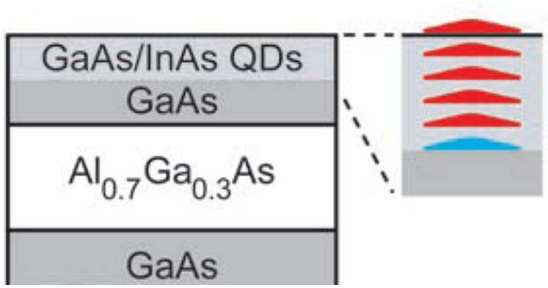
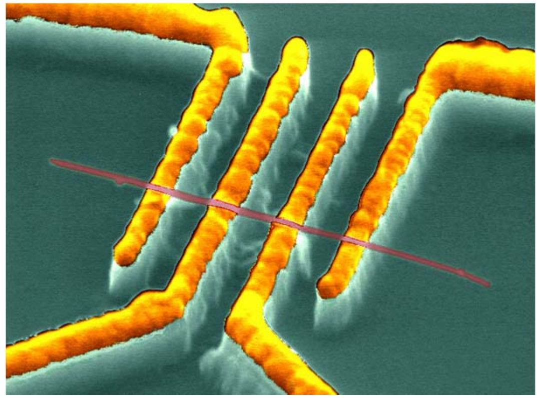
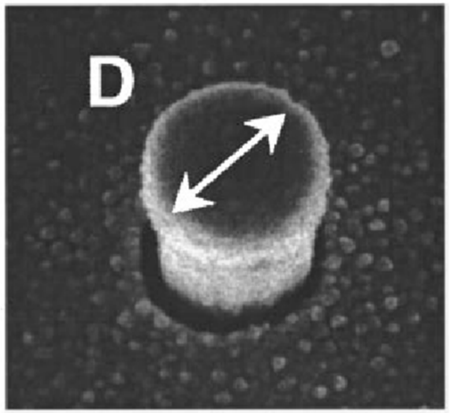
Quantum Dots



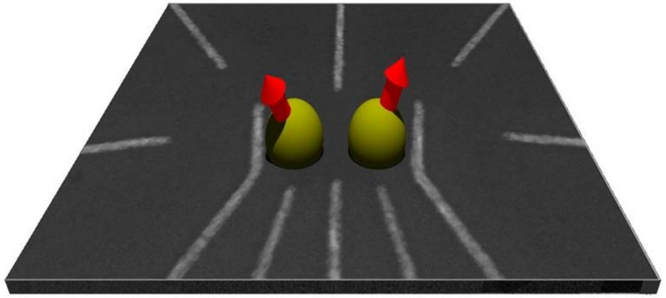
MBE grown



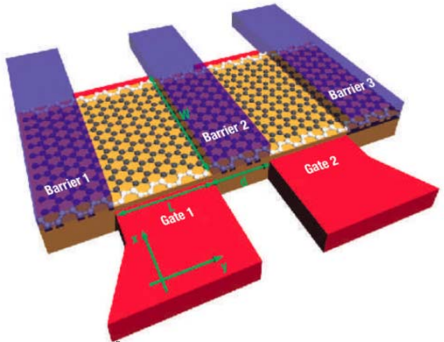
carbon nanotube / nanowires



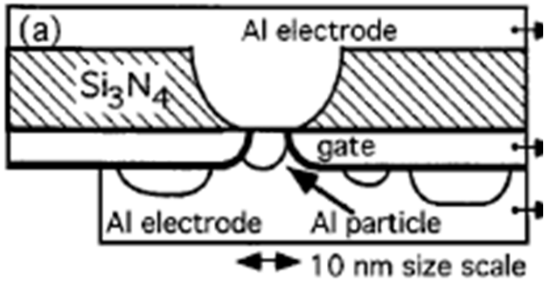
GaAs vertical dot



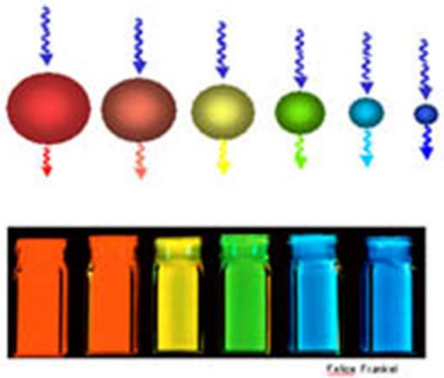
GaAs lateral dot



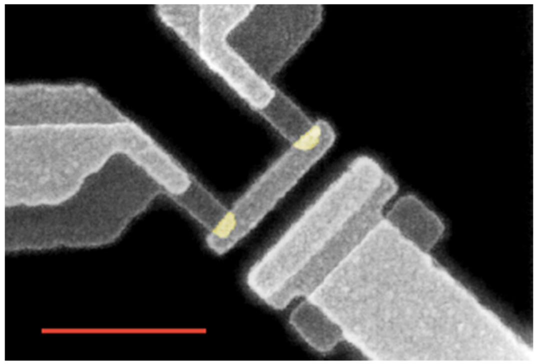
graphene



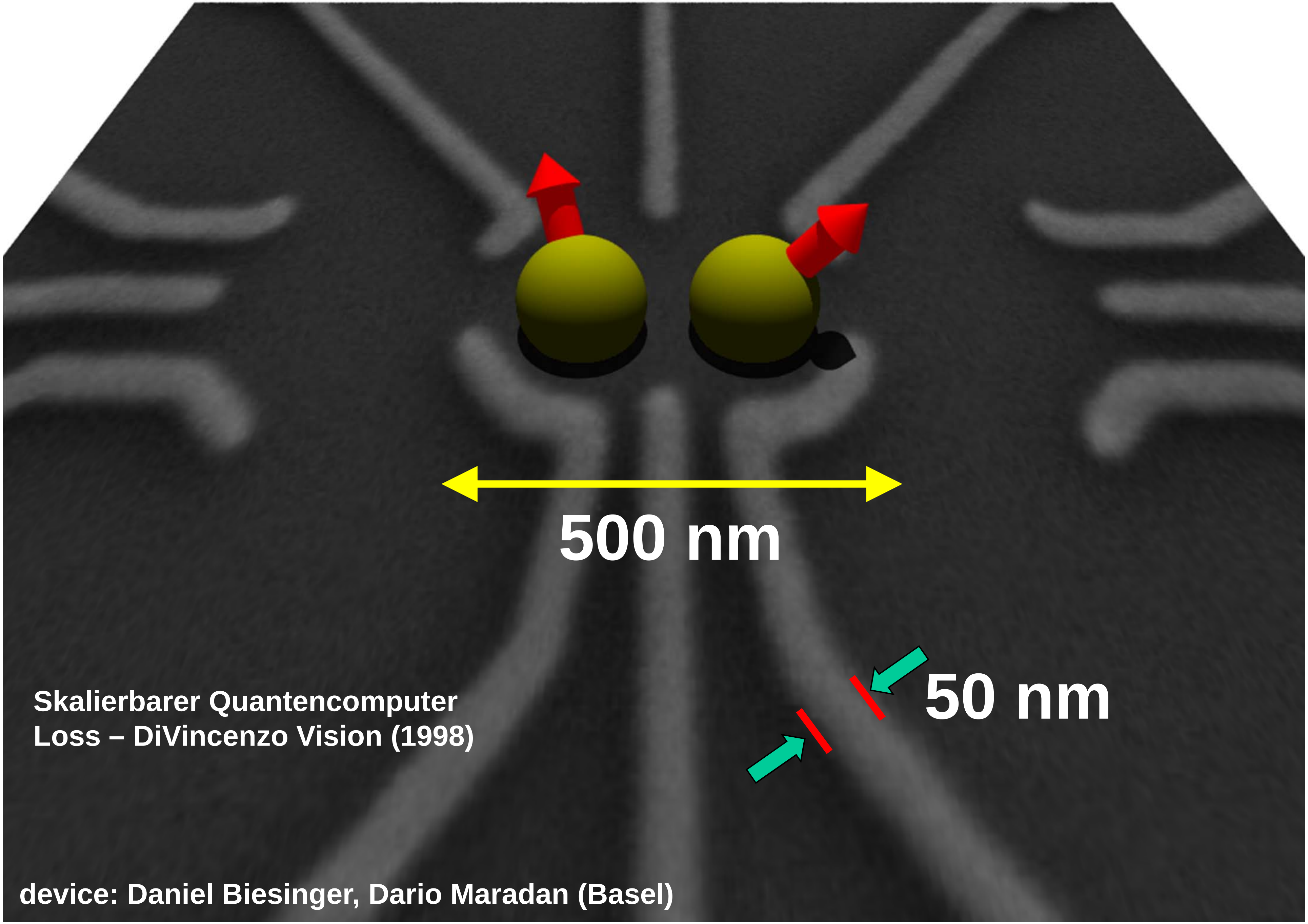
metal grain



III-V self assembled



metallic SET

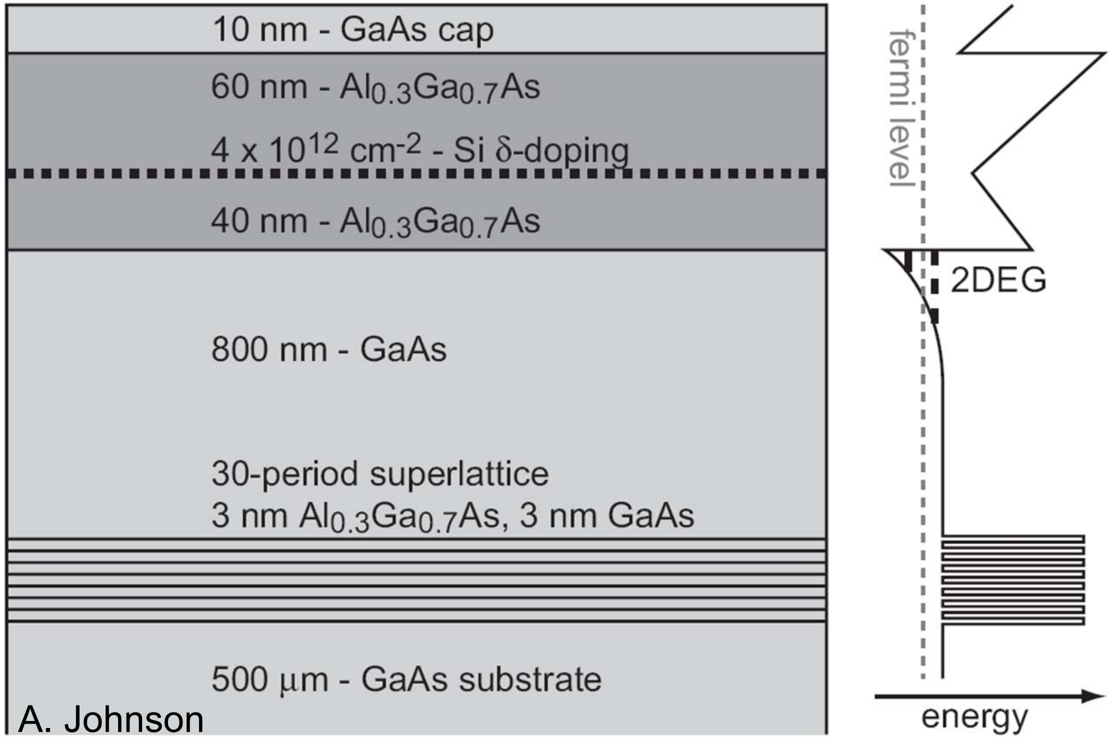

500 nm
50 nm
Skalierbarer Quantencomputer
Loss – DiVincenzo Vision (1998)
device: Daniel Biesinger, Dario Maradan (Basel)

Part 1: Quantum Dot Basics

- GaAs 2D electron gas (2DEG)
  conductance quantization in QPCs
- Coulomb blockade and charging energy $E_C = e^2/C$
  quantum confinement energy Δ
- Constant interaction model and
  Coulomb diamonds
- electronic transport via
  - sequential tunneling Γ
  - cotunneling Γ^2 / E_c (elastic / inelastic)
  - cotunneling assisted sequential tunneling
- singlet & triplet states,
  exchange splitting $J = E_T - E_S$
- Pauli Spin blockade

next week:	part 2	putting the basics to work g-factor, ST transition, spin entanglement
	part 3	charge sensing charge & spin tunneling, spin relaxation, charge fluctuations

GaAs 2D Electron Gas (2DEG)



density $n = 2 \times 10^{11}$ cm^{-2}

mobility $\mu \sim 200'000$ $cm^2/(Vs)$

Fermi wavelength $\lambda_F \sim 50$ nm

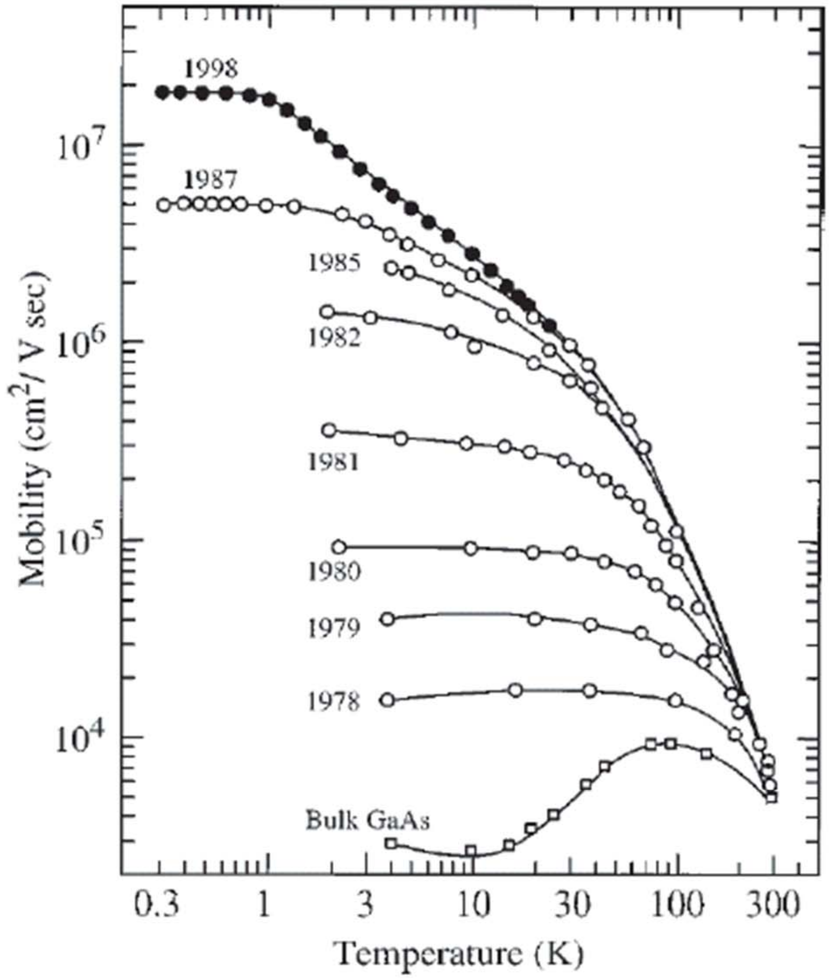
mean free path ~ micron

J. Zimmerman and A. C. Gossard, UC Santa Barbara

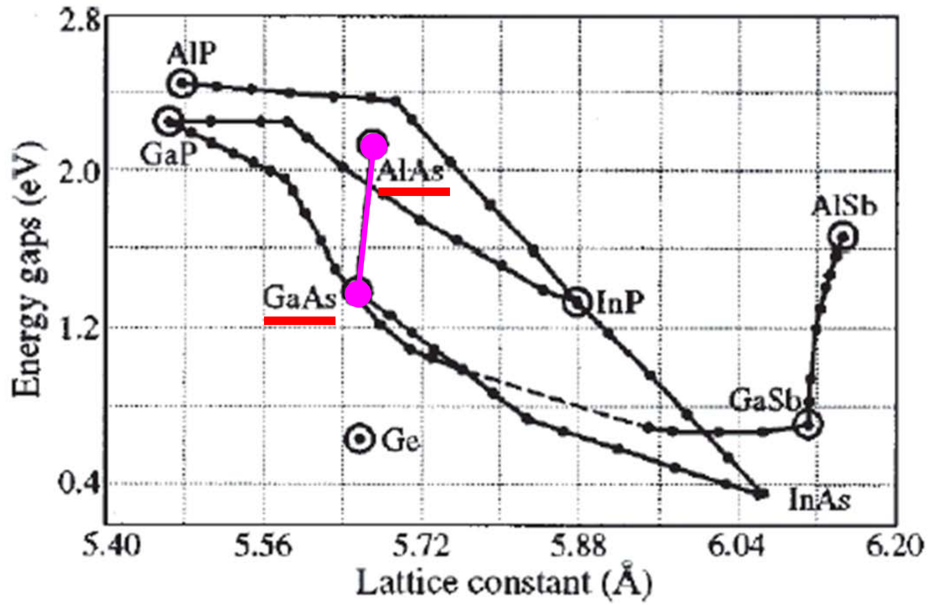
C. Reichl and W. Wegscheider, ETHZ

GaAs 2D Electron Gas (2DEG)

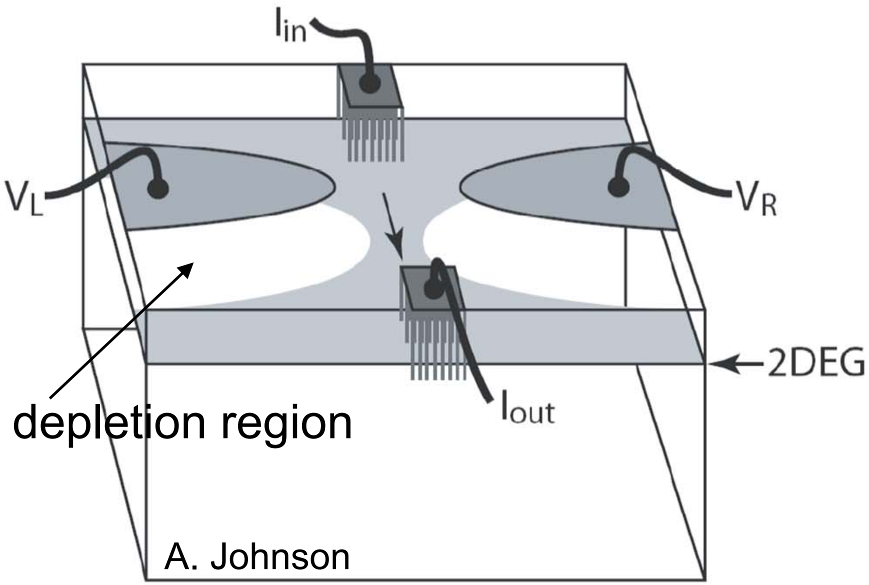
mobility

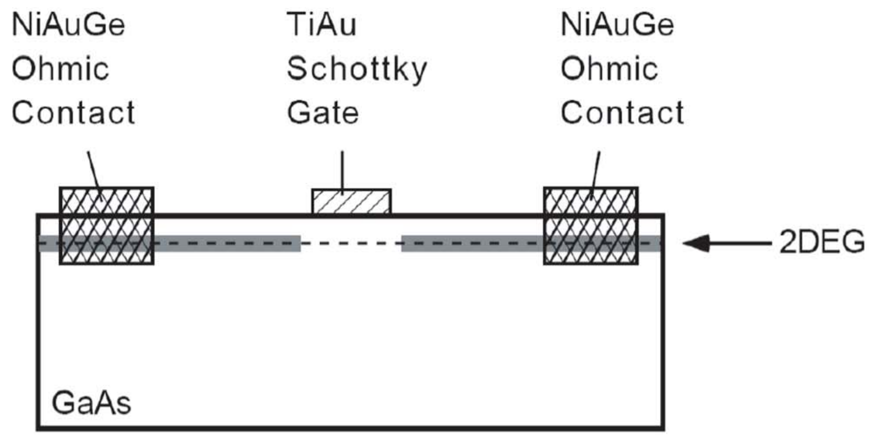


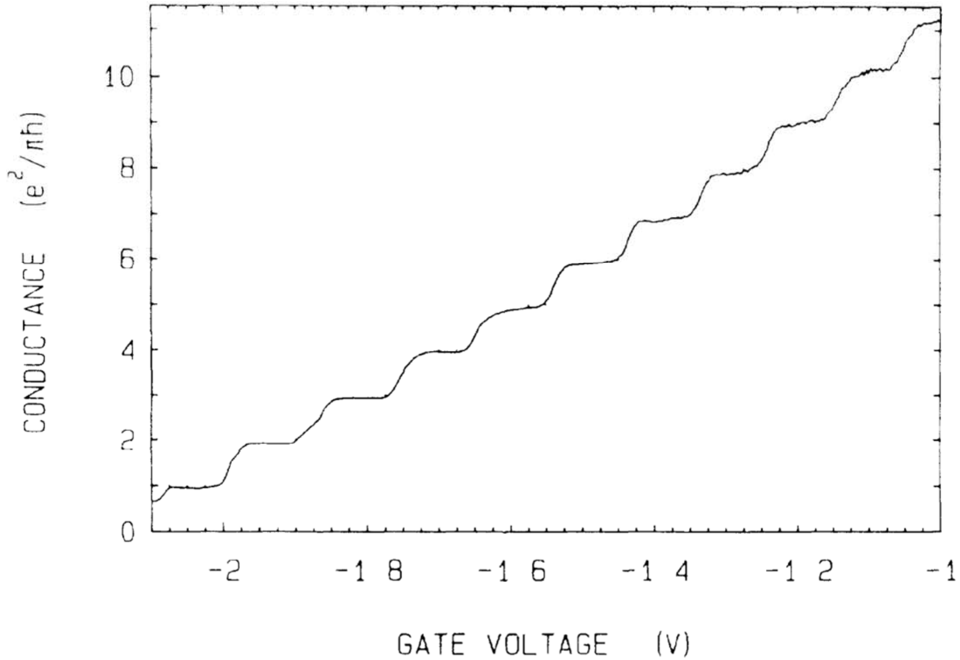
gaps vs lattice constants



Lateral Depletion Gating, Ohmic Contacts







voltage adjustable
depletion area

quantum point
contact

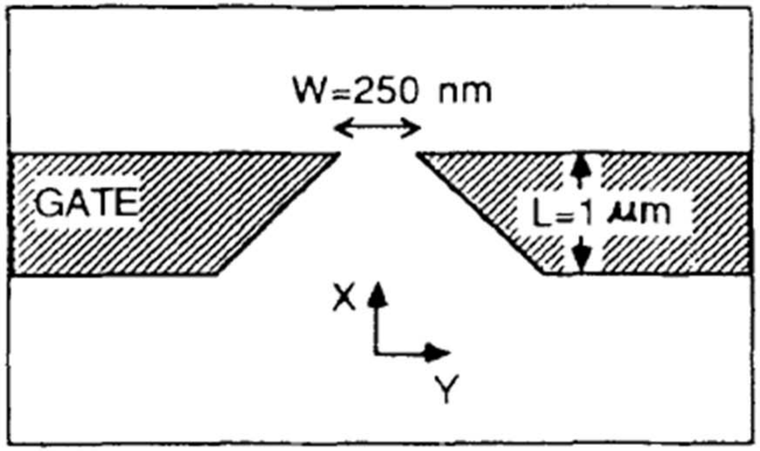
B. van Wees et al., PRL 1988
D. Wharam et al., JPCMD 1988

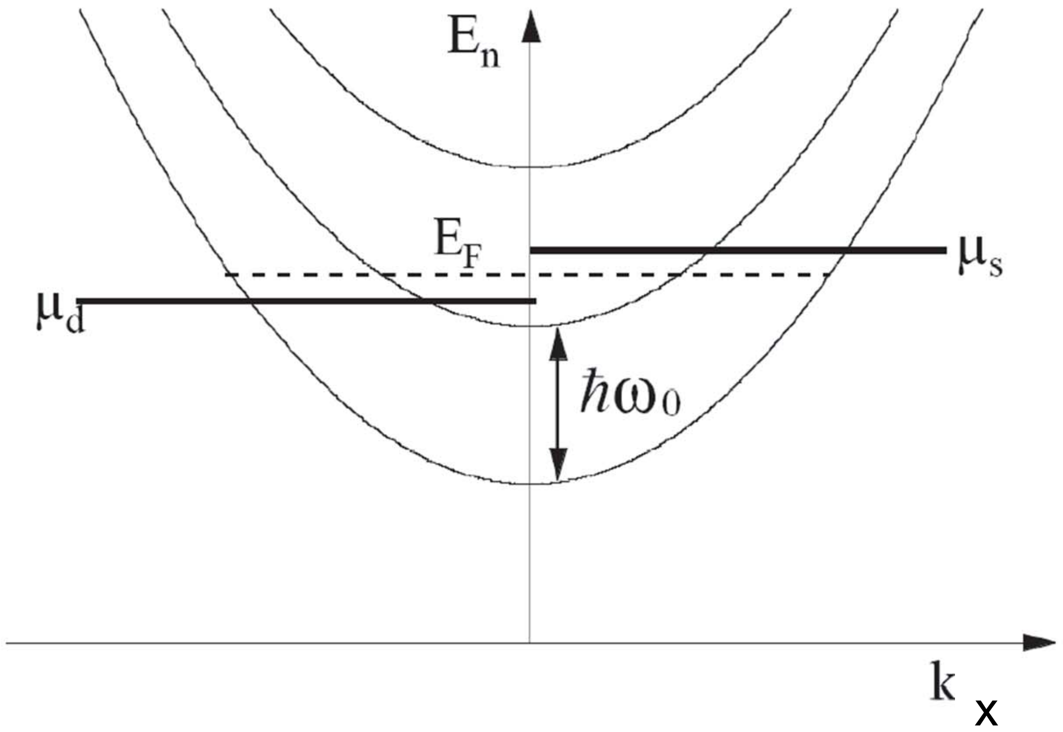
Quantum Point Contact (QPC)

$$\left[\frac{\hbar^2 k^2}{2m^*} + \frac{p_y^2}{2m^*} + \frac{1}{2}m^*\omega_0^2 y^2\right]\chi(y) = \chi(y)$$

$$\chi_{n,k}(y) \quad = \quad u_n(q) \quad \text{where} \quad q = \sqrt{m^*\omega_0/\hbar}\, y$$

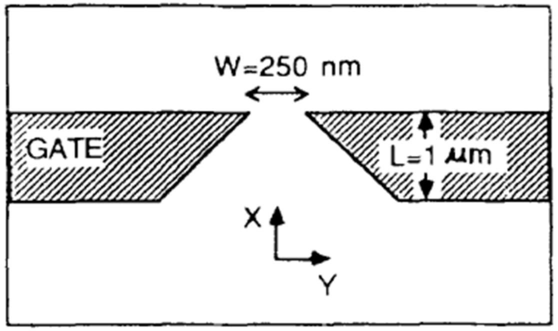
$$v(n,k) = \frac{1}{\hbar}\frac{\partial E(n,k)}{\partial k} = \frac{\hbar k}{m^*}$$





Conductance Quantization in 1D (QPC)

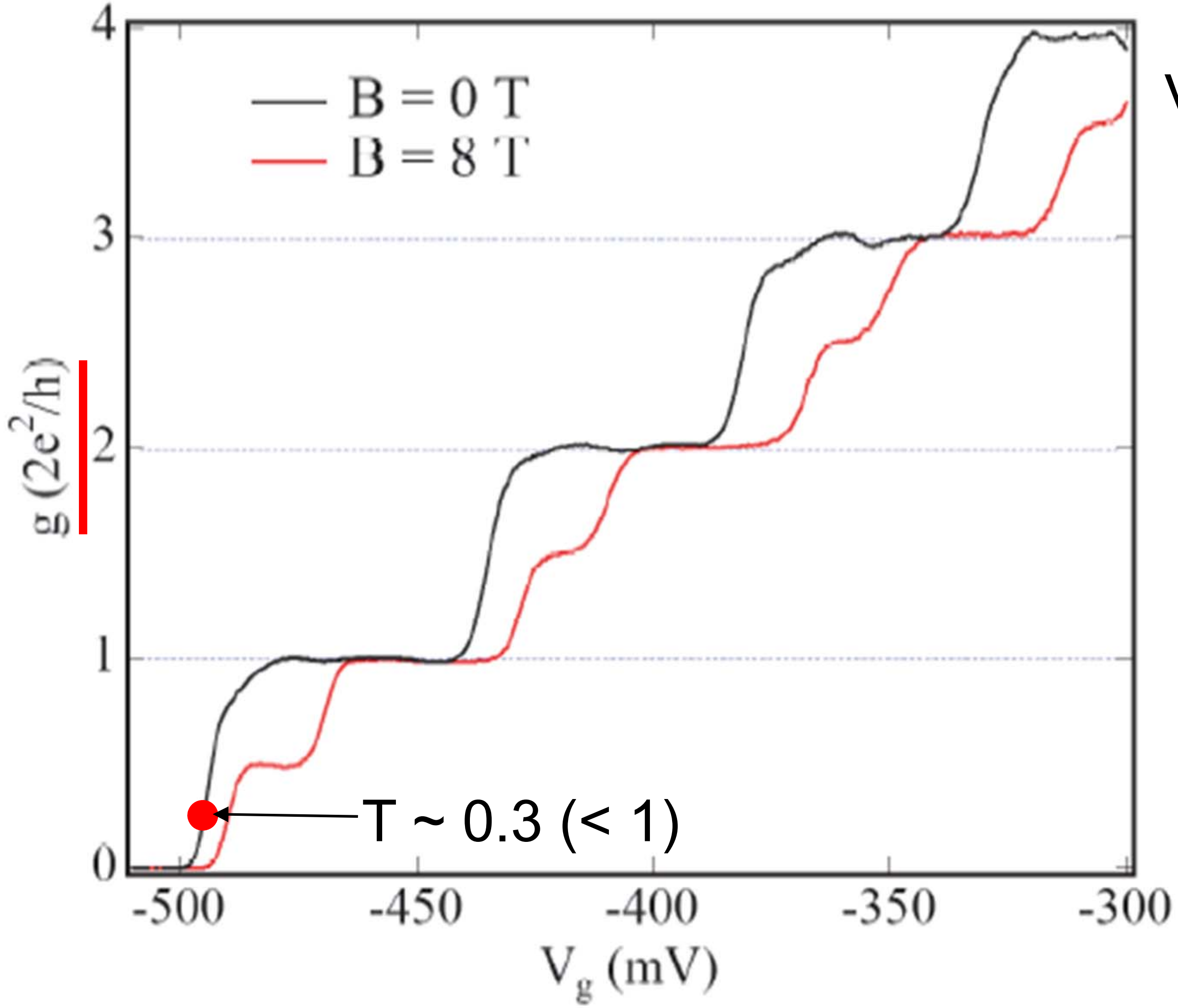
$$I = e\sum_{n=1}^{N}\int_{\mu_d}^{\mu_s} dE \frac{1}{2}\rho_n(E)v_n(E)T_n(E),$$

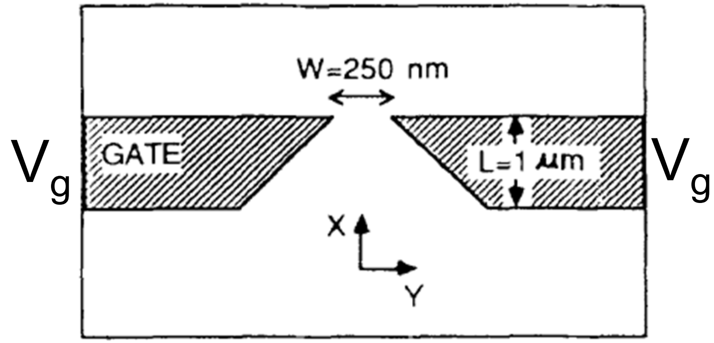


$$I = e\sum_{n=1}^{N}\int_{\mu_d}^{\mu_s} dE \frac{1}{2}\frac{2}{\pi}\left(\frac{\partial E_n}{\partial k_x}\right)^{-1}\frac{1}{\hbar}\frac{\partial E_n}{\partial k_x}T_n(E_F)$$

$$G = \frac{2e^2}{h}\sum_{n=1}^{N}T_n(E_F) \qquad G = \frac{2e^2}{h}N,$$

Conductance Quantization in 1D (QPC)

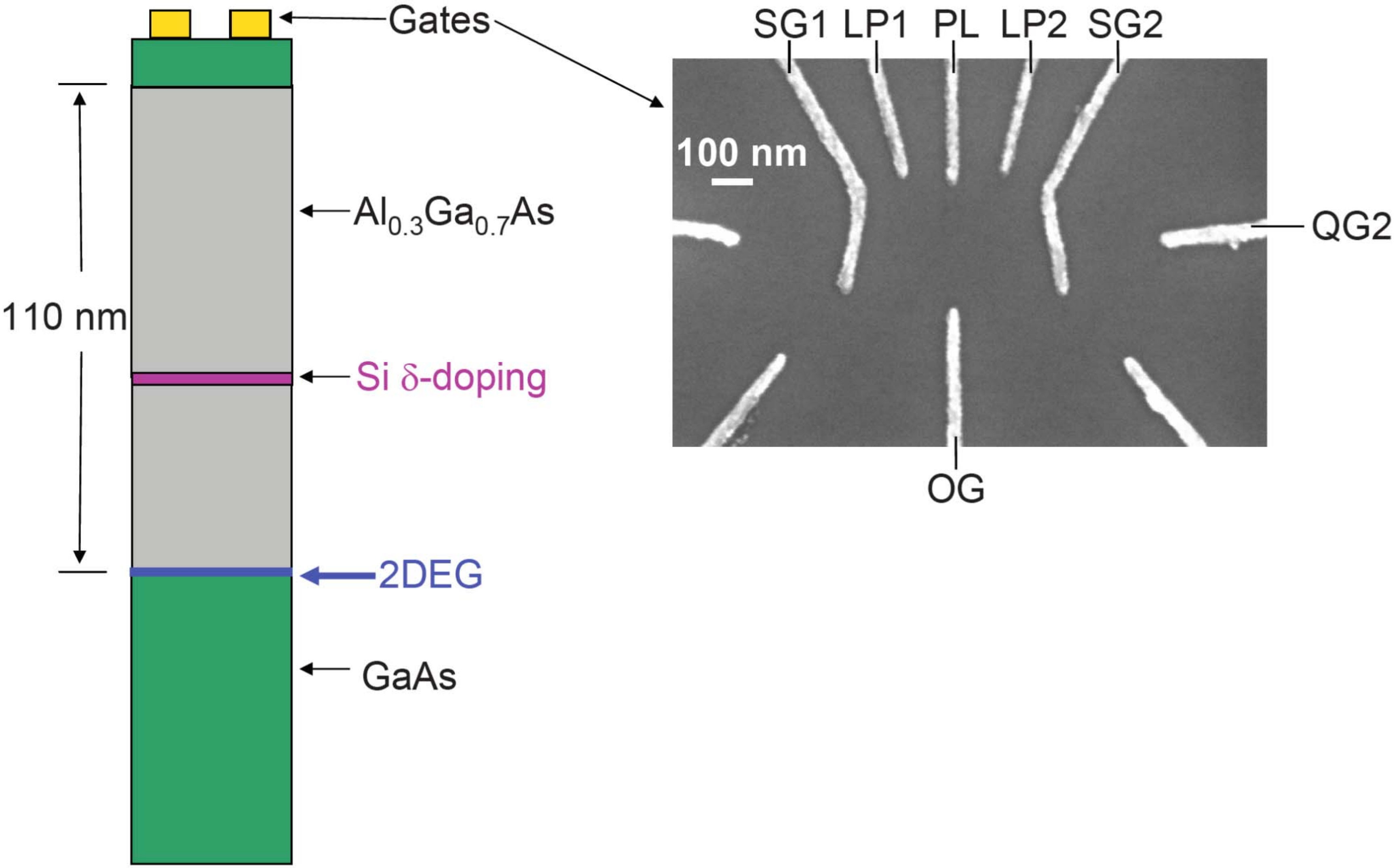




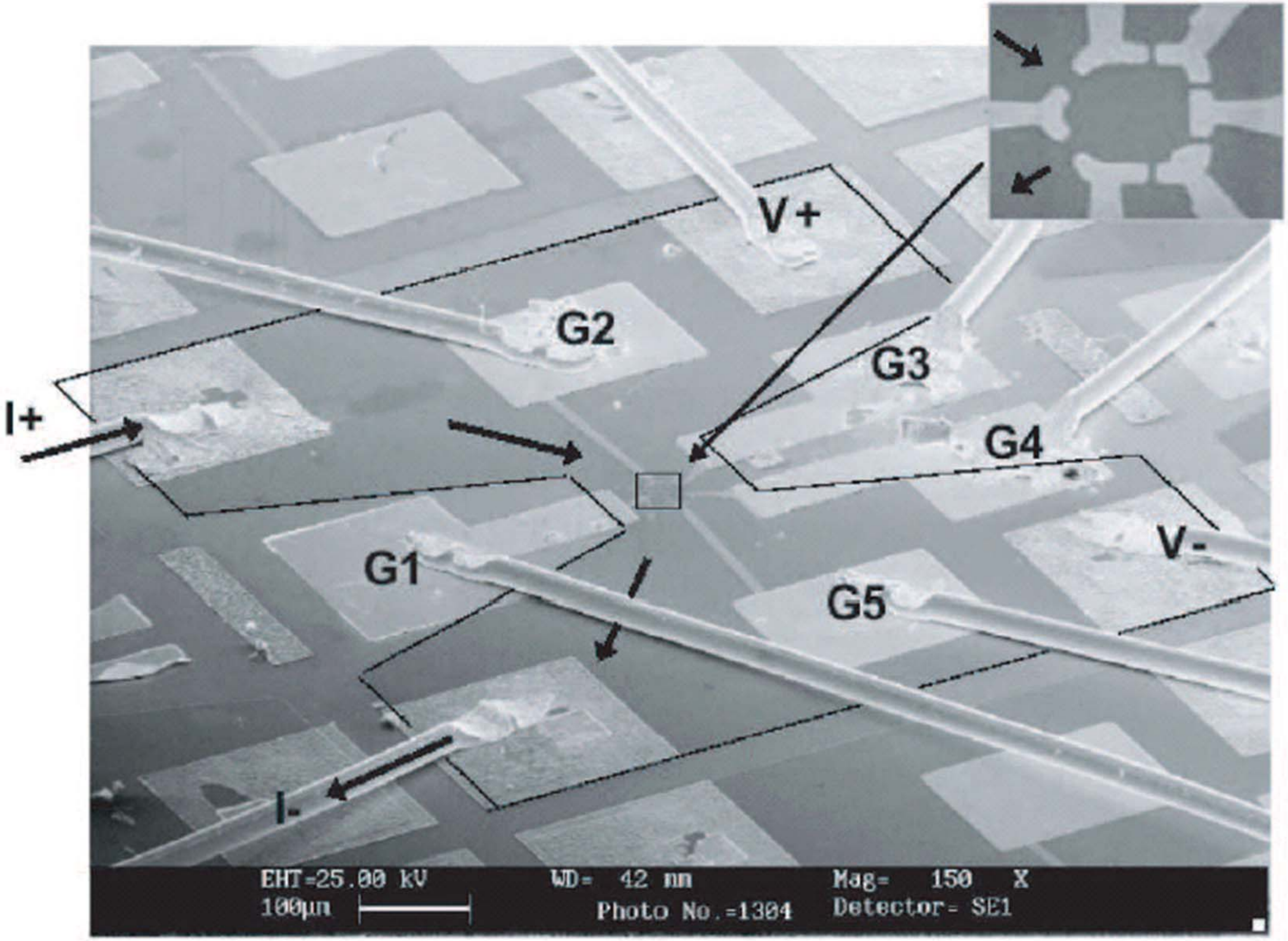
$$G = \frac{2e^2}{h}N,$$

factor 2: spin degeneracy, $E_Z = g\ \mu_B\ B = 25\ \mu eV/T\ \ B$ with $|\ g\ | = 0.44$ GaAs

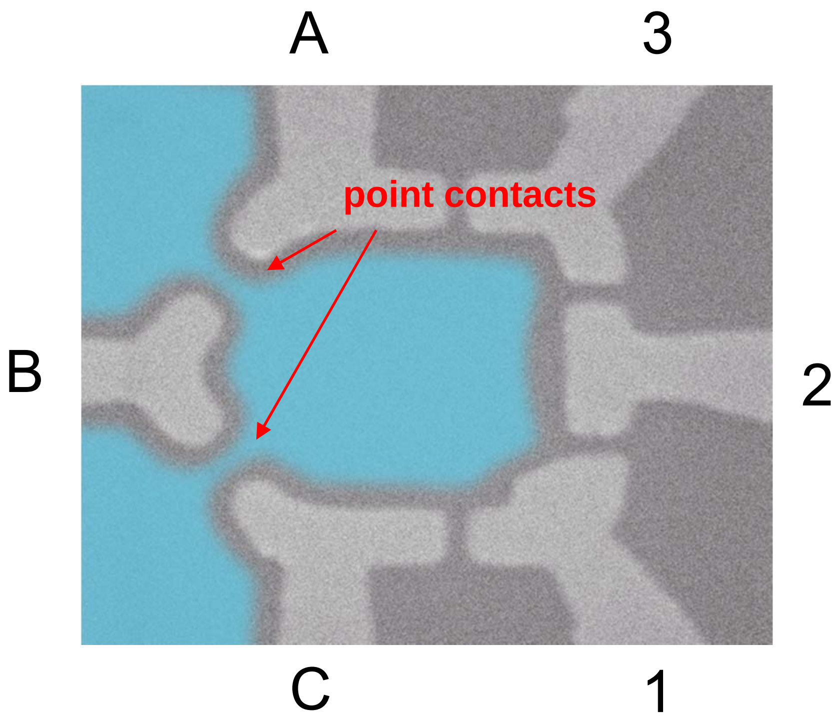
Forming a Quantum Dot

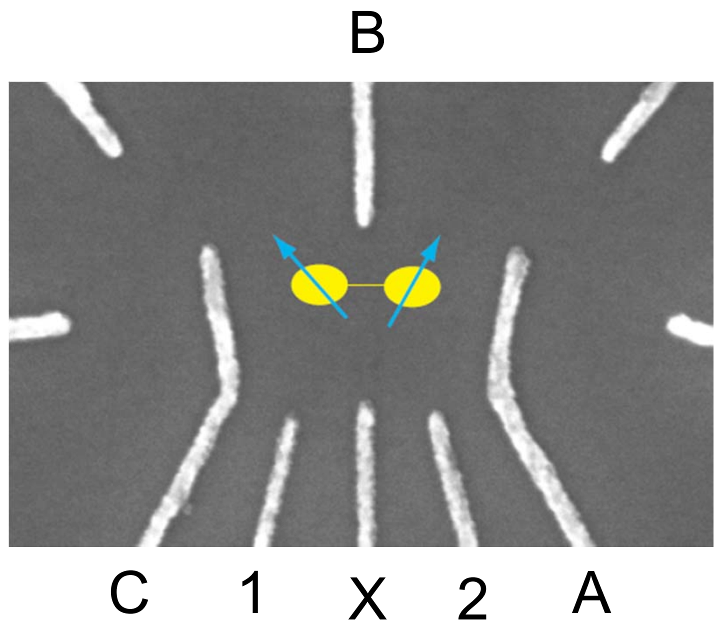


Device Integration



gate defined dots





A,B,C : control quantum point contacts transmission to reservoirs

1,2,3: control confinement potential / energy levels only

X control dot-internal tunneling rate

Quantum Dot Basics

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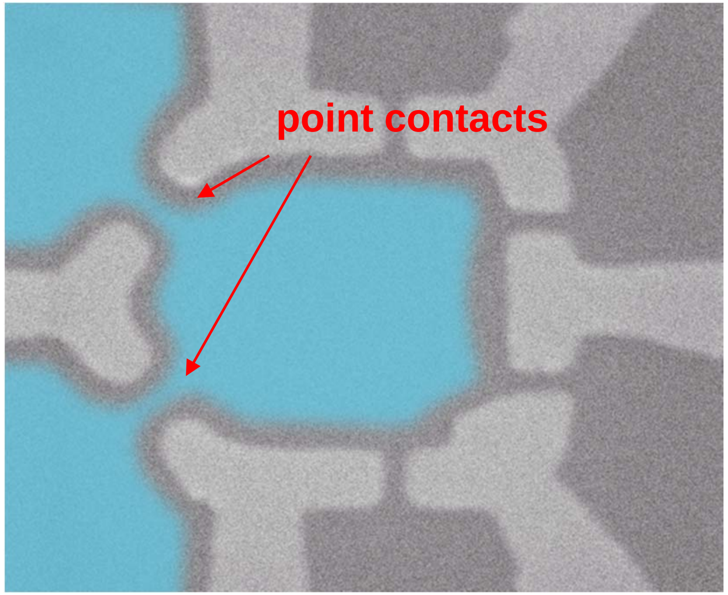
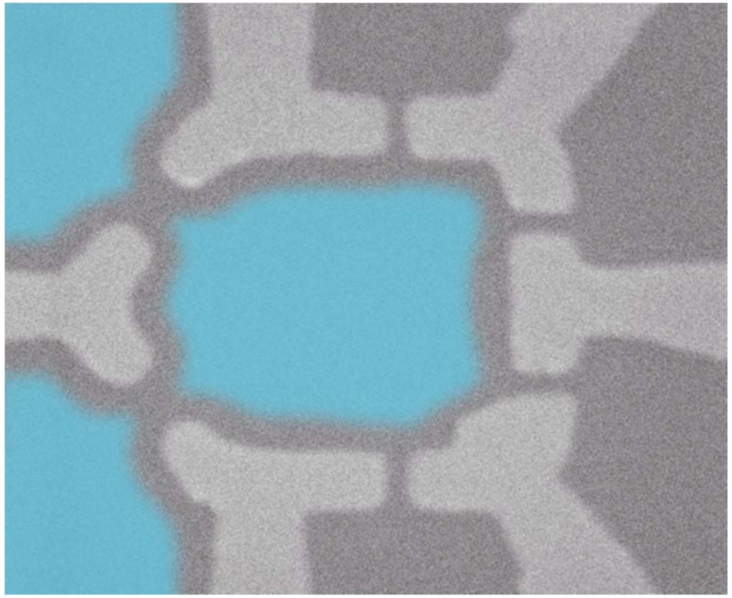
Open vs. Closed

Open Dot



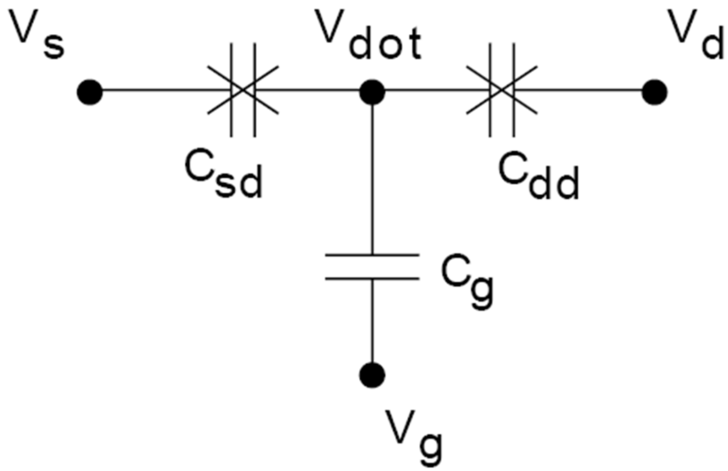
- V_{gate} set to allow ≥ $2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

Closed Dot



- V_{gate} set to require tunnelling across point contacts
- Dot is isolated from reservoirs, contains discrete energy levels
- Transport measurements exhibit Coulomb Blockade

Electrostatic Energy



apply voltages

what is potential on dot?

voltage divider...

$$C_\Sigma = C_{sd} + C_{dd} + C_{g1} + C_{g2} + \ldots$$

$$V_{dot} = \sum_i \alpha_i V_i$$

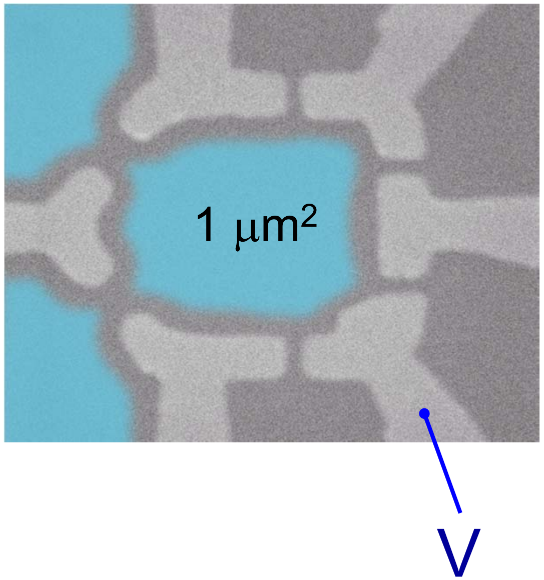
$$\alpha_i = \frac{C_i}{C_\Sigma}$$

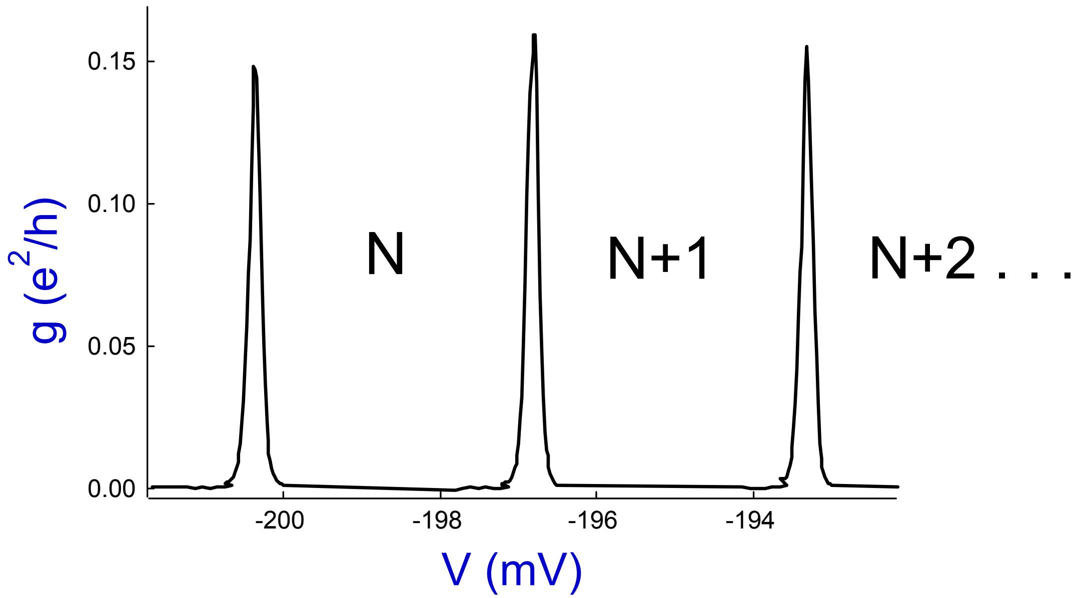
can use V_g to shift dot energy!!

Coulomb Blockade in Closed Dots (SET)

Finite energy $E_c = e^2/C_{dot}$ is needed to add an additional electron to the dot. When $kT \ll E_c$ charging blocks conduction in valleys.

Coulomb blockade peaks:
resonant transport through dot levels





Charging Energy

capacitance of dot to world = C

$$C = \epsilon_0 \epsilon \frac{A}{d}$$

total energy U stored in C

$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

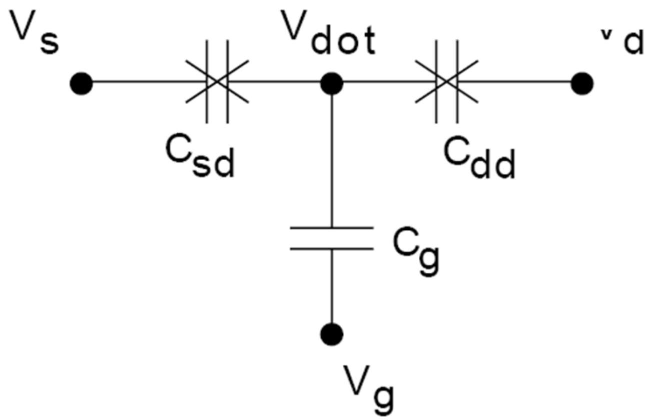
Incremental E to add one electron

charging energy

$$E_C = \frac{e^2}{C_\Sigma}$$

can range from
~0 to many meV

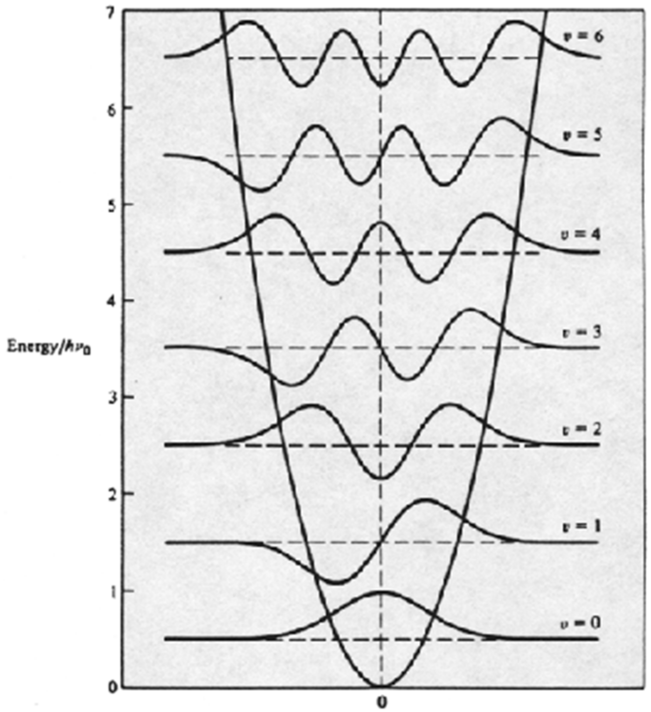
$$C_\Sigma \gtrsim 10\,\mathrm{aF}$$



Classical Effect, NOT quantum

Confinement Energy

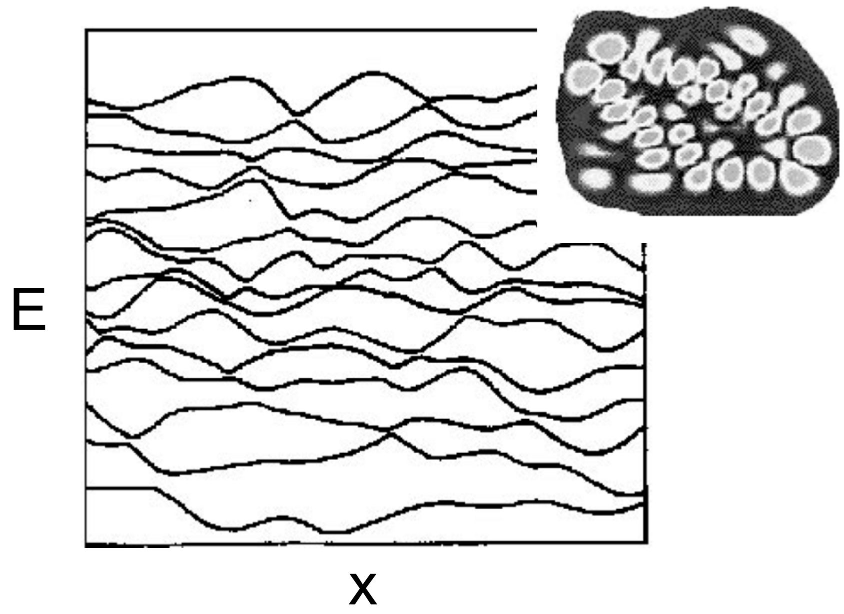
harmonic potential



$$E_n = \left[n + \frac{1}{2}\right]\hbar\omega$$

μeV to meV

complicated potential



average level spacing

$$\Delta = \frac{2\pi\hbar^2}{m^* A}$$

quantum mechanical effect!!

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- Pauli Spin blockade

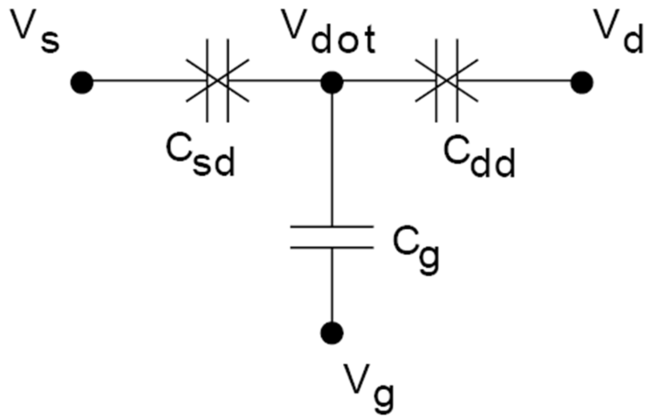
Capacitor Model

$$E(N) = [Q_{tot}]^2 / (2C_\Sigma) + \sum_{k=1}^{N} \epsilon_k$$

total dot energy

$$E(N) = \left[e(N - N_0) - \sum_{k=1}^{N} C_k V_k \right]^2 / (2C_\Sigma) + \sum_{k=1}^{N} \epsilon_k$$

offset charge



Constant Interaction Model

$$E_i = \sum_{k=1}^{N} q_k \phi_k$$

$q_k = -e$

ϕ_k : interaction of electron k with rest

constant inteaction: model ϕ_k with C_Σ

$\phi_k = -(k-1)e/C_\Sigma$

$$\begin{aligned} E_i &= \frac{e^2}{C_\Sigma} \sum_{k=1}^{N} (k-1) \\ &= \frac{N(N-1)e^2}{2C_\Sigma} \end{aligned}$$

$$\begin{aligned} E(N) &= E_{\mathrm{QM}} + E_i + E_e \\ &= \sum_{n=1}^{N} \epsilon_n + \frac{N(N-1)e^2}{2C_\Sigma} - Ne \sum_{i=1}^{6} \alpha_i \mathrm{V}_i \end{aligned}$$

total dot energy

Chemical Potential / Addition Energy

$$\mu_{\rm dot}(N) \equiv E(N) - E(N-1)$$

energy at which an electron can be added

μ=0: change N current flows

constant interaction model:

$$\mu_{dot}(N) = \epsilon_N + (N-1)\frac{e^2}{C} - e\sum_i \alpha_i V_i$$

addition energy

$$(\mu_{\rm dot}(N+1) - \mu_{\rm dot}(N))|_{\rm fixed\ V_{\it i}}$$

Temperature Regimes

$$\Delta, \frac{e^2}{C} \ll kT$$

no charging effects, no Coulomb blockade

$$g_\infty = \left(\frac{1}{g_L} + \frac{1}{g_R}\right)^{-1}$$

$$\Gamma, \Delta \ll kT \ll \frac{e^2}{C}$$

classical Coulomb blockade (metallic CB)
temperature broadened
transport through several quantum dot energy levels

$$g \sim \frac{g_\infty}{2} \cosh^{-2}\left(\frac{\epsilon}{2.5kT}\right)$$

peak conductance idependent of T
FWHM ~ 4.35kT

$\Gamma = \Gamma_L + \Gamma_R$ escape broadening (tunneling rates)

Beenakker, PRB44, 1646 (1991)

Temperature Regimes

$$\Gamma \ll kT \ll \Delta \ll \frac{e^2}{C}$$

quantum Coulomb blockade
temperature broadened regime
resonant tunneling
transport through only one dot level

$$g \sim \frac{e^2}{h}\frac{\gamma}{4kT}\cosh^{-2}\left(\frac{\epsilon}{2kT}\right)$$

peak conductance 1/T
FWHM ~ 3.5kT

$$\gamma = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$$

$$kT \ll \Gamma, \Delta \ll \frac{e^2}{C}$$

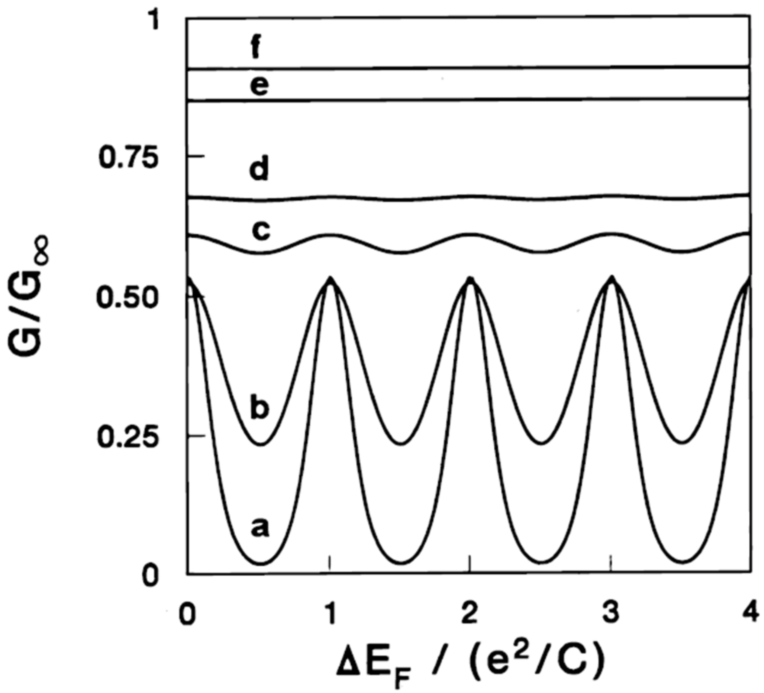
quantum Coulomb blockade
lifetime broadened regime
transport through only one dot level

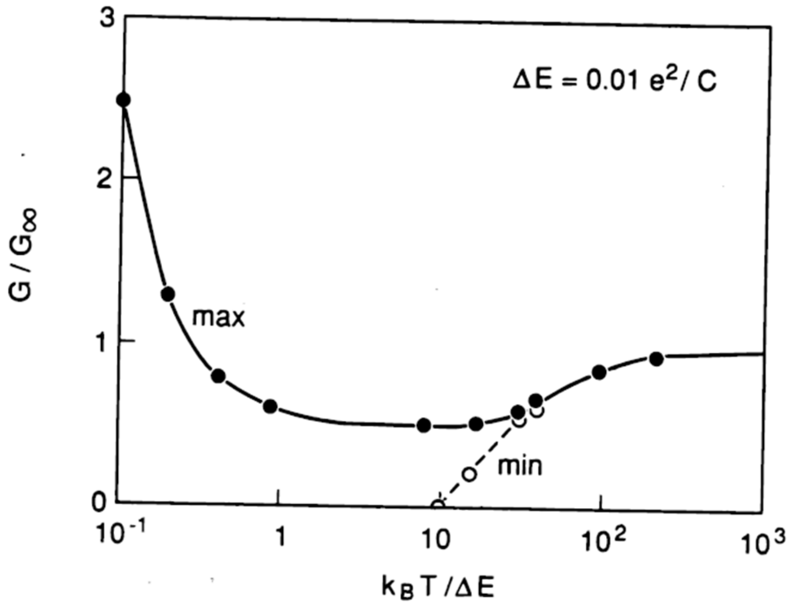
$$g_{BW} \sim \frac{e^2}{h}\frac{\gamma\Gamma}{(\epsilon/\hbar)^2 + (\Gamma/2)^2}$$

peak conductance e^2/h indep. of T
FWHM ~ Γ

Beenakker, PRB44, 1646 (1991)

Temperature Dependence: Theory





Δ= 0.01 e^2/C

kT / e^2C

a 0.075

b 0.15

c 0.3

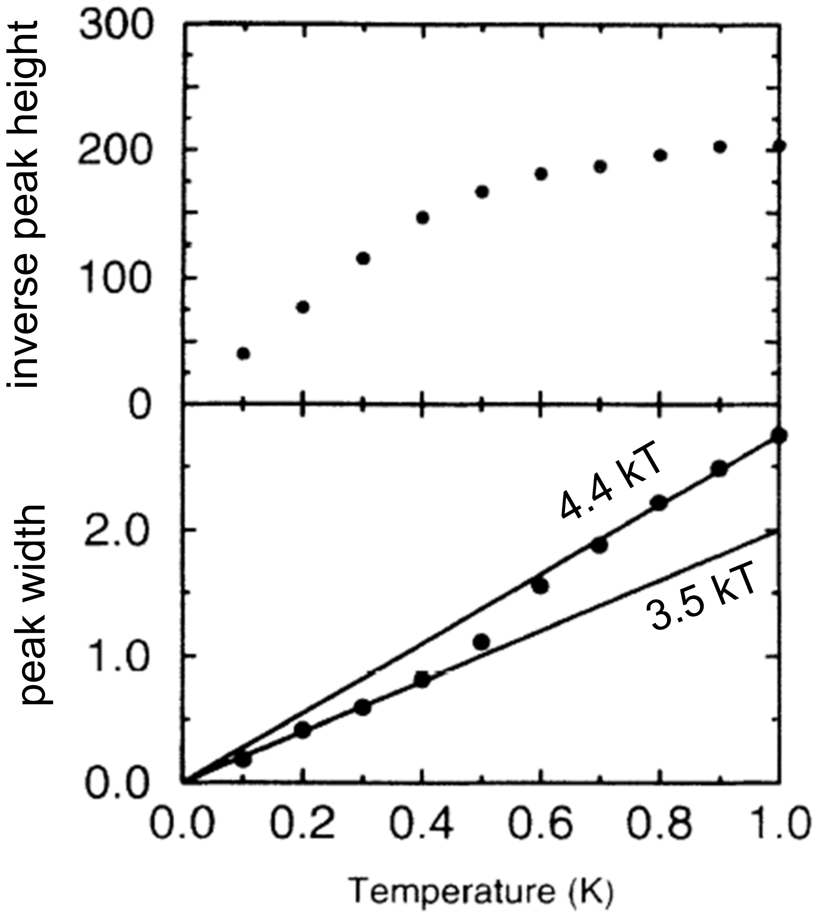
d 0.4

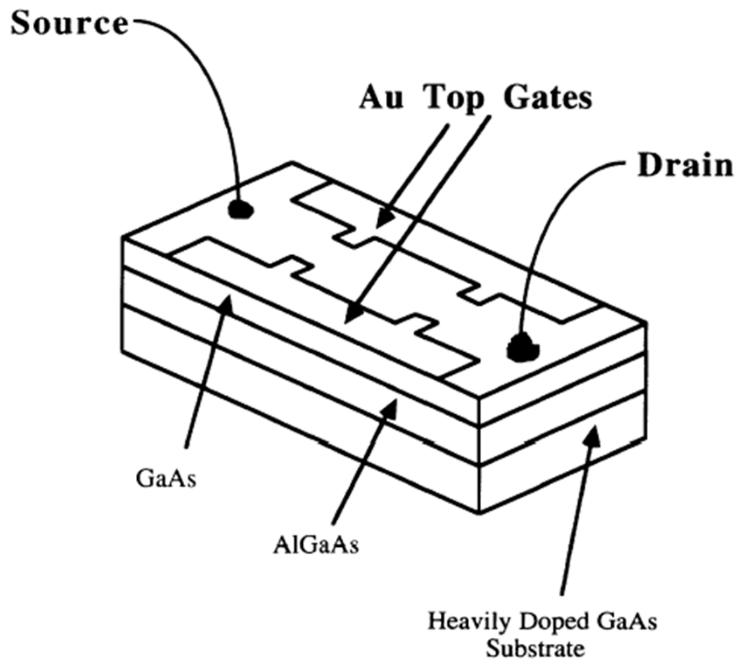
e 1

f 2

van Houten, Beenakker & Staring, NATO ASI Series

Temperature Dependence: Experiment



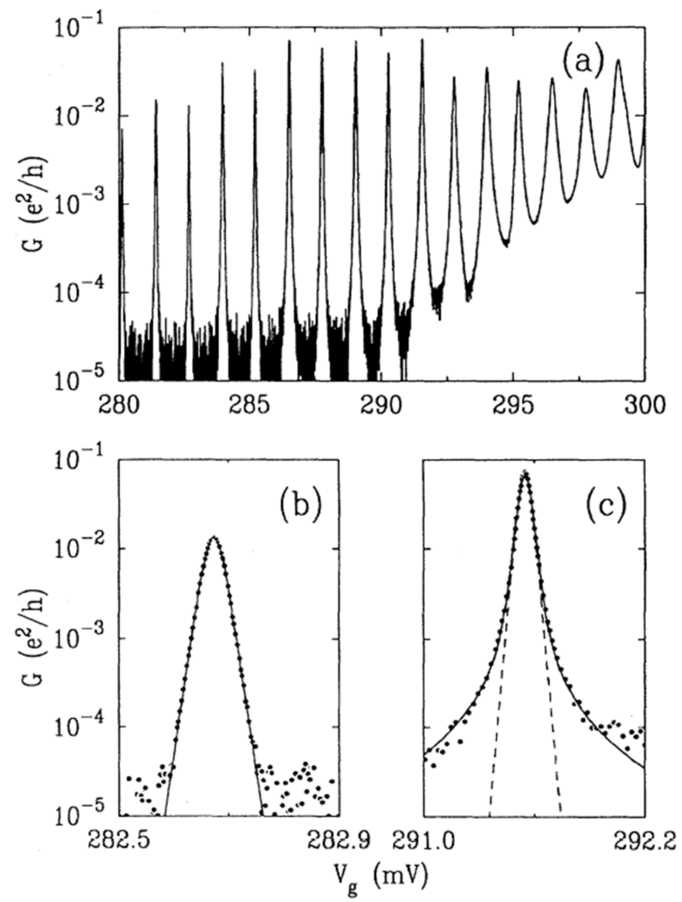


crossover 3.5 to 4.3kT peak width

peak g
1/T dependence: quantum regime
T independent: cassical regime

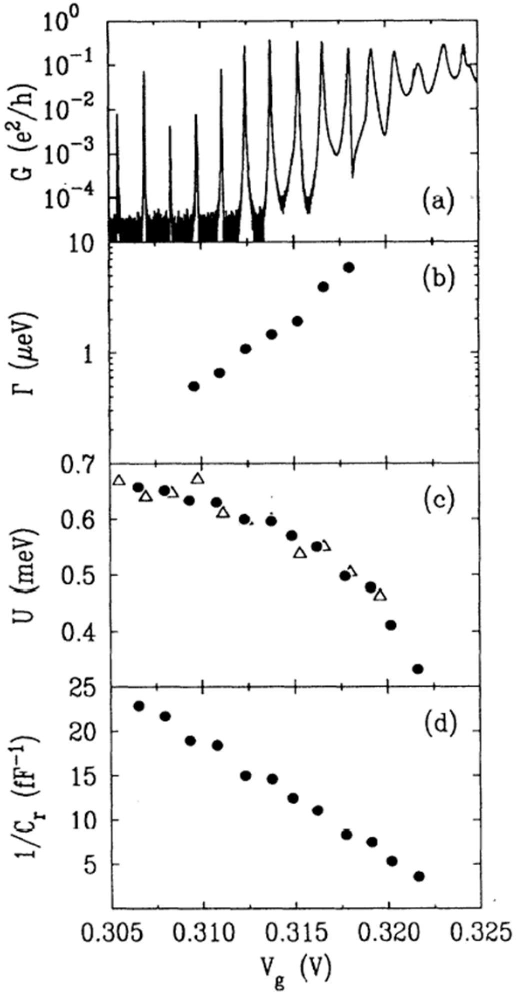
Foxman et al., PRB50, 14193 (1994)

Line Shapes: Experiments



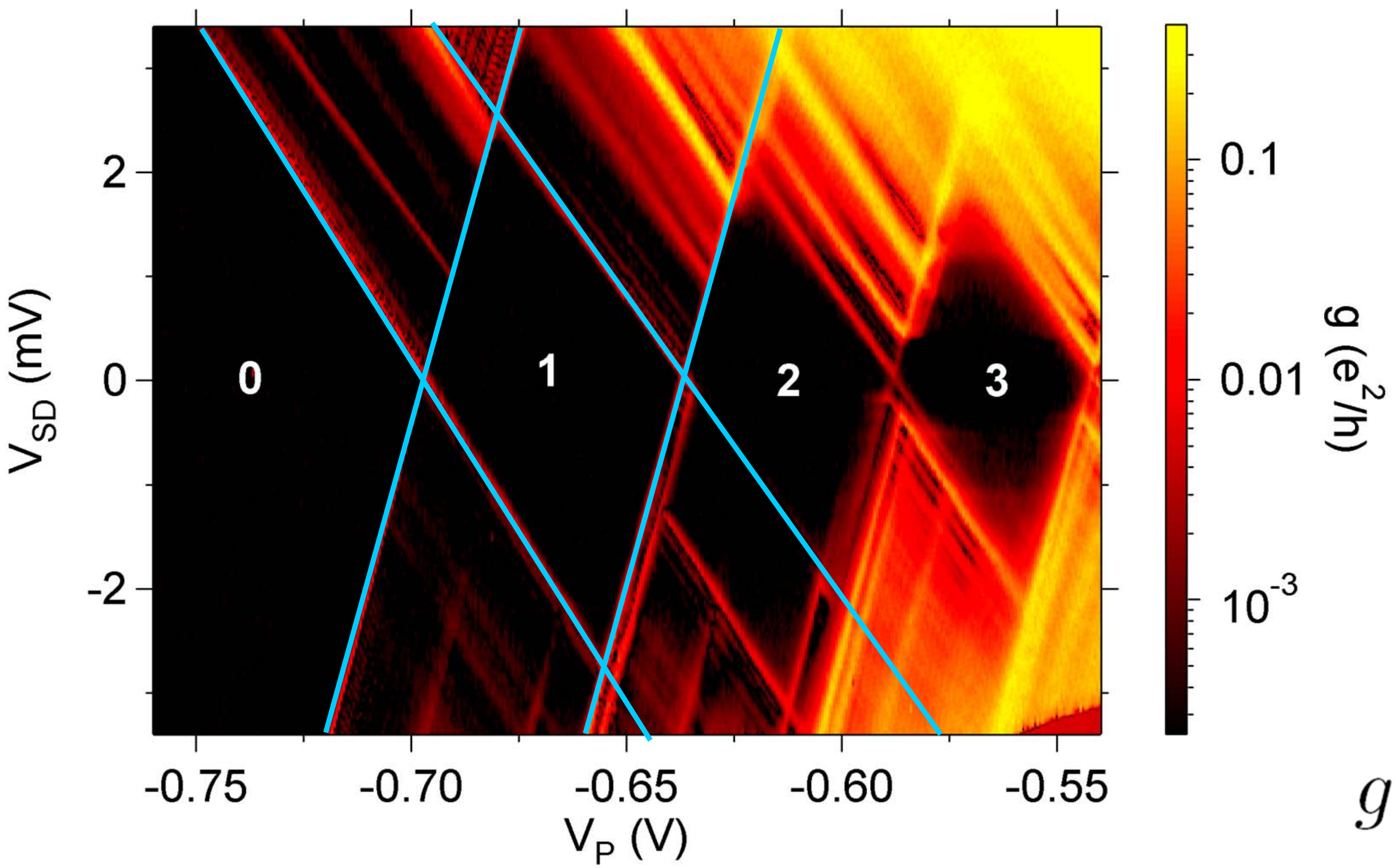
T-broadened

lifetime broadened



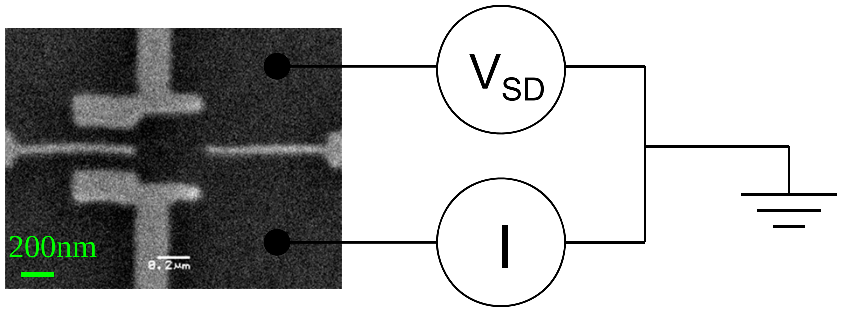
Foxman et al., PRB47, 10020 (1993)

Coulomb Diamonds



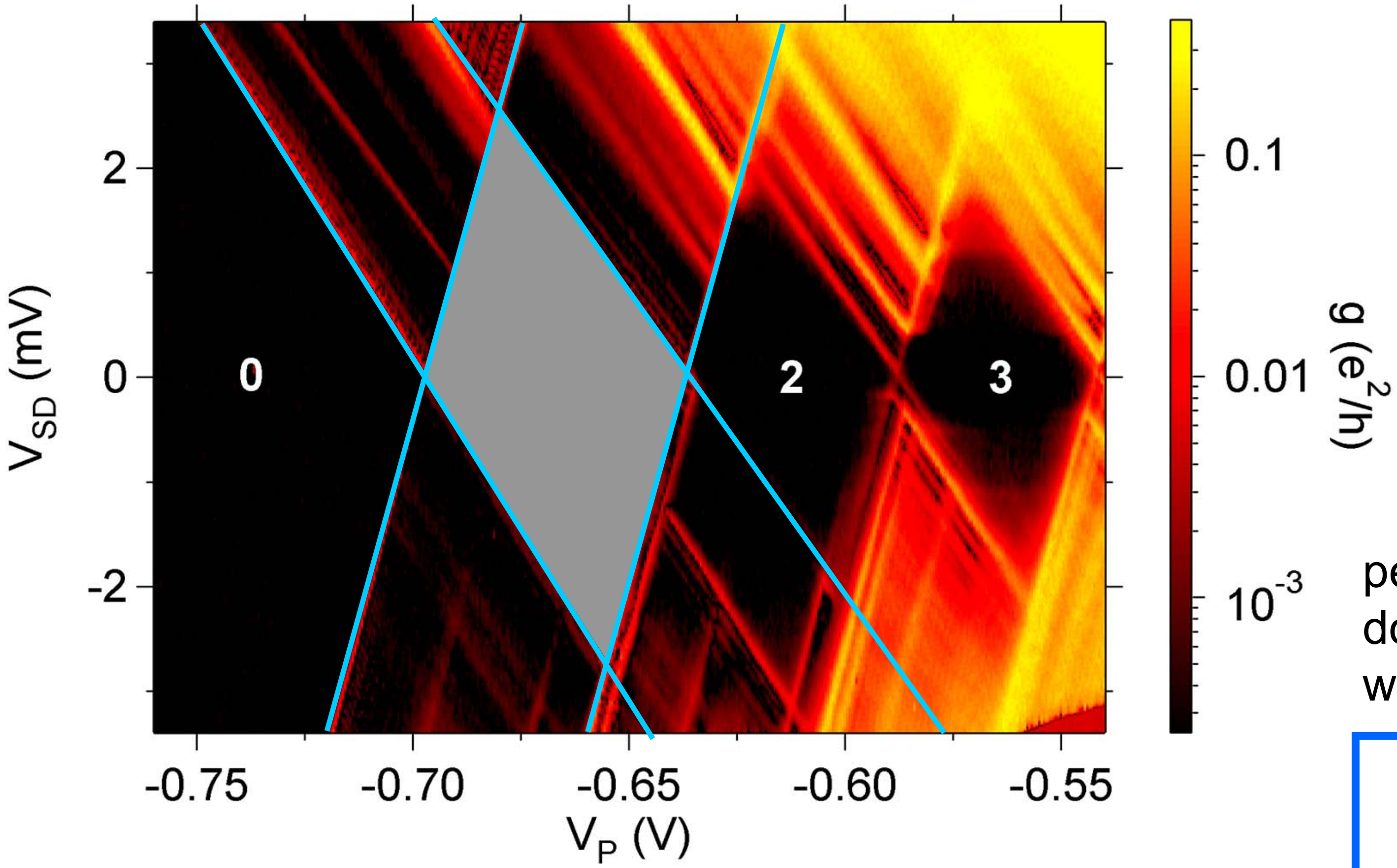
kT ~ μeV
Δ ~ meV
Ec ~ meV

$$g = \frac{dI}{dV}$$

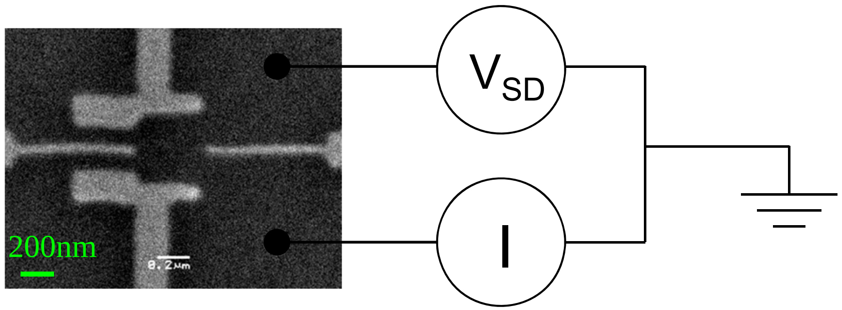


differential conductance:
peaks when current through dot is changing

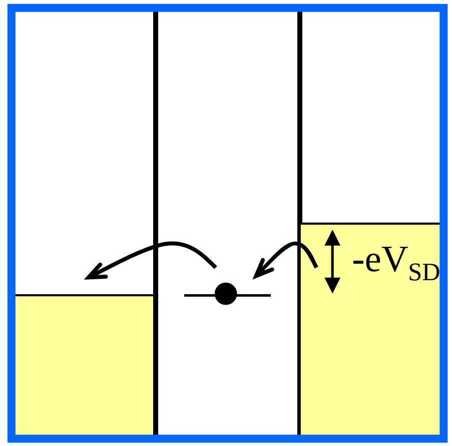
Coulomb Diamonds



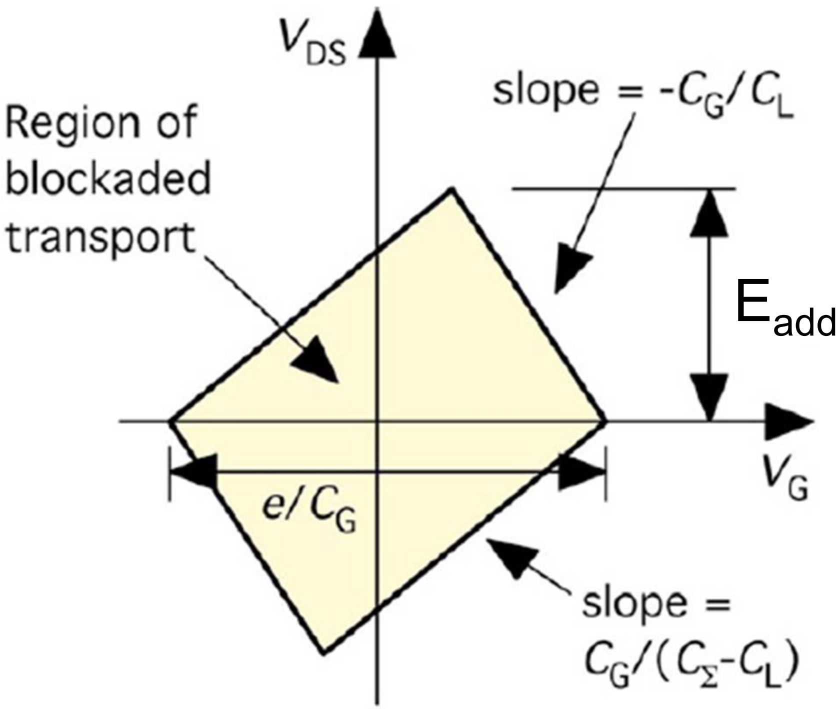
kT ~ μeV
Δ ~ meV
Ec ~ meV



peaks:
dot level aligned
with source / drain



Coulomb Diamond Slopes



two slopes, each associated with its respective dot-lead capacitance

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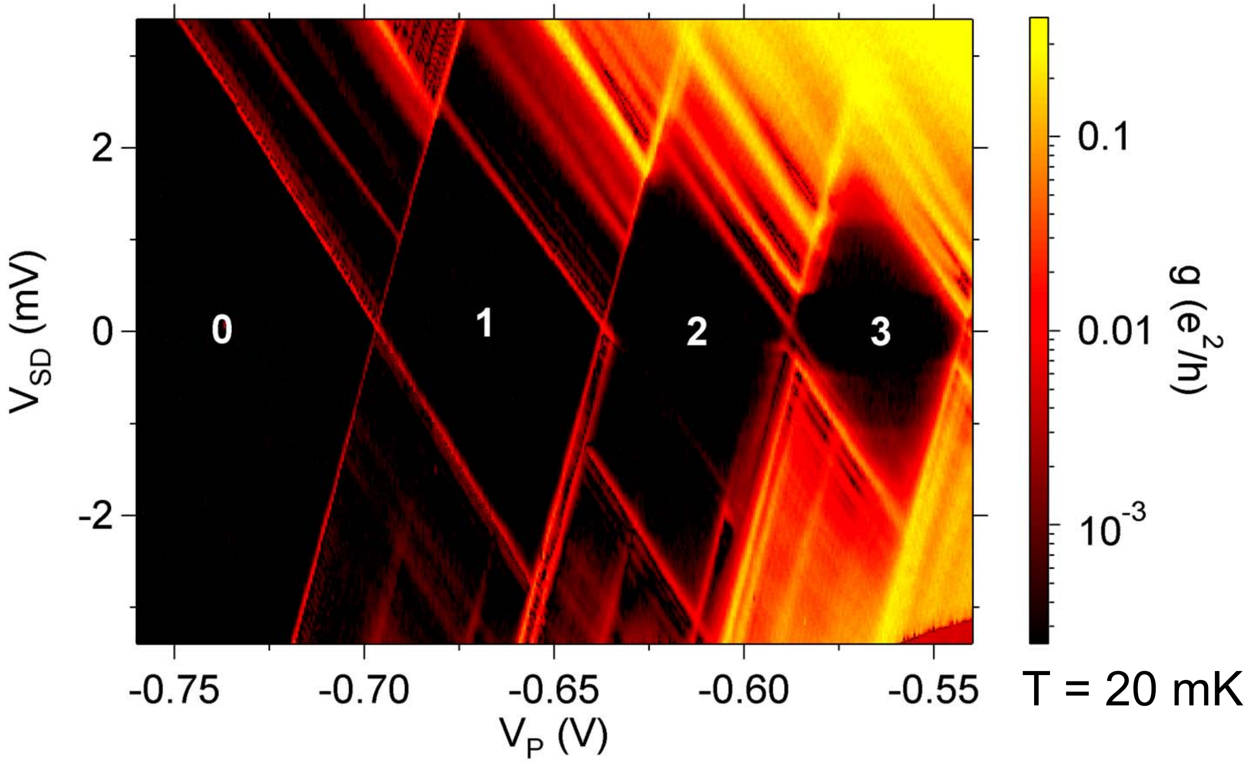
Fundamental Quantum Dot Transport Mechanisms

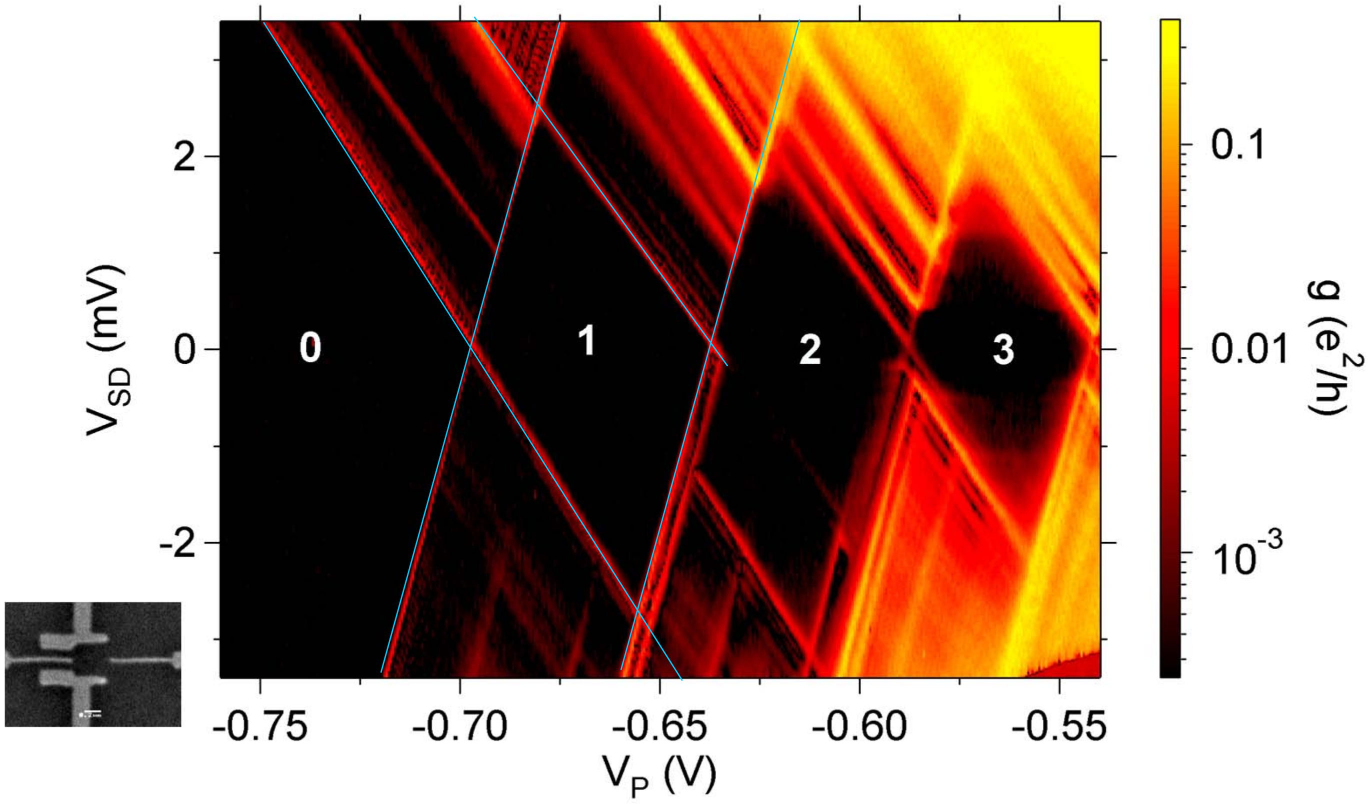


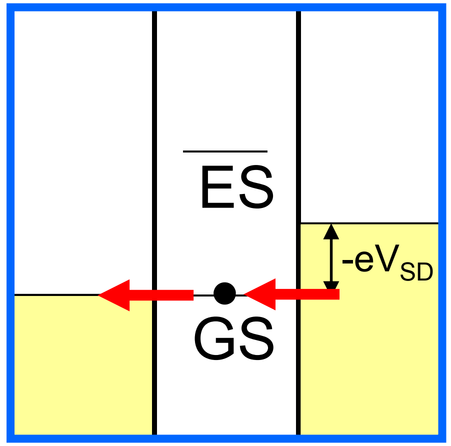
electron transport

- sequential tunneling
- cotunneling - elastic

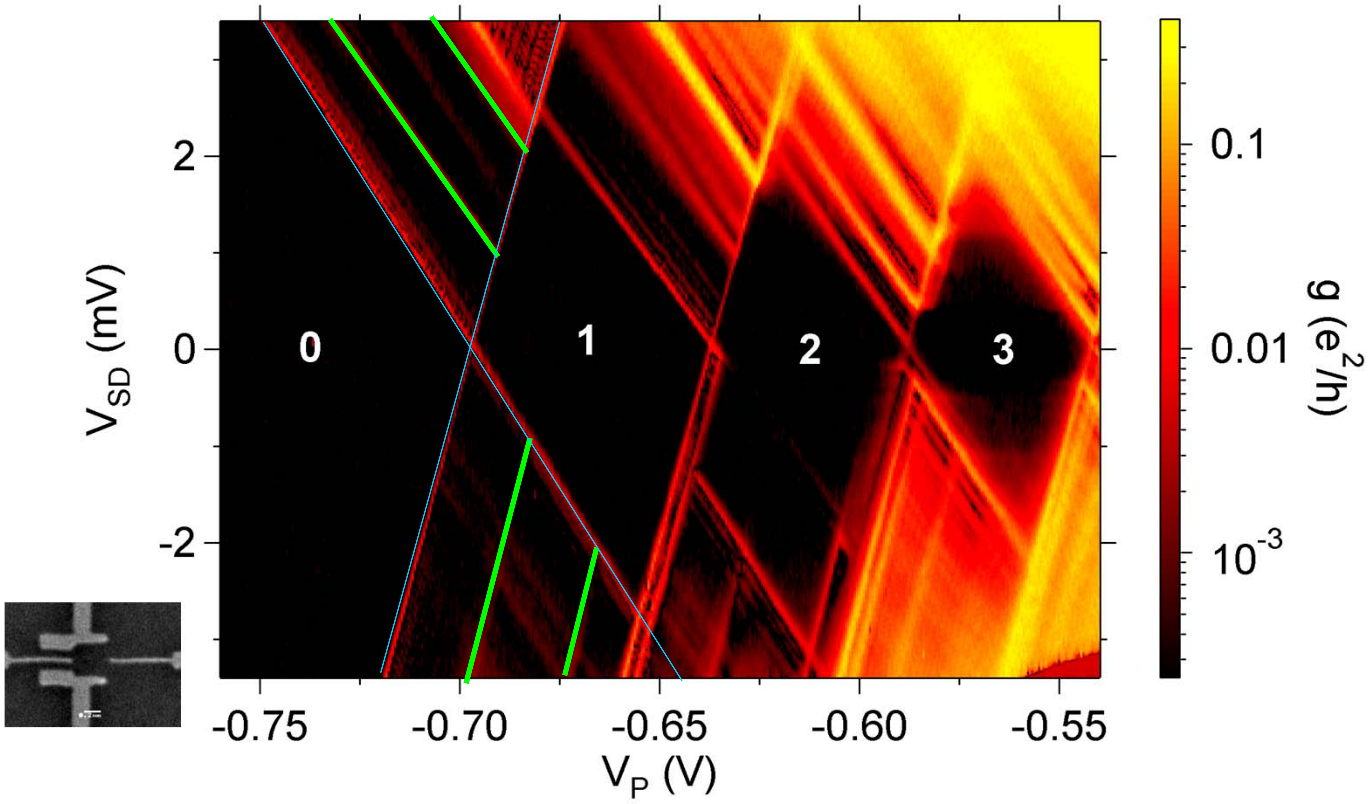
  - inelastic
- cotunneling assisted sequential tunneling (CAST)

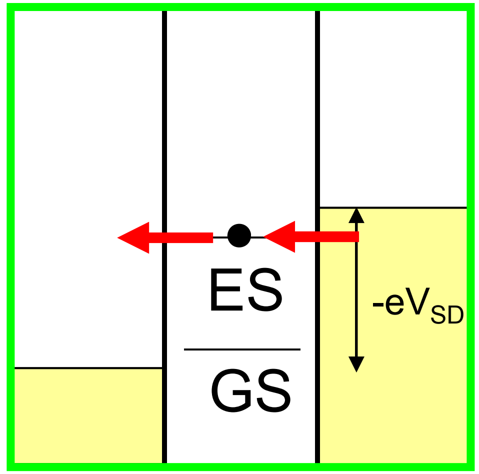
Ground State Sequential Tunneling Transport



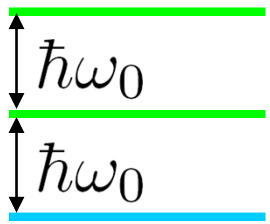


Excited State Sequential Tunneling Transport



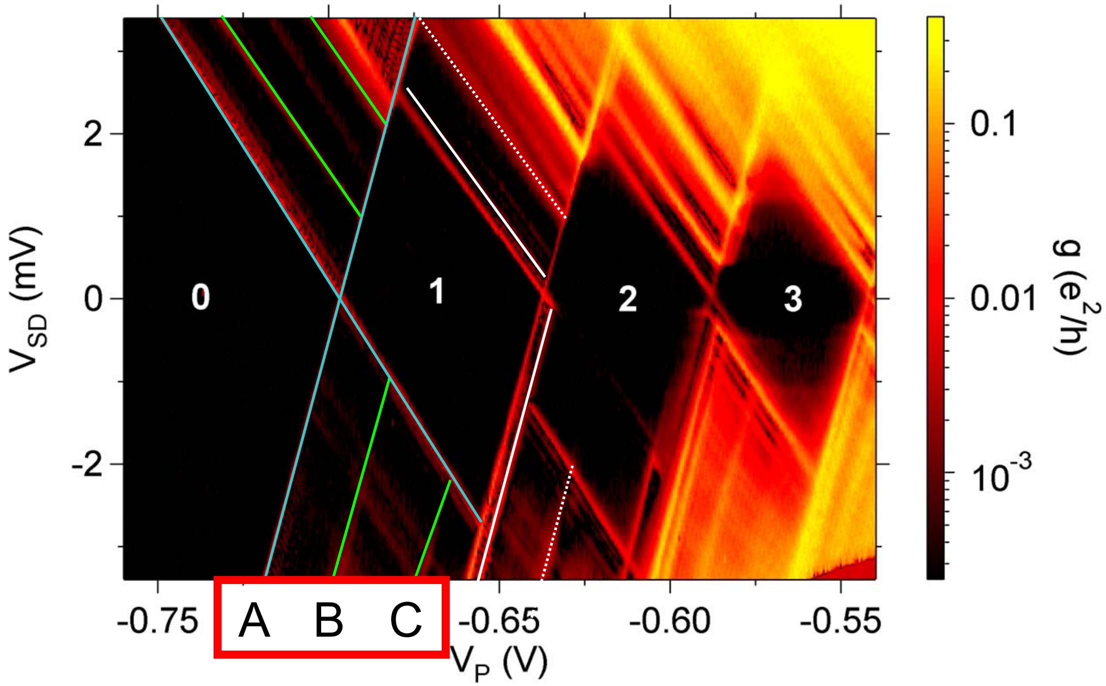


one electron states



harmonic oscillator

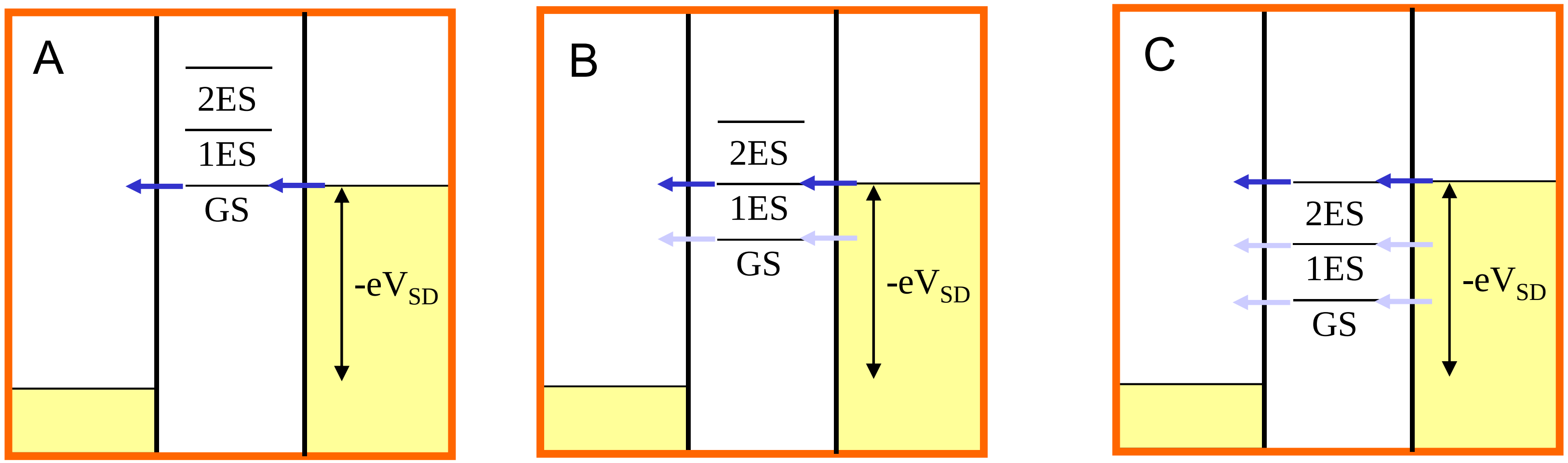
Sequential Tunneling Transport



lab to investigate quantum levels in device!!

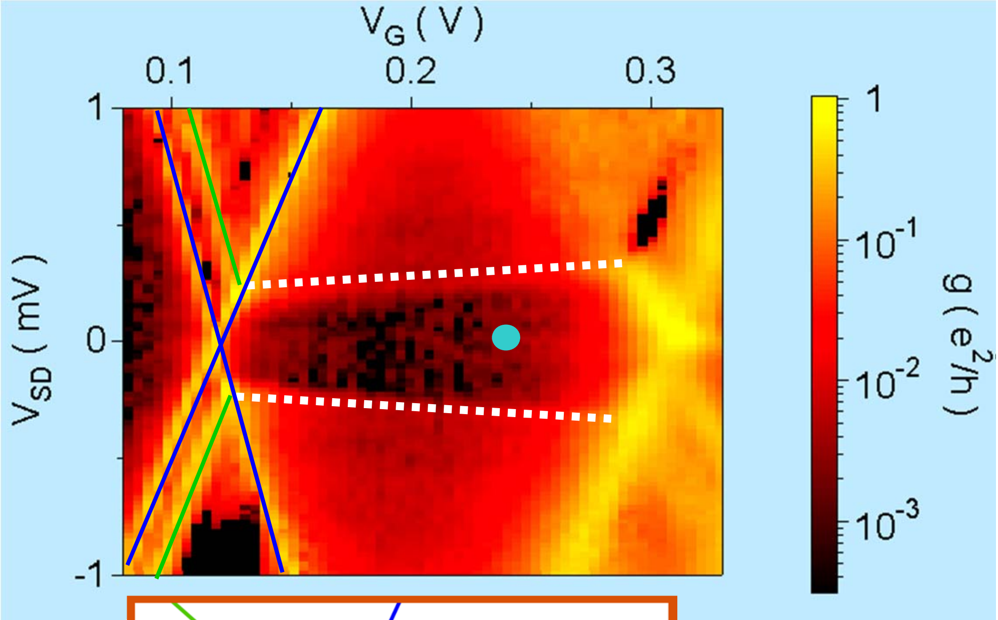
quantum confinement energies

internal excitations (spin)



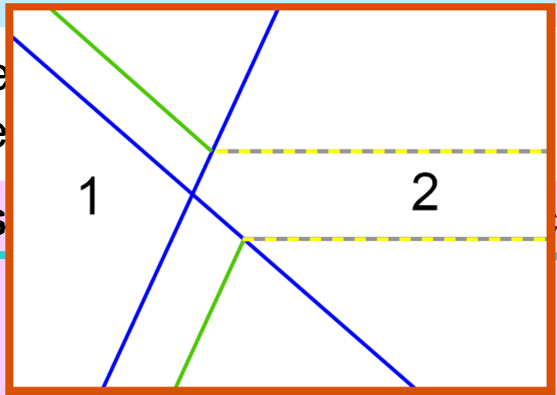
only one excess electron can be on dot (charging energy)

Cotunneling: elastic / inelastic

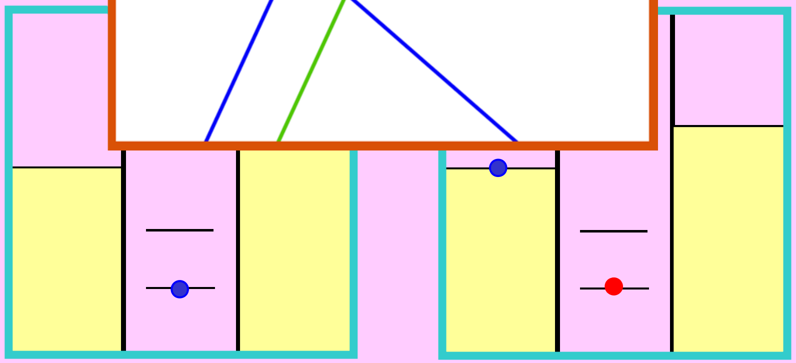


highe

two e … e energy



elas … le)

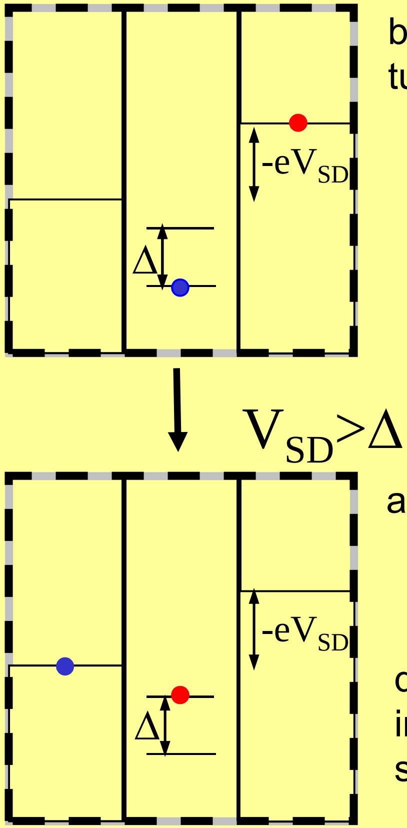


dot energy unchanged

before tunneling

after

inelastic cotunneling
(white lines)
dot energy changes
only possible for V_{SD}>D



before tunneling

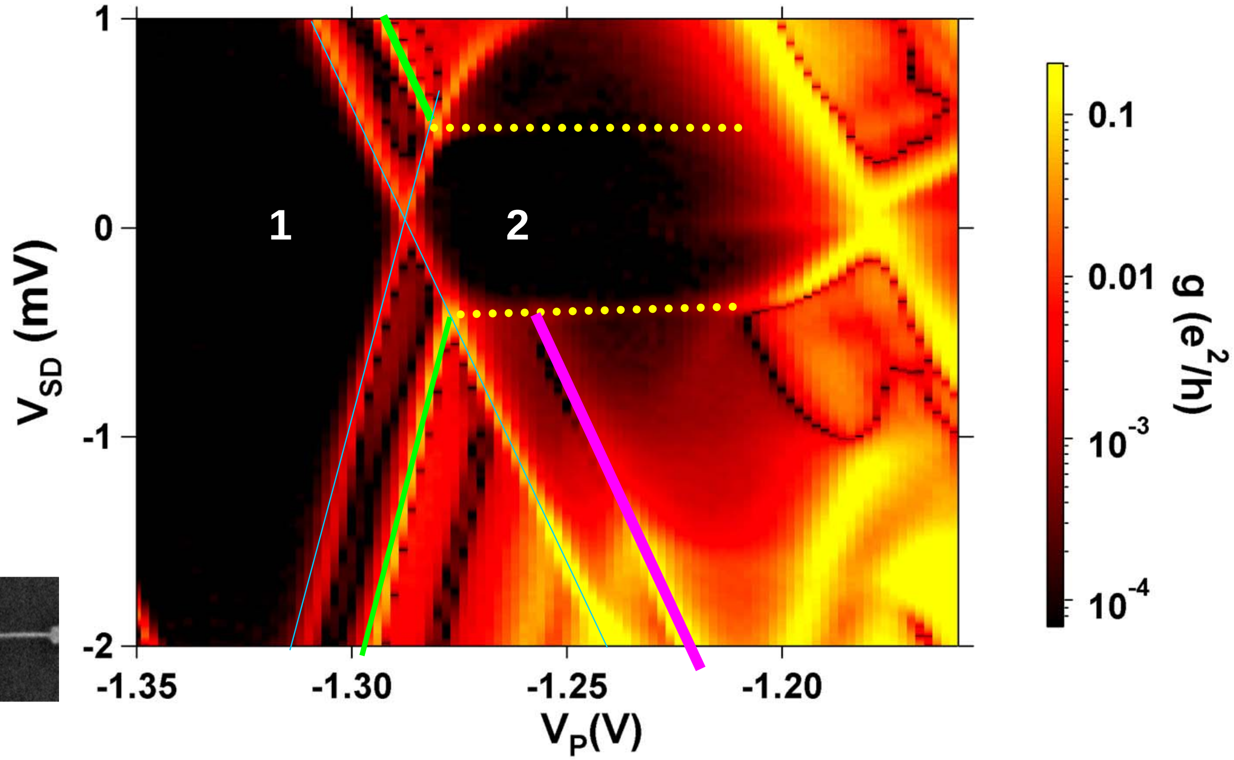
after

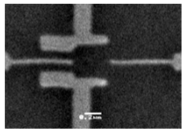
dot now in excited state

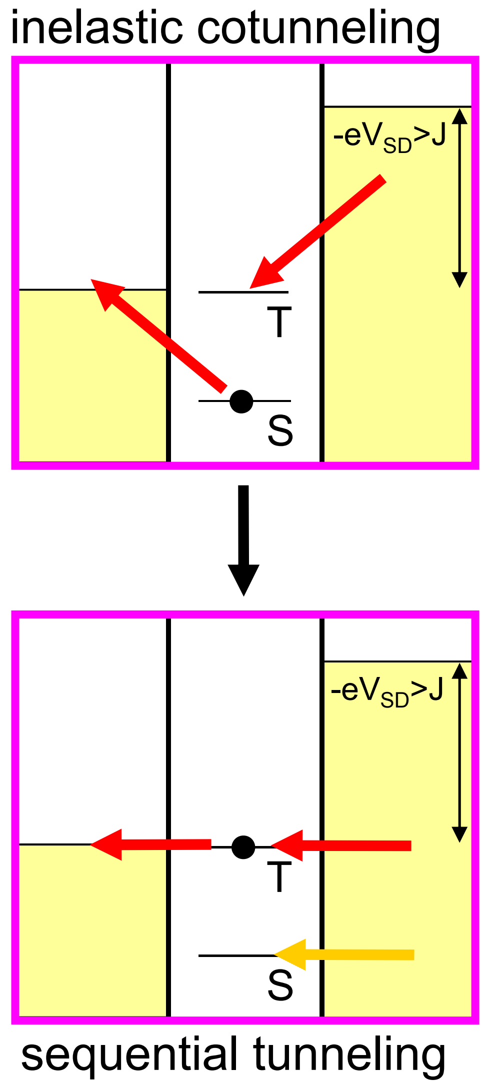
Quantum Dot Basics

- GaAs 2D electron gas (2DEG)
  conductance quantization in QPCs
- Coulomb blockade and charging energy $E_C = e^2/C$
  quantum confinement energy Δ
- Constant interaction model and
  Coulomb diamonds
- electronic transport via
  - sequential tunneling Γ
  - cotunneling Γ^2 / E_c (elastic / inelastic)
  - cotunneling assisted sequential tunneling
- singlet &triplet states,
  exchange splitting $J = E_T - E_S < \Delta$ (interactions)
- Pauli Spin blockade

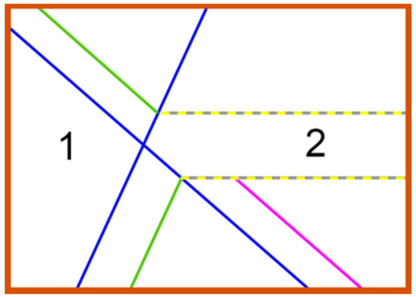
Additional Line inside Coulomb Diamond





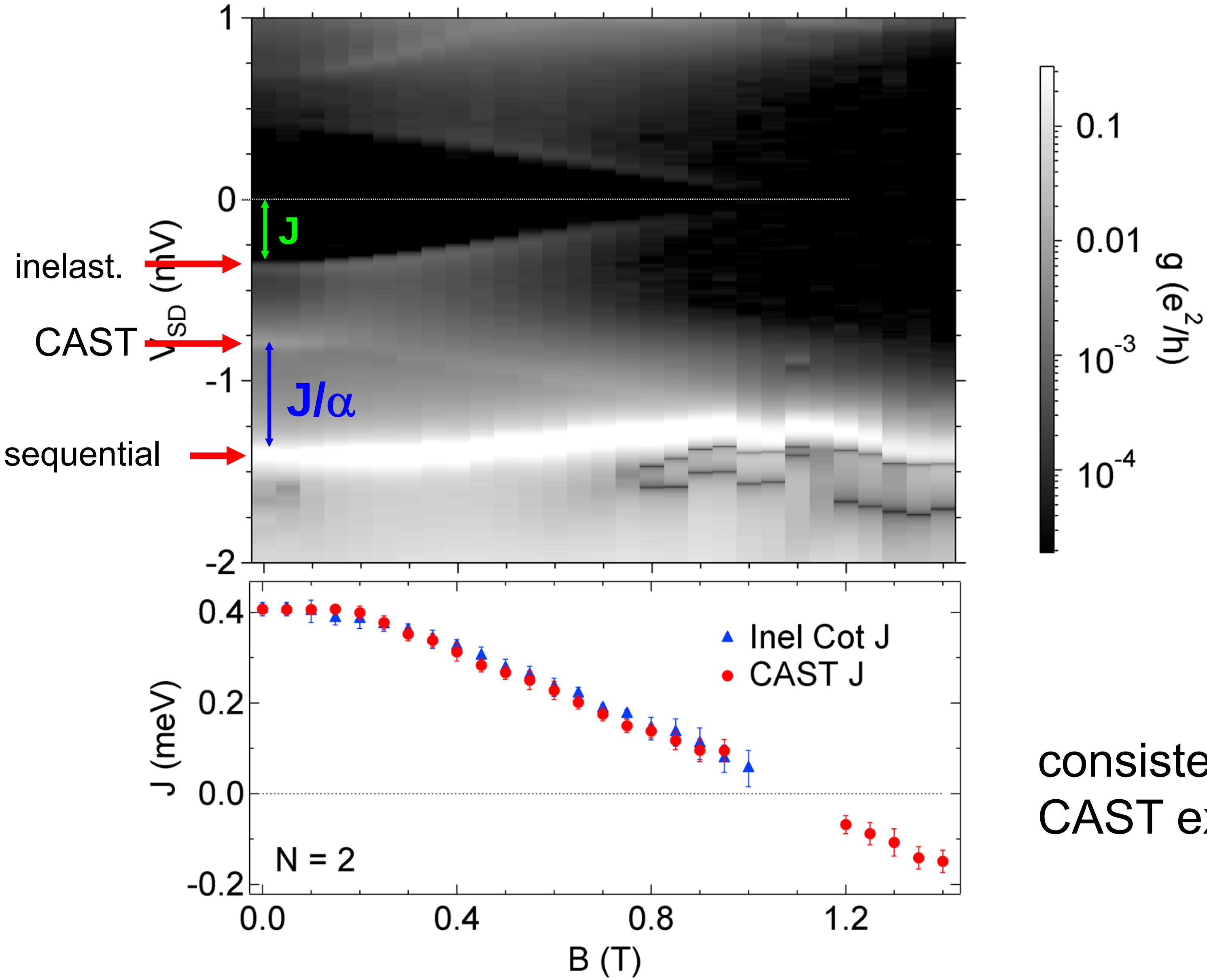


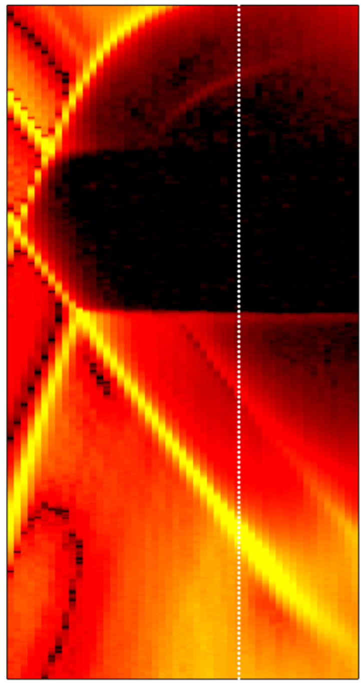
hypothesis:
cotunneling assisted
sequential tunneling
Golovach and Loss, PRB 2004



many electron dot
Schleser et al., PRL 2005

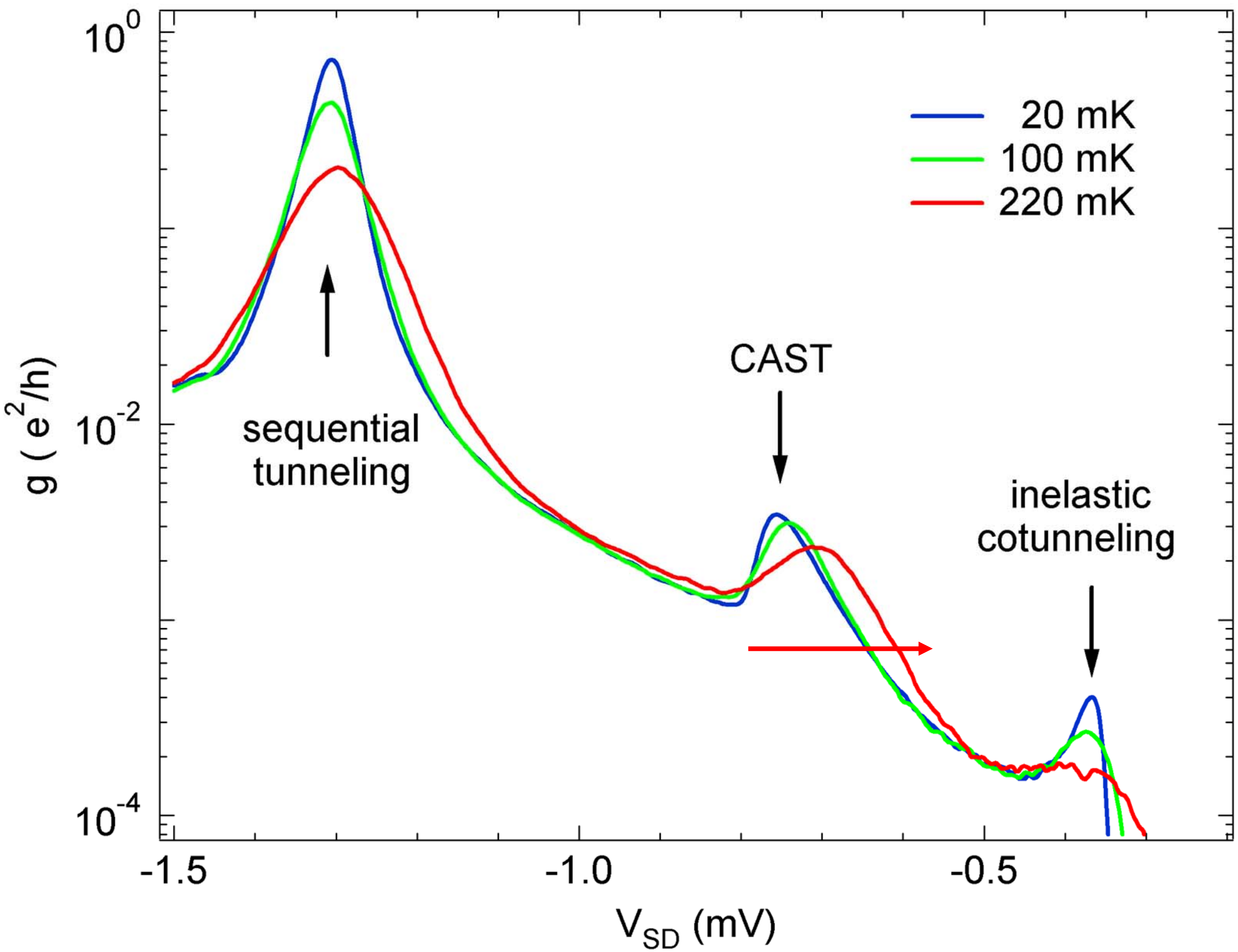
Magnetic Field Dependence



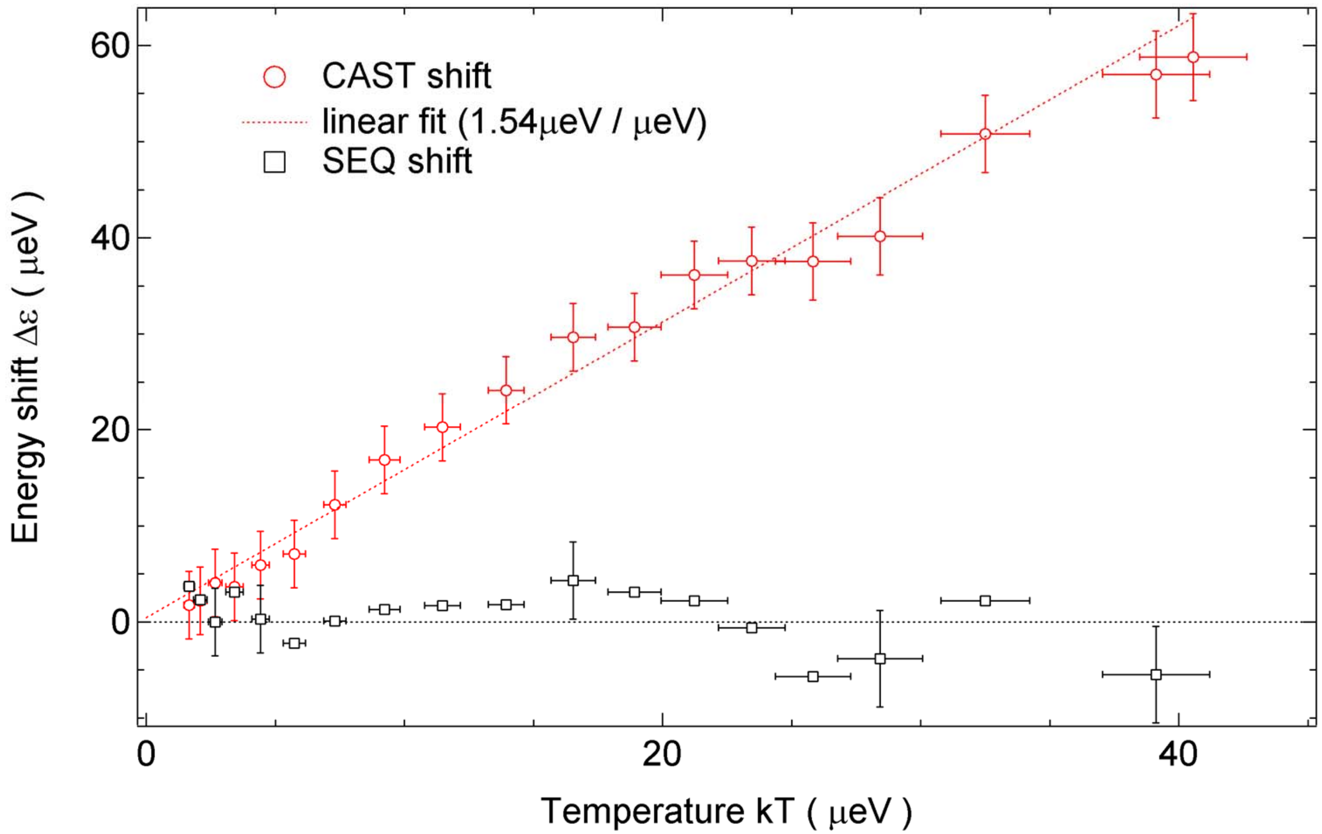


consistent with CAST explanation

Temperature dependence



Shift of Cotunneling Assisted Peak Position with Temperature



peak moves lineary with temperature, as expected for CAST

$$\Delta\epsilon \sim kT \ln\left[\frac{J\Gamma_L^{-1}}{(1+\eta/2)}\right]$$

for $T \ll T_0 = \frac{k^{-1}J}{\ln(J/\Gamma_L)} \sim 2\,\mathrm{K}$

Golovach &Loss, PRB **69**, 245327 (2004)

Part 1: Quantum Dot Basics

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  exchange splitting $J = E_T - E_S$
- Pauli Spin blockade

part 2	putting the basics to work g-factor, ST transition, spin entanglement
part 3	charge sensing charge & spin tunneling, spin relaxation, charge fluctuations