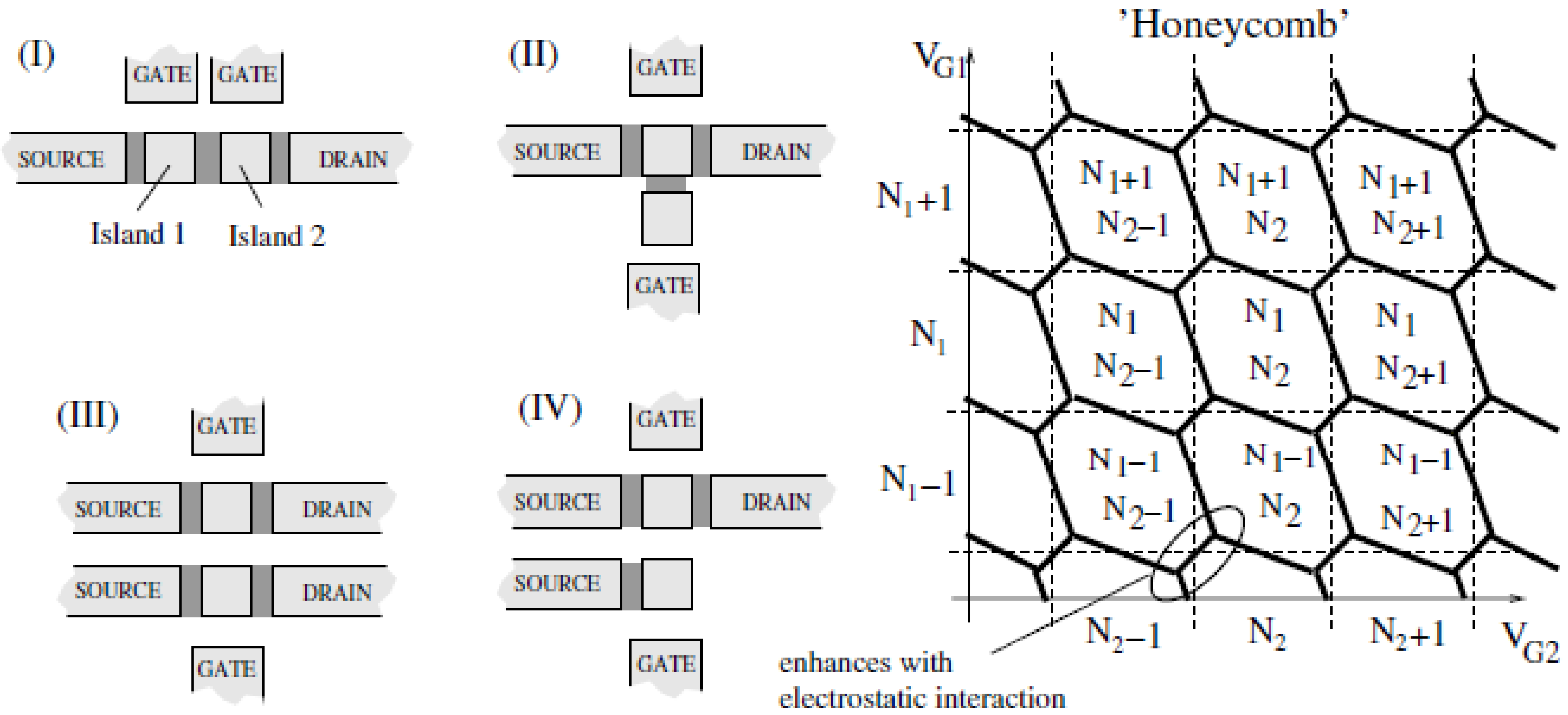


Fig. 16. Charge stability diagram valid for the two-island arrangements (I) to (IV) for $V_{\mathrm{DS}} = 0$ – denoted as 'honeycomb' pattern.

Charge-diagram of two islands

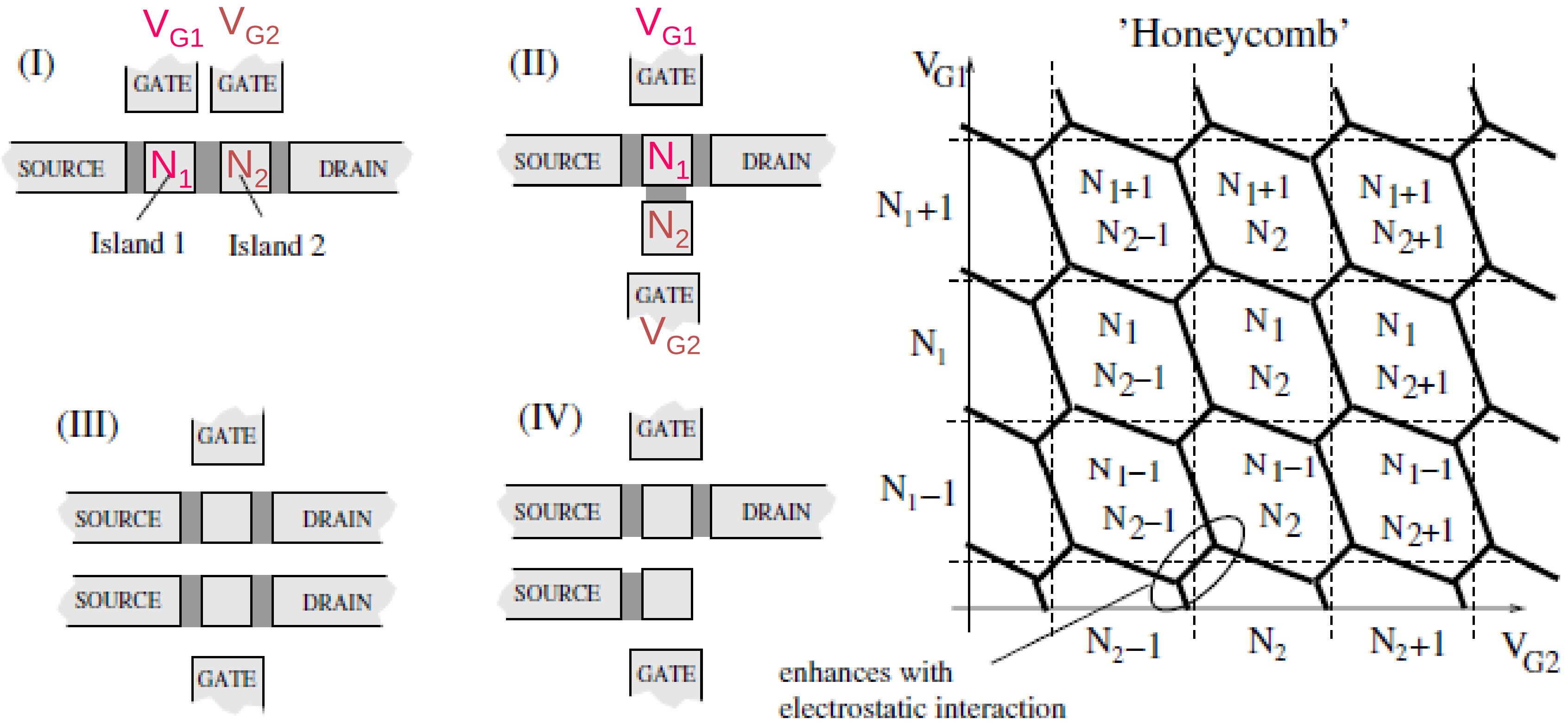


Fig. 16. Charge stability diagram valid for the two-island arrangements (I) to (IV) for $V_{\rm DS} = 0$ – denoted as 'honeycomb' pattern.

Exercise 7: assume that we study the linear-response conductance (small source-drain bias) of the above systems. The charge-stability diagram on the right is in principle valid for all four cases. Indicate in the charge stability diagram where one can expect a finite source-drain conductance. a) in case (I) assume that the electrostatic interaction is zero (charge stability diagram consists of squares). b) in case (II) assume that there is substantial interaction. c) for case (III) assume that the two source contacts are connected together and similarly for the two drain contacts.

Single-electron pump

Single-Electron Pump Based on Charging Effects.

H. Pothier, P. Lafarge, C. Urbina, D. Esteve and M. H. Devoret

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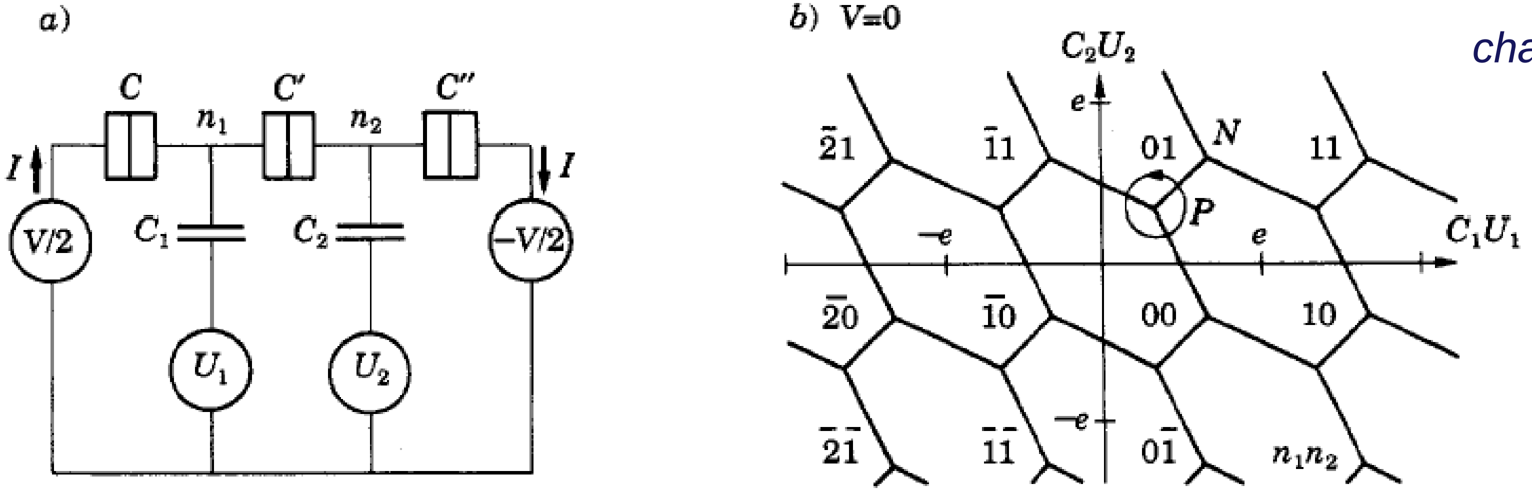
Abstract. – We have designed and operated a device consisting of three nanoscale tunnel junctions biased below the Coulomb gap. Phase shifted r.f. voltages of frequency f applied to two gates «pump» one electron per cycle through the device. This is shown experimentally by plateaus in the current-voltage characteristic at $I = \pm ef$, the sign of the current depending on the relative phase of the r.f. voltages and not on the sign of the bias voltage.



charge-stability diagram

Fig. 1. – Principle of reversible transfer of a single electron using a «pump» controlled by two gate voltages U_1 and U_2. *a*) Circuit schematic: the nanoscale junctions constituting the pump are represented by double-box symbols. *b*) Stable configuration diagram for $V = 0$ and $C = C' = C''$. One turn around a triple point such as P or N, obtained by modulating the gate voltages by two phase-shifted signals, induces one electron to go around the circuit.

Single-electron pump

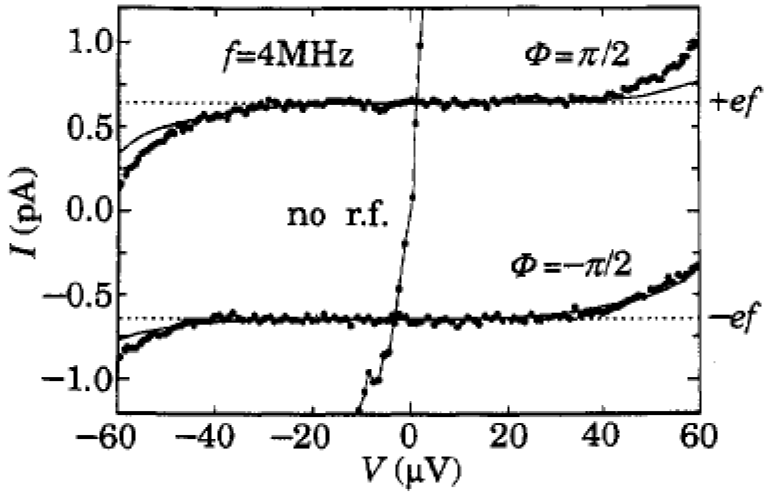


Fig. 3.

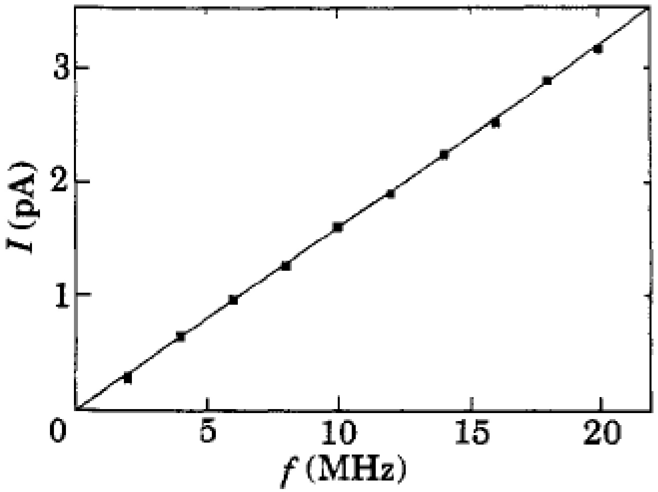
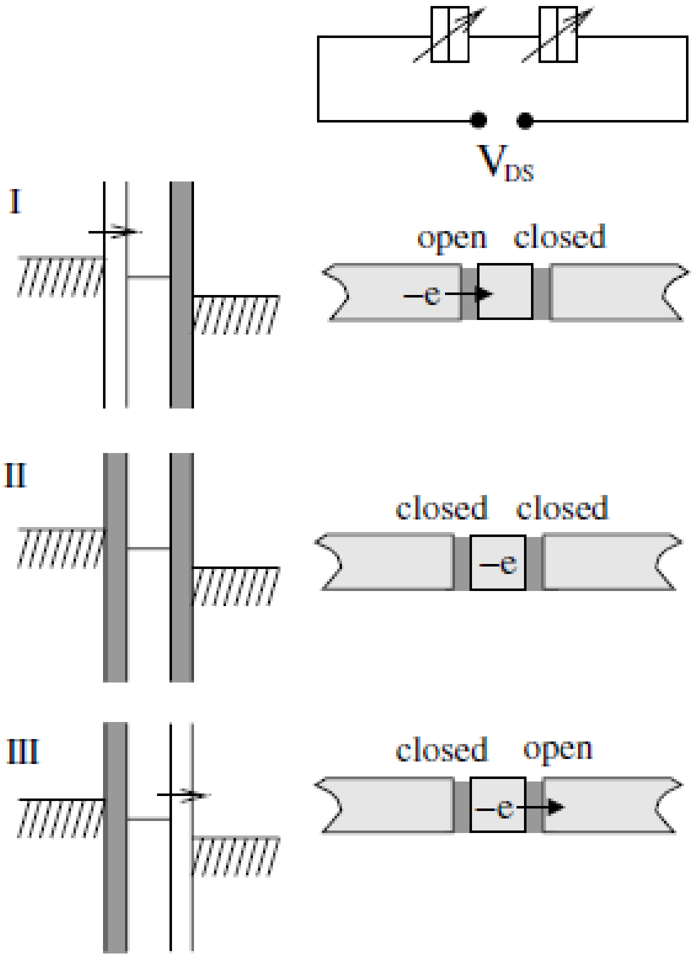


Fig. 4.

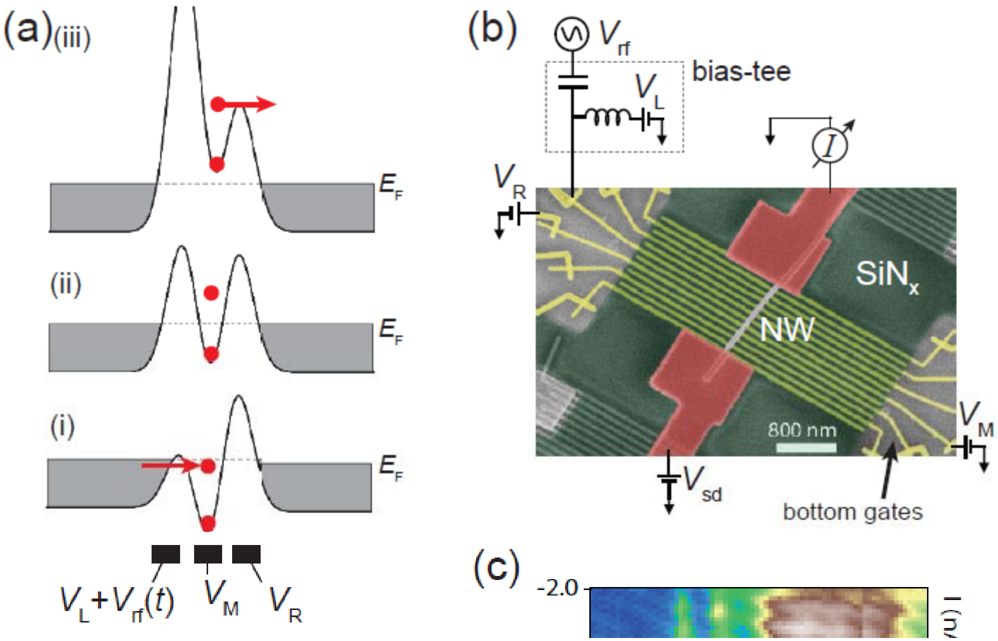
Fig. 3. – Current-voltage characteristics with and without a $f = 4$ MHz gate voltage modulation around a «P-type» triple point. The U_1 and U_2 r.f. amplitudes were 1 mV and 0.6 mV, respectively. Current plateaus are seen at $I = \pm ef$ (marked with dashed lines), the sign depending on the phase shift Φ between the two r.f. signals. Results of numerical simulations including co-tunnelling events are shown in full lines.

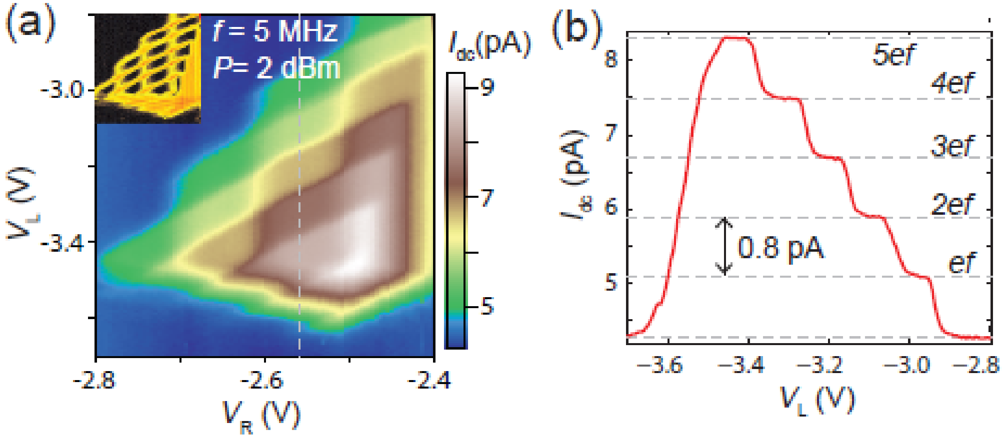
Fig. 4. – Current at the inflexion point of the plateaus of fig. 3 *vs.* r.f. frequency. Full line: theoretical prediction $I = ef$.

Single-electron turnstyle (pump)



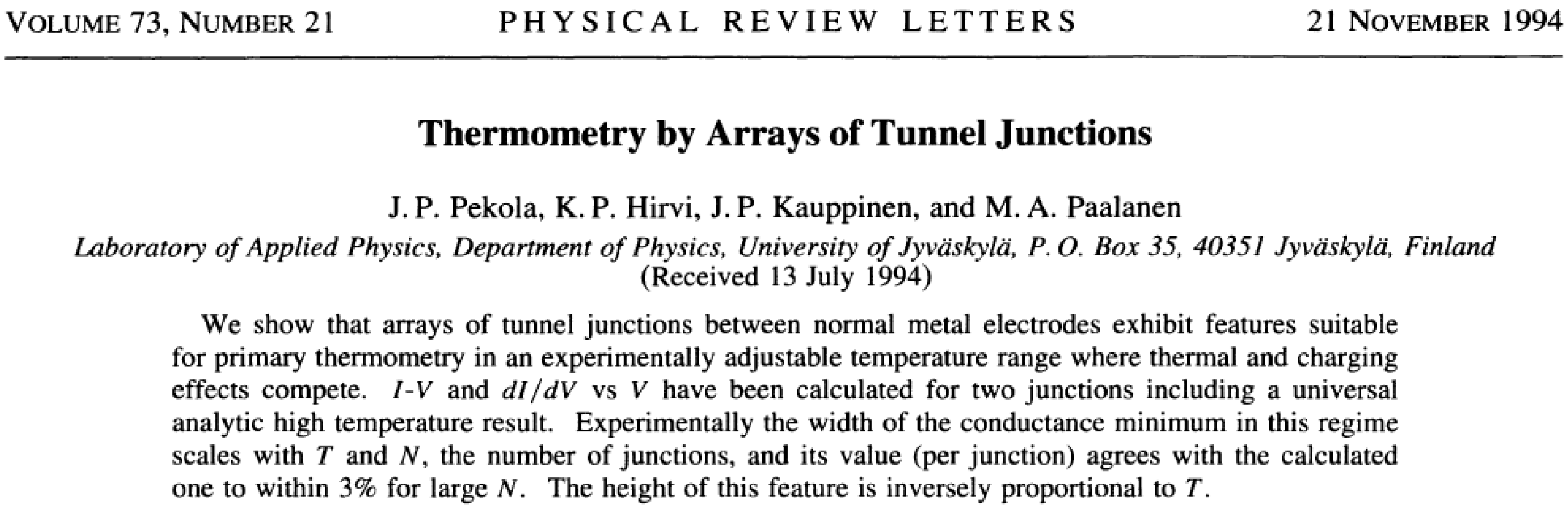
works by tuning the barriers





S. d'Hollosy et al. (CS group), to be published

SET thermometer



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Thermometry by Arrays of Tunnel Junctions

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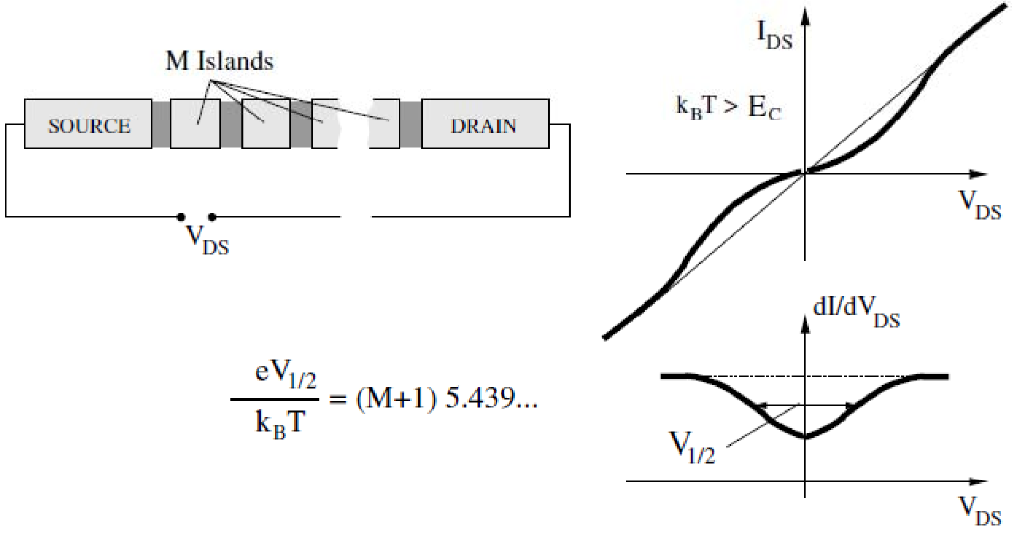
(Received 13 July 1994)

We show that arrays of tunnel junctions between normal metal electrodes exhibit features suitable for primary thermometry in an experimentally adjustable temperature range where thermal and charging effects compete. I-V and dI/dV vs V have been calculated for two junctions including a universal analytic high temperature result. Experimentally the width of the conductance minimum in this regime scales with T and N, the number of junctions, and its value (per junction) agrees with the calculated one to within 3% for large N. The height of this feature is inversely proportional to T.

Casparis et al. Rev. Sci. Instr.. 83, 083903 (2012)



$$\frac{eV_{1/2}}{k_BT} = (M+1)\,5.439...$$

Fig. 18. Primary thermometer.

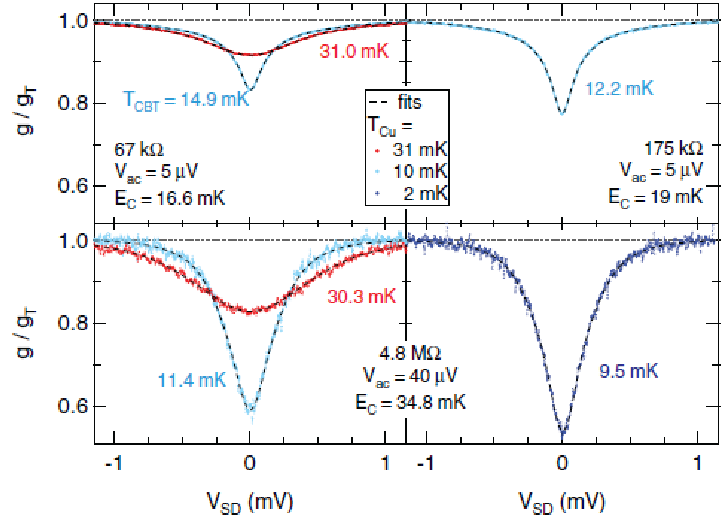


FIG. 2. CBT normalized differential conductance g/g_T versus source-drain dc bias V_{SD} for various NR temperatures T_{Cu} as color-coded, with resulting T_{CBT} (δg method, see text) given adjacent to each trace. Data from a 67 kΩ, 175 kΩ, and 4.8 MΩ CBT is shown. Dashed curves are fits to a model (see text). Note lower noise in low-R sensors due to larger resulting currents.

SET thermometer

aivon

Coulomb Blockade Thermometer (CBT) sensors for primary thermometry

Coulomb Blockade thermometer (CBT) is a primary thermometer for cryogenic temperatures based on change of electric conductance of tunnel junction arrays. In CBT, the differential conductance of tunnel junction array is a bell-shaped curve (Figure 1). It has been shown [1], that the full-width of the curve depends only on the (electron) temperature of the sensor and on some constants of nature.

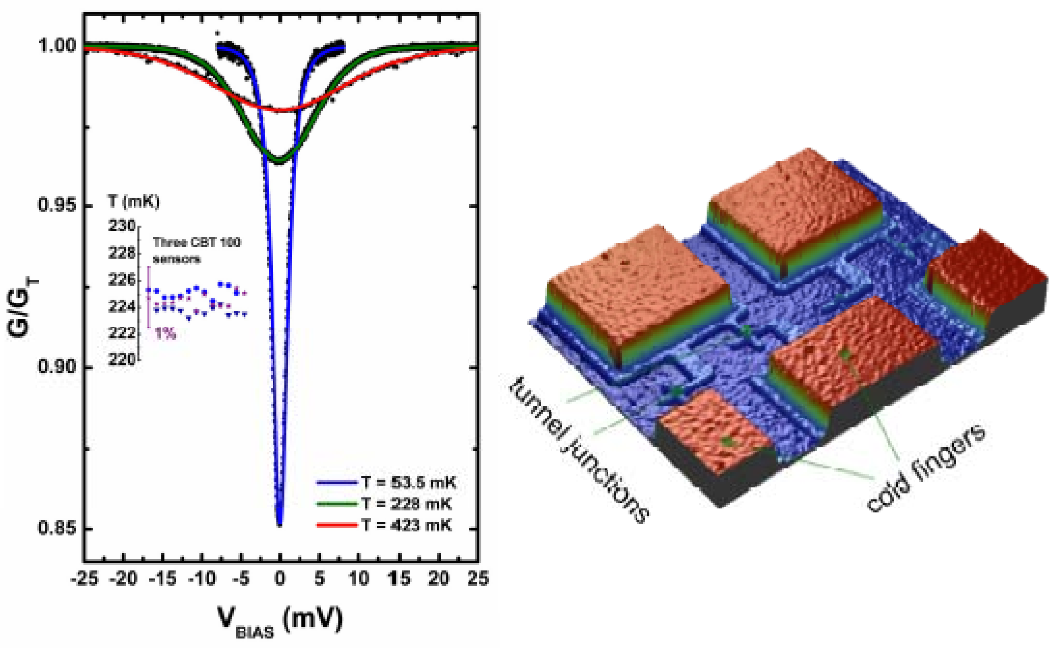
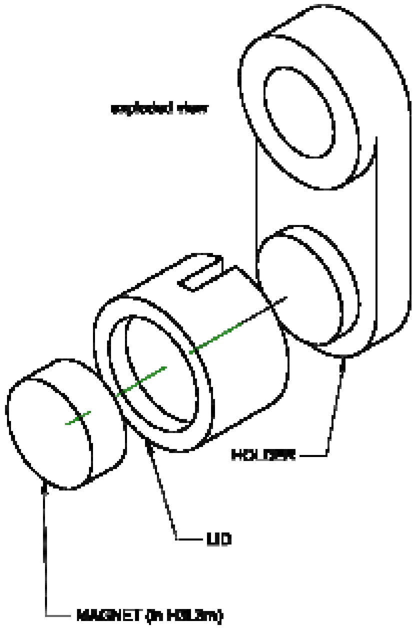
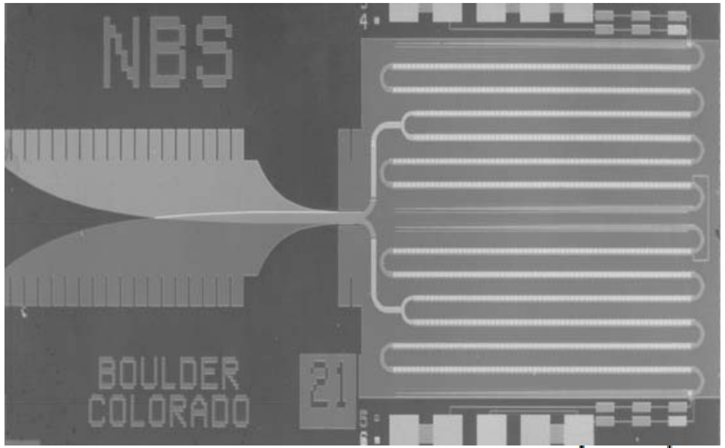
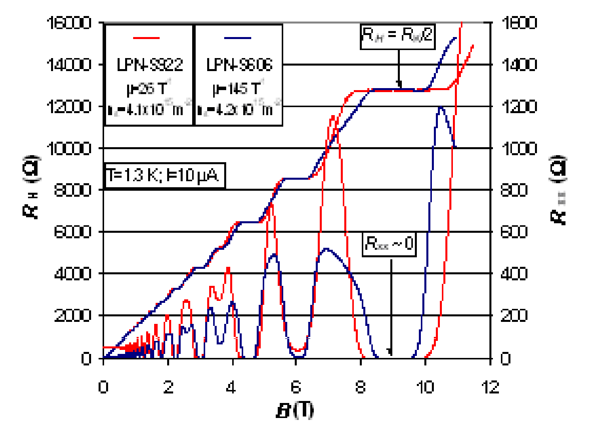


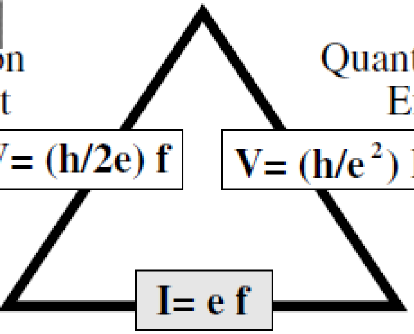
Figure 1: Left panel: Differential conductance as a function of bias voltage of CBT sensor measured at three different temperatures. G_T^{-1}=47600 Ohm. Solid line is a fit to theoretical formula [1] with two parameters. The full-width at half maximum FWHM depends only on temperature T. The depth of the conductance dip depends on temperature T and charging energy Ec, which can be calculated once T is known from FWHM. Small insert shows readings of three different sensors measured taken from several subsequent measurements. Right panel: Pseudo-color image of CBT tunnel junction array. Courtesy of VTT Technical Research Center of Finland.



Metrological triangle







V [Volt]

Josephson Effect

Quantum Hall Effect

$V = (h/2e)\,f$

$V = (h/e^2)\,I$

f [Hertz]

$I = e\,f$

I [Ampere]

Single Electron Charging Effect