Quantum Transport FS 2015
Problem Set 2, Quantum Dots

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1. Quantum vs Coulomb energy

Derive the average quantum level spacing of a circular quantum dot of area πR^2 starting from the 2D density of states. Further, find an expression for the charging energy, assuming an infinitely thin flat disc of radius R. Compare qualitatively the size dependence of the charging energy and the quantum level spacing.

2. Sequential Tunneling through a Single-Level Quantum Dot

Consider a quantum dot coupled to two reservoirs with Fermi-Dirac distributions at temperature T and tunneling rates Γ_S and Γ_D through the source and drain barriers, respectively. Assume the temperature broadened regime $h\Gamma_{S,D} \ll k_B T$. A source-drain bias $eV_{SD} = \mu_S - \mu_D$ is applied, where μ_S and μ_D are the chemical potentials of the source and drain reservoir. Assume that the dot has only a single quantum level at energy ϵ above μ_D. *Hint:* Assume $\mu_D = 0$ throughout this exercise for simplicity.

(a) Draw a sketch of the situation with reservoirs, dot, energy level and tunnel barriers.

(b) Derive an expression for the sequential tunneling current I through the dot as a function of T, V_{SD}, Γ_S, Γ_D and for arbitrary level energy ϵ.

(c) How can this dot be used as a thermometer?

(d) From the current I, find the differential conductance g as a function of the same parameters as for the current. What is the line shape as a function of gate voltage?

3. Double dot anticrossing due to tunneling

Take a simple model of a double quantum dot where where the left dot level is at energy E_L and the right dot at energy E_R with detuning $\Delta E = E_R - E_L$ in absence of tunneling. Calculate both the level energies and the eigenfunctions in presence of non-zero tunneling of strength t as a function of detuning. Plot the energies as a function of detuning ΔE for $t = 0$ and $t \neq 0$.

4. Double dot capacitances and energy scales

Read the paper "Determination of energy scales in few-electron double quantum dots" by Taubert et al., Rev. Sci. Instr. **82**, 123905 (2011). Explain how the various capacitances and energy scales in a double quantum dot can be extracted from transport data.

5. Anisotropic quantum harmonic oscillator model of a quantum dot

Suppose that in your experiment, you have figured out the energies Δ_1 and Δ_2 of two excited states of a single electron quantum dot from bias-spectroscopy at a number of perpendicular magnetic field $B_\perp$ values, as given in the table below. Assuming an anisotropic harmonic oscillator (see B. Schuh, J. of Phys. A: Math. and Gen. **18**, 803 (1985), eq. 28), determine both confinement energies ω_X and ω_Y. Finally, give error bars on your estimate of the confinement energies assuming experimental error bars of 3% on Δ_1 and 6% on Δ_2.

Hint: the data in the table is taken from Zumbühl et al., Phys. Rev. Lett. **93**, 256801 (2004), Figure 4(a).

$B_\perp$ [T]	Δ_1 [meV]	Δ_2 [meV]
0	1.2	2.4
0.25	1.2	
0.50	1.17	
0.75	1.13	
1.00	1.07	2.03
1.25	1.03	1.95
1.55	0.98	1.8

6. Singlet-Triplet transition in a circular quantum dot

In their subsequent paper *Excitation Spectra of Circular, Few-Electron Quantum Dots* in Science **278**, 1788 (1997), Kouwenhoven et al. describe the excited state spectra. Read this paper, and explain

(a) How are the excited states visible in this experiment?

(b) Do they agree with the simple Fock-Darwin model?

(c) What is the source of various additional B-field dependent features in the spectra besides the Fock-Darwin excited states?

(d) Estimate the magnetic fields required to perform similar spectroscopy in a 'traditional' atom such as H or He.

(e) Finally, the triplet excited state energy J is here rather close to the $N = 1$ orbital excited state energy Δ (see Fig. 5A), quite unlike the situation described in the lecture in lateral quantum dots, where we found $J \ll \Delta$. What could be the reason for this?