Quantum Transport FS 2015
Problem Set Quantum Dots

1. Quantum level spacing and dimensionality

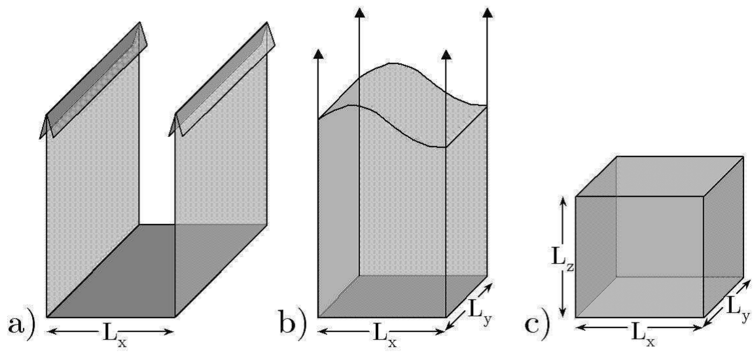
Imagine that you can produce semiconductor quantum structures with dimensional control of plus or minus one monolayer, i.e. approximately 0.25 nm. Assume that the structures are made of GaAs ($m^* = 0.067\, m_0$) and that the barrier height is infinite. State both the nominal value of the energy level and its possible range of values in light of the dimensional uncertainty. Describe qualitatively what happens when the barriers become finite.

Calculate the energy above the conduction band edge for ...

(a) two quantum wells which are 10 nm wide and 30 nm wide, respectively, for the ground states as well as for the first and second excited states ($n = 1, 2, 3$).

(b) a quantum wire with $(10\,\text{nm})^2$ ("width and length").

(c) a quantum dot that is $(10\,\text{nm})^3$.



(d) a quantum wire which has the same $n = 1$ energy level as the quantum well described in (a) ($\rightarrow$ 10 nm). What are the dimensions of this wire?

(e) a quantum dot which has the same $n = 1$ energy level as the quantum well described in (a) ($\rightarrow$ 10 nm). What are the dimensions of this dot?

2. Constant interaction model

Starting from the total energy of a quantum dot based on electrostatic and quantum confinement terms (see lecture, constant interaction model), derive the expressions for

(a) the dot chemical potential,

(b) the addition energy and

(c) the charging energy.

Give a qualitative description of these quantities.

3. Coulomb Diamonds

Consider a Coulomb blockade diamond where the differential conductance $g = dI/dV$ is plotted as a function of plunger gate voltage V_G (horizontal axis) and source-drain voltage V_{SD} (vertical axis). Use a simple capacitive model of the dot with gate capacitance C_G, left and right lead capacitance $C_{L,R}$ and total capacitance C_Σ.

(a) Draw the dot level diagram with left/right reservoirs, dot level and tunneling barriers at a few relevant locations in the CB diamond.

(b) Derive the slopes of the diamond lines.

(c) Finally, qualitatively sketch some Coulomb diamonds for the limiting case $C_L \lesssim C_\Sigma$.

4. Quantum vs Coulomb energy

Derive the average quantum level spacing of a circular quantum dot of area πR^2 starting from the 2D density of states. Further, find an expression for the charging energy, assuming an infinitely thin flat disc of radius R. Compare qualitatively the size dependence of the charging energy and the quantum level spacing.

5. Sequential Tunneling through a Single-Level Quantum Dot

Consider a quantum dot coupled to two reservoirs with Fermi-Dirac distributions at temperature T and tunneling rates Γ_S and Γ_D through the source and drain barriers, respectively. Assume the temperature broadened regime $h\Gamma_{S,D} \ll k_B T$. A source-drain bias $eV_{SD} = \mu_S - \mu_D$ is applied, where μ_S and μ_D are the chemical potentials of the source and drain reservoir. Assume that the dot has only a single quantum level at energy ϵ above μ_D. *Hint:* Assume $\mu_D = 0$ throughout this exercise for simplicity.

(a) Draw a sketch of the situation with reservoirs, dot, energy level and tunnel barriers.

(b) Derive an expression for the sequential tunneling current I through the dot as a function of T, V_{SD}, Γ_S, Γ_D and for arbitrary level energy ϵ.

(c) How can this dot be used as a thermometer?

(d) From the current I, find the differential conductance g as a function of the same parameters as for the current. What is the line shape as a function of gate voltage?

6. Shell filling in circular 2D quantum dots

In their seminal paper *Shell Filling and Spin Effects in Few Electron Quantum Dots* in Physical Review Letters **77**, 3613 (1996) (more than 1'000 citations), Tarucha et al. identify the shell filling in 2D circular few electron quantum dots and find new "magic numbers". Read this paper, and derive the magic numbers based on the isotropic quantum harmonic oscillator model (Fock-Darwin spectrum).

7. Singlet-Triplet transition in a circular quantum dot

In their subsequent paper *Excitation Spectra of Circular, Few-Electron Quantum Dots* in Science **278**, 1788 (1997), Kouwenhoven et al. describe the excited state spectra. Read this paper, and explain

(a) How are the excited states visible in this experiment?

(b) Do they agree with the simple Fock-Darwin model?

(c) What is the source of various additional B-field dependent features in the spectra besides the Fock-Darwin excited states?

(d) Estimate the magnetic fields required to perform similar spectroscopy in a 'traditional' atom such as H or He.

(e) Finally, the triplet excited state energy J is here rather close to the $N = 1$ orbital excited state energy Δ (see Fig. 5A), quite unlike the situation described in the lecture in lateral quantum dots, where we found $J \ll \Delta$. What could be the reason for this?