Part 1: Quantum Dot Basics

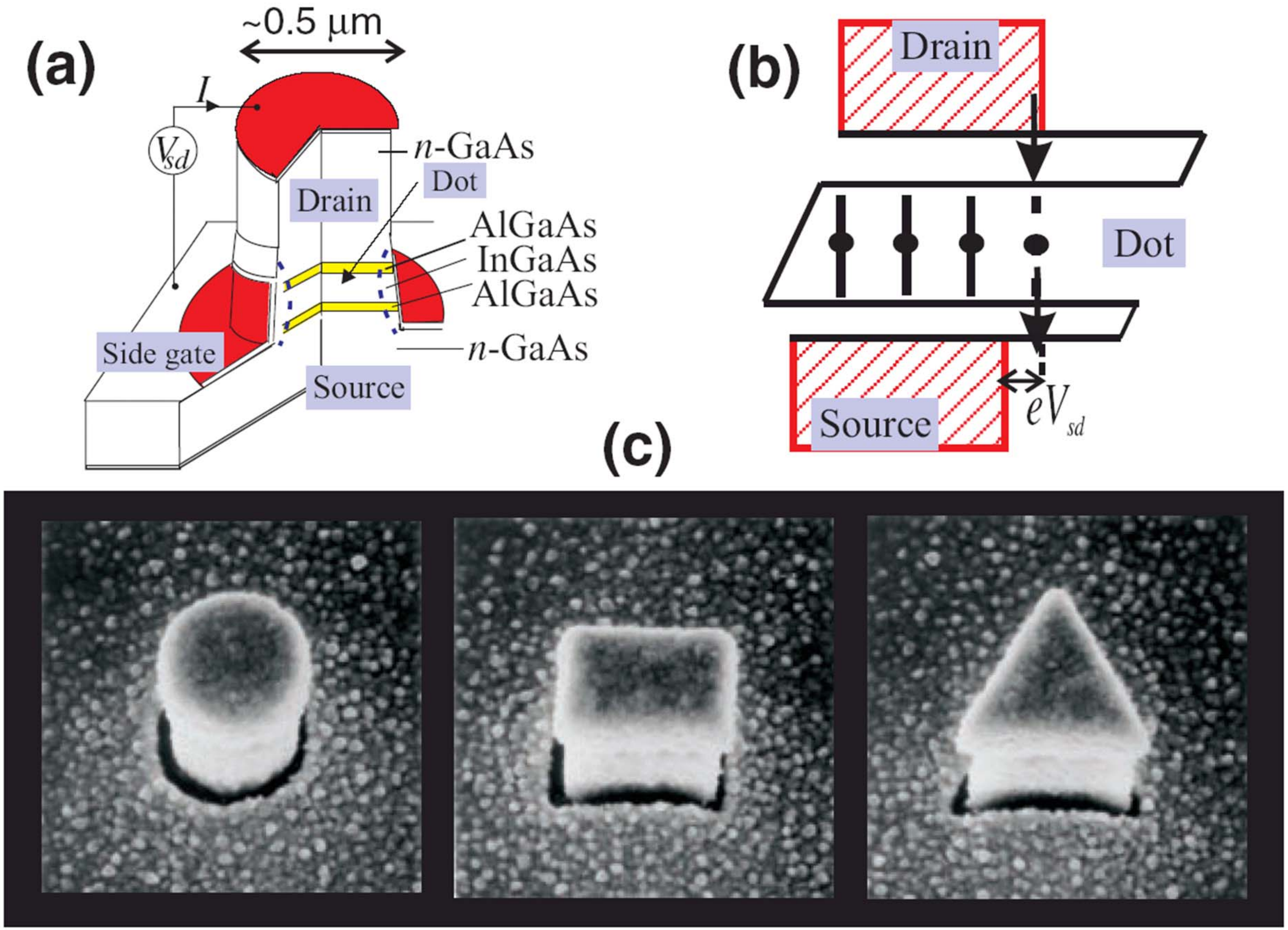
- GaAs 2D electron gas (2DEG)
  conductance quantization in QPCs
- Coulomb blockade and charging energy $E_C = e^2/C$
  quantum confinement energy Δ
- Constant interaction model and
  Coulomb diamonds
- electronic transport via
  - sequential tunneling Γ
  - cotunneling Γ^2 / E_c (elastic / inelastic)
  - cotunneling assisted sequential tunneling
- singlet & triplet states,
  exchange splitting $J = E_T – E_S$
- Pauli Spin blockade

Quantum Dots Part 2

1. Quantum Dot Basics
2. **Few Electron Dots**
3. Double Quantum Dots and Pauli Spin Blockade
4. Kondo Effect
5. Charge Sensing and Spin Relaxation

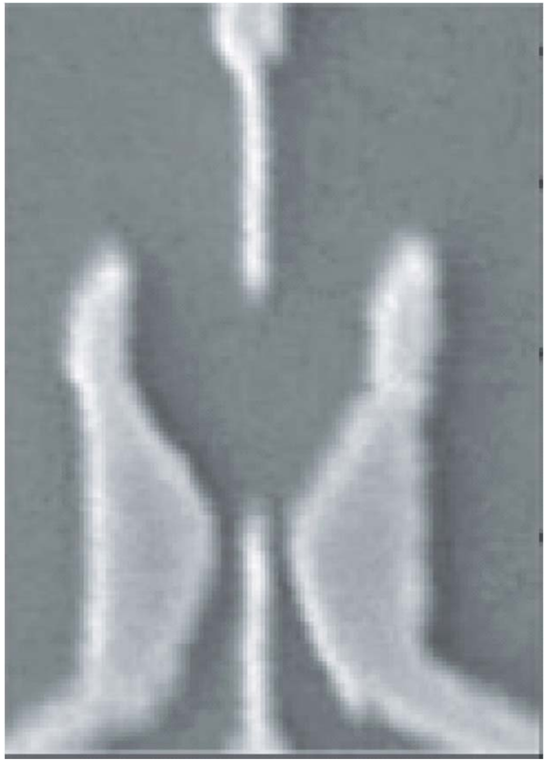
Kouwenhoven, Austing and Tarucha, RPP 64, 701 (2002)
Tarucha et al., PRL77, 3613 (1996)
Kouwenhoven et al., Science 278, 1788 (1997)

Few Electron Quantum Dots: Vertical

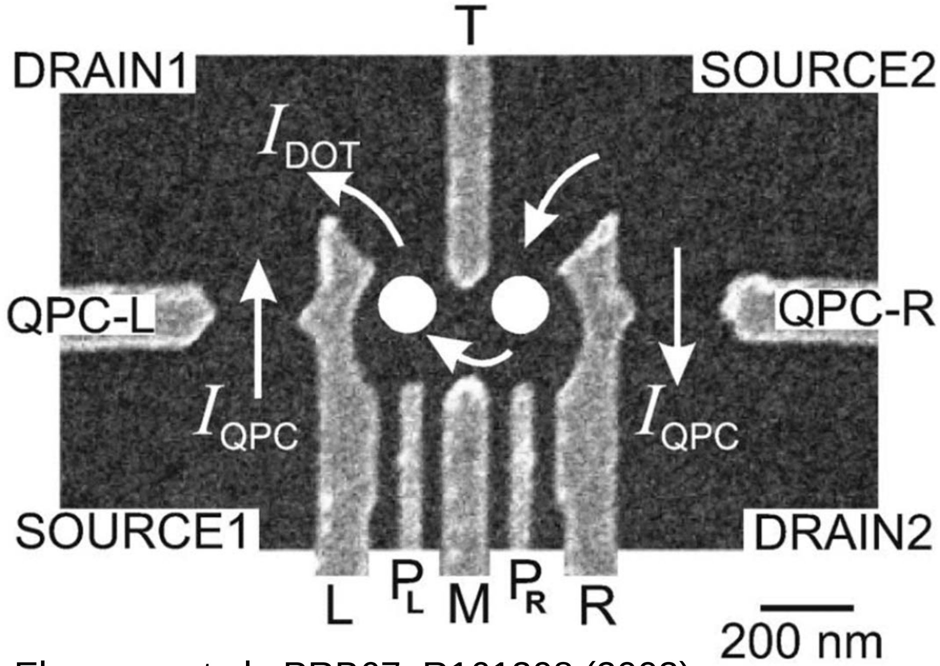


Kouwenhoven, Austing and Tarucha, RPP 64, 701 (2001)

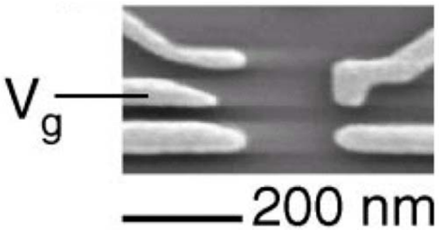
Few Electron Quantum Dots: Lateral



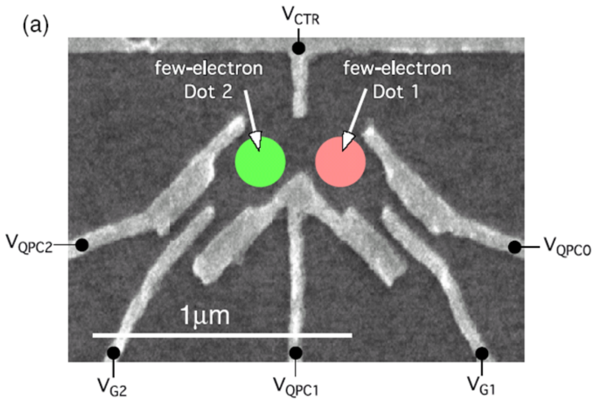
Ciorga et al., PRB61, R16315 (2000)



Elzerman et al., PRB67, R161308 (2003)



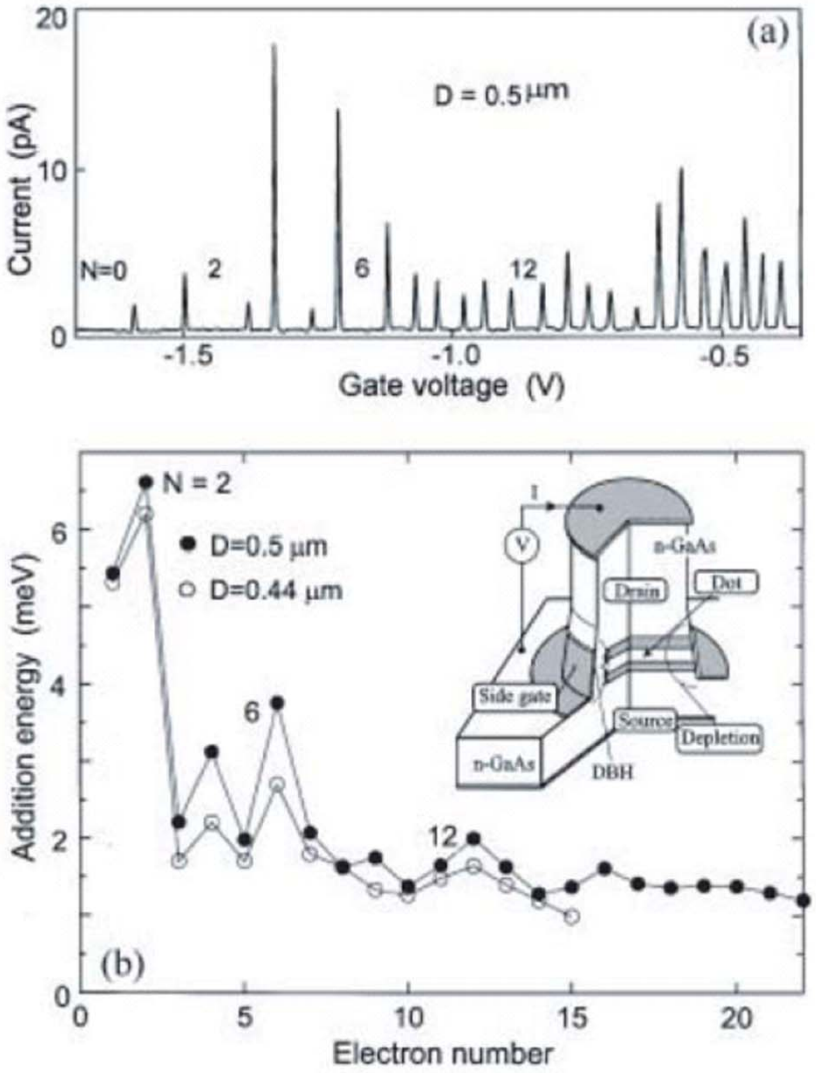
Zumbuhl et al., PRL93, 256801 (2004)



Chan et al., Nanotech. 15, 609 (2004)

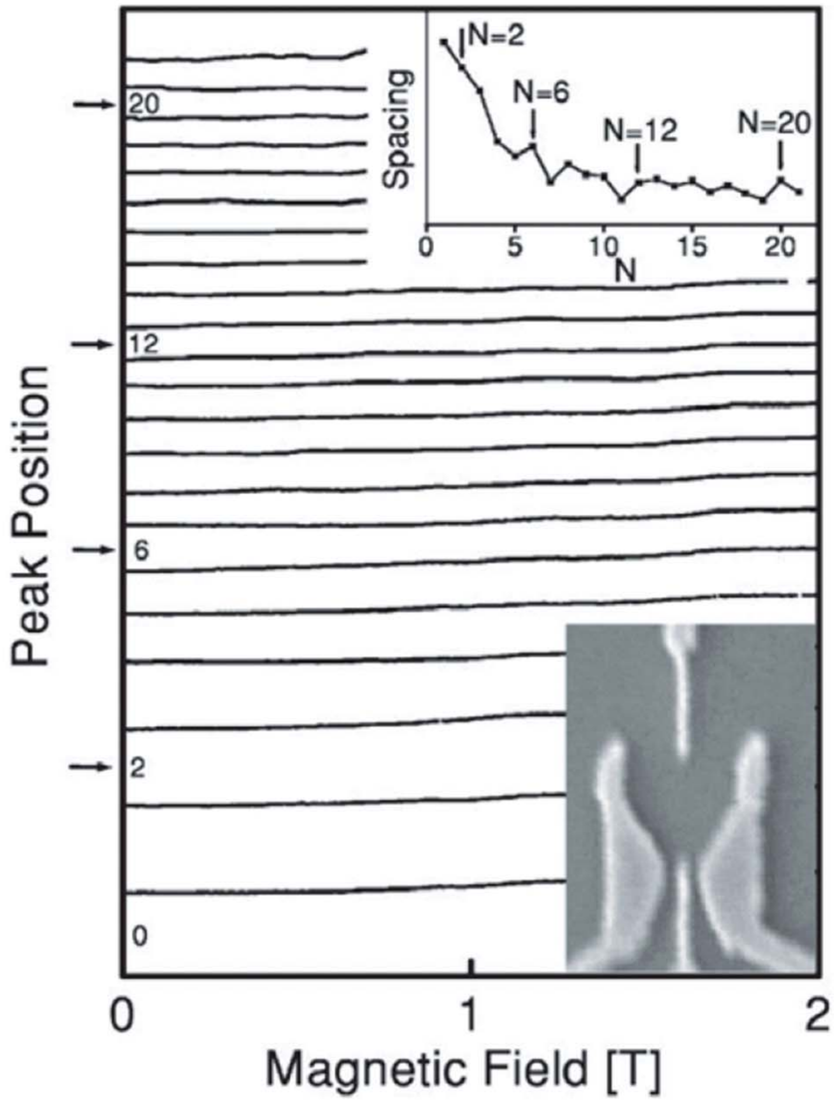
Rotation Symmetry and Angular Momentum

circular symmetry: 2D shell filling



Tarucha et al., PRL77, 3613 (1996)

circular symmetry broken



Ciorga et al., PRB61, R16315 (2000)

Isotropic Quantum Harmonic Oscillator: Fock-Darwin Spectrum

$$H = \frac{p_x^2 + p_y^2}{2m^*} + \frac{1}{2} m^* \omega_0 (x^2 + y^2)$$

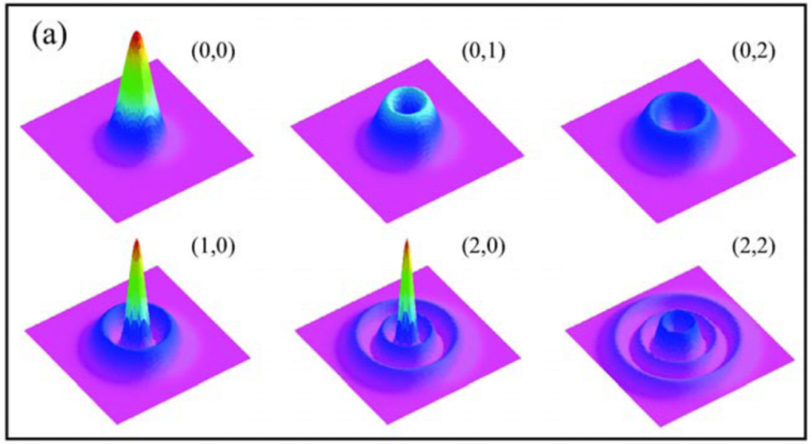
rotation symmetry ↔ angular momentum conservation

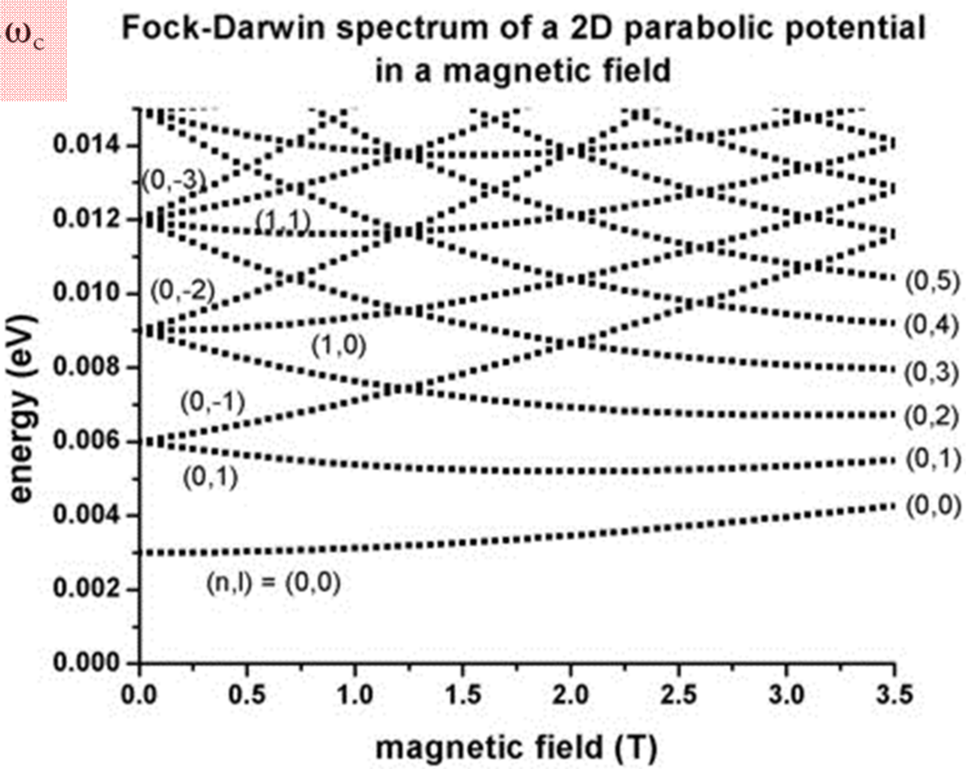
Fock-Darwin Energies

$$E_{n,\ell} = (2n + |\ell| + 1)\hbar \sqrt{\left(\frac{1}{4}\omega_c^2 + \omega_o^2\right)} - \frac{1}{2}\ell\hbar\omega_c$$

$n = 0, 1, 2, \ldots$ radial

$l = 0, \pm 1, \pm 2, \ldots$ angular momentum

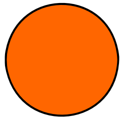




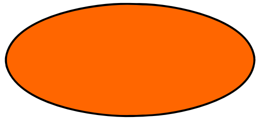
Quantum Harmonic Oscillator: **anisotropic**

$$H = \frac{p_x^2}{2m^*} + \frac{1}{2}m^*\omega_x^2 x^2 + \frac{p_y^2}{2m^*} + \frac{1}{2}m^*\omega_y^2 y^2$$

isotropic, circular symmetry: $\omega_x = \omega_y$



anisotropic, no rotation symmetry: $\omega_x \neq \omega_y$



energy levels:

$$E_{p,q} = \left(p + \tfrac{1}{2}\right)\hbar\omega_x + \left(q + \tfrac{1}{2}\right)\hbar\omega_y$$

in magnetic field

$$\epsilon_{jk} = j\frac{\hbar}{2}\sqrt{\omega_c^2 + (\omega_a + \omega_b)^2} + k\frac{\hbar}{2}\sqrt{\omega_c^2 + (\omega_a - \omega_b)^2}$$

$j \in \{1, 2, ...\}$ and $k \in \{j-1, j-3, ..., -j+1\}$

B. Schuh, J. Phys A: Math. Gen. 18, 803 (1985)

Quantum Harmonic Oscillator: anisotropic (2)

energy levels:

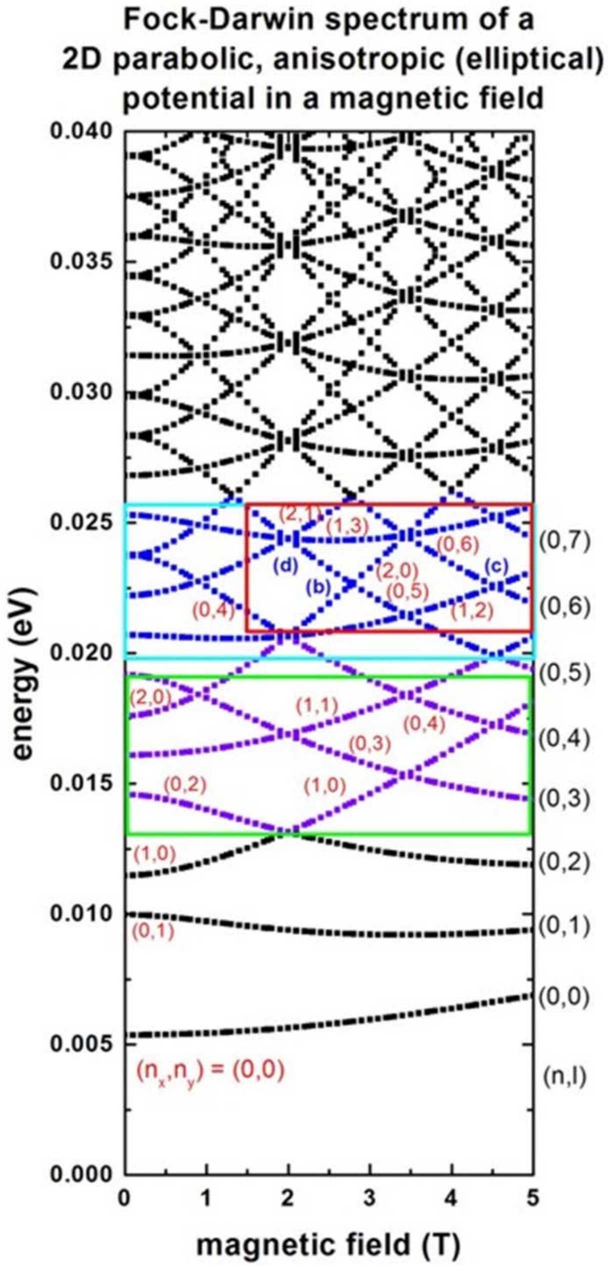
$$E_{p,q} = \left(p + \tfrac{1}{2}\right)\hbar\omega_x + \left(q + \tfrac{1}{2}\right)\hbar\omega_y$$

in magnetic field

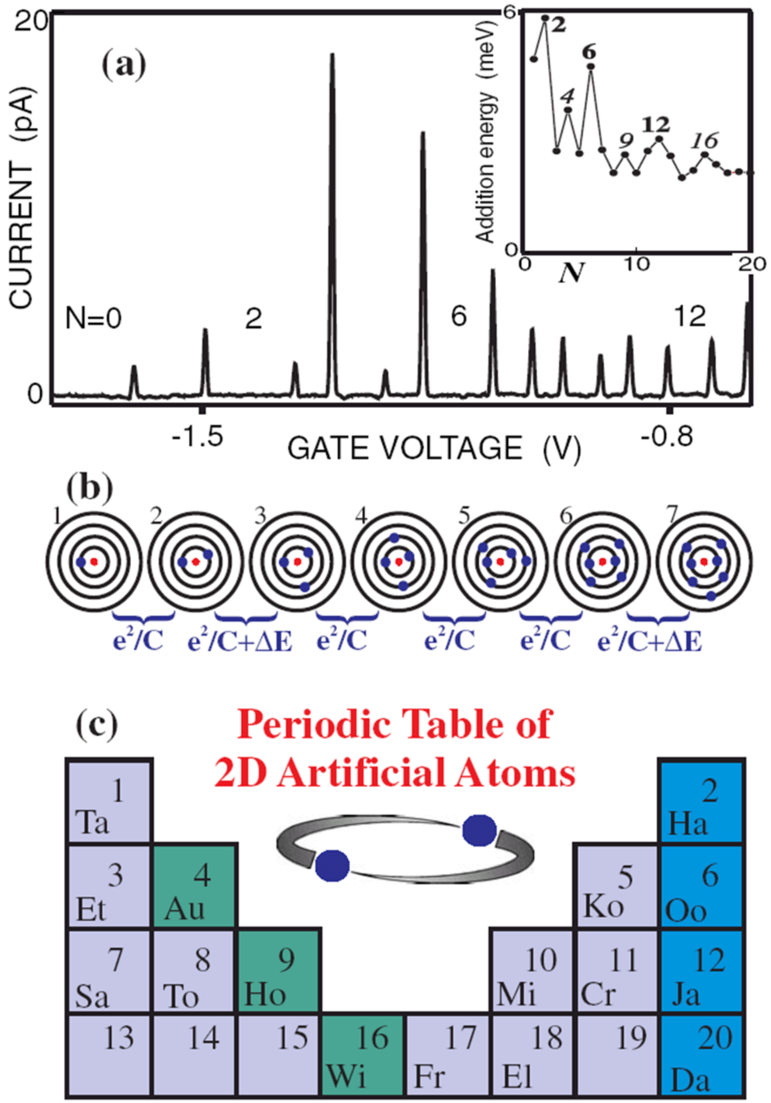
$$\epsilon_{jk} = j\frac{\hbar}{2}\sqrt{\omega_c^2 + (\omega_a + \omega_b)^2} + k\frac{\hbar}{2}\sqrt{\omega_c^2 + (\omega_a - \omega_b)^2}$$

$$j \in \{1, 2, ...\} \text{ and } k \in \{j-1, j-3, ..., -j+1\}$$

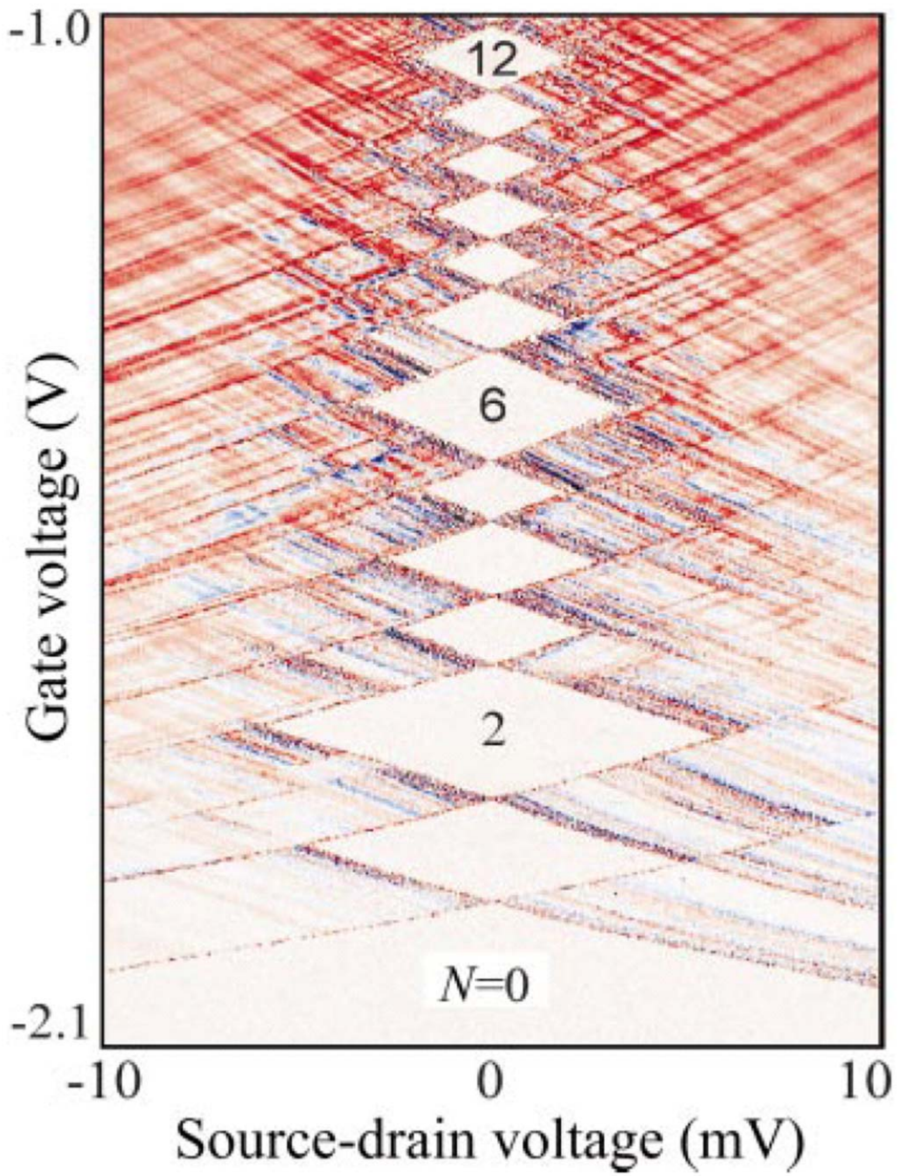
B. Schuh, J. Phys A: Math. Gen. 18, 803 (1985)

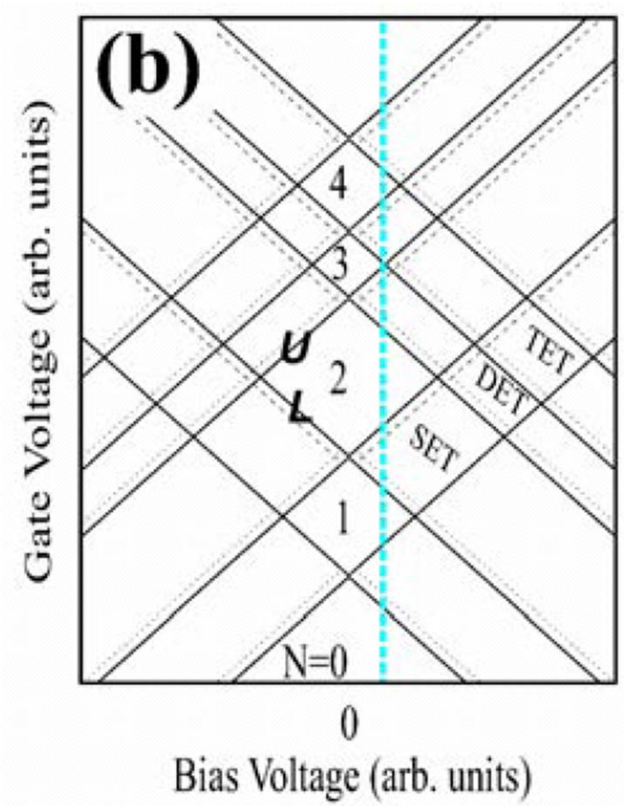


2D Periodic Table of Elements (symmetric potential)



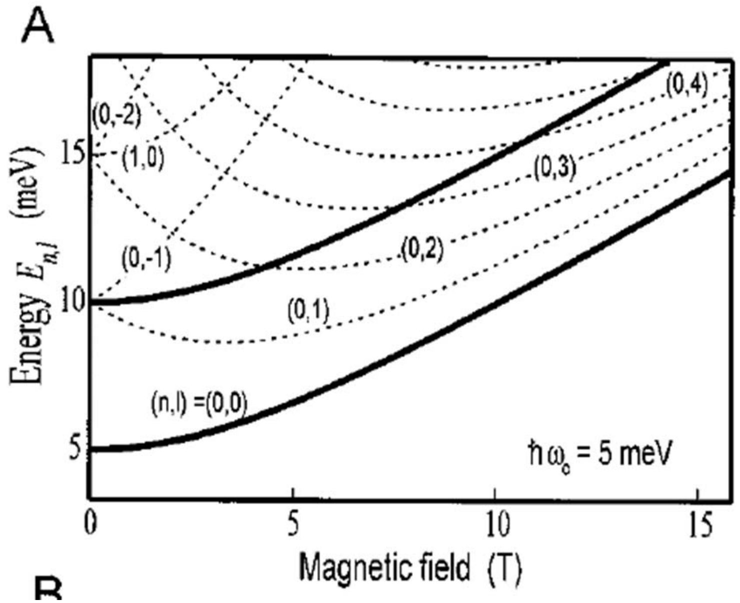
Excitation Spectra of Circular, Few Electron Dots

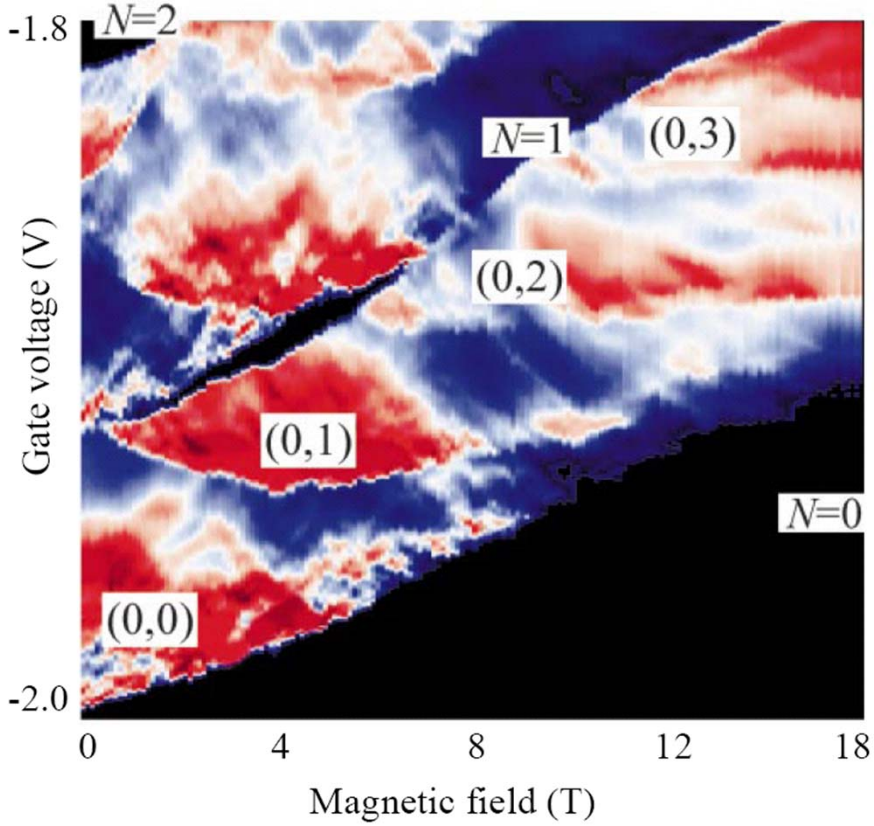




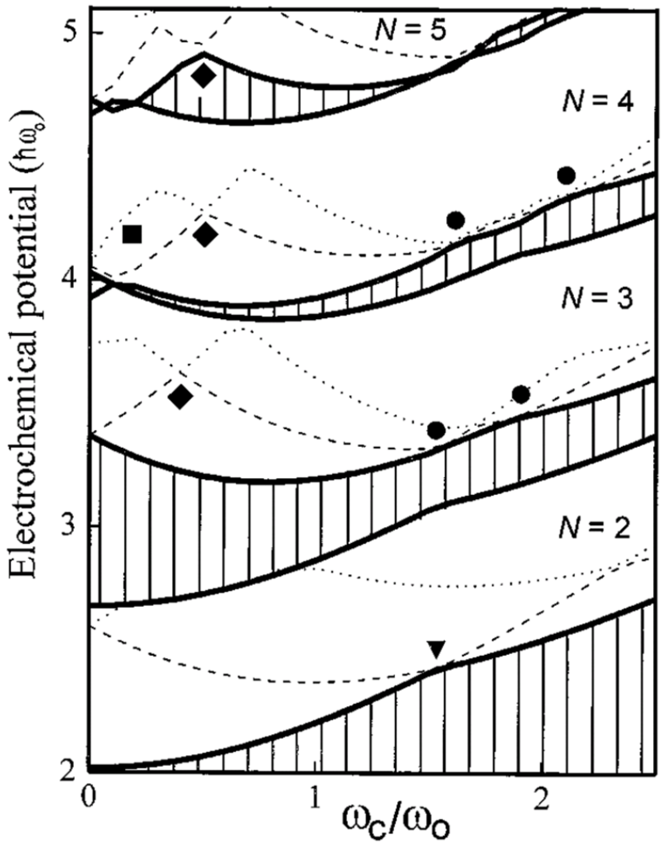
Kouwenhoven et al., Science 1997

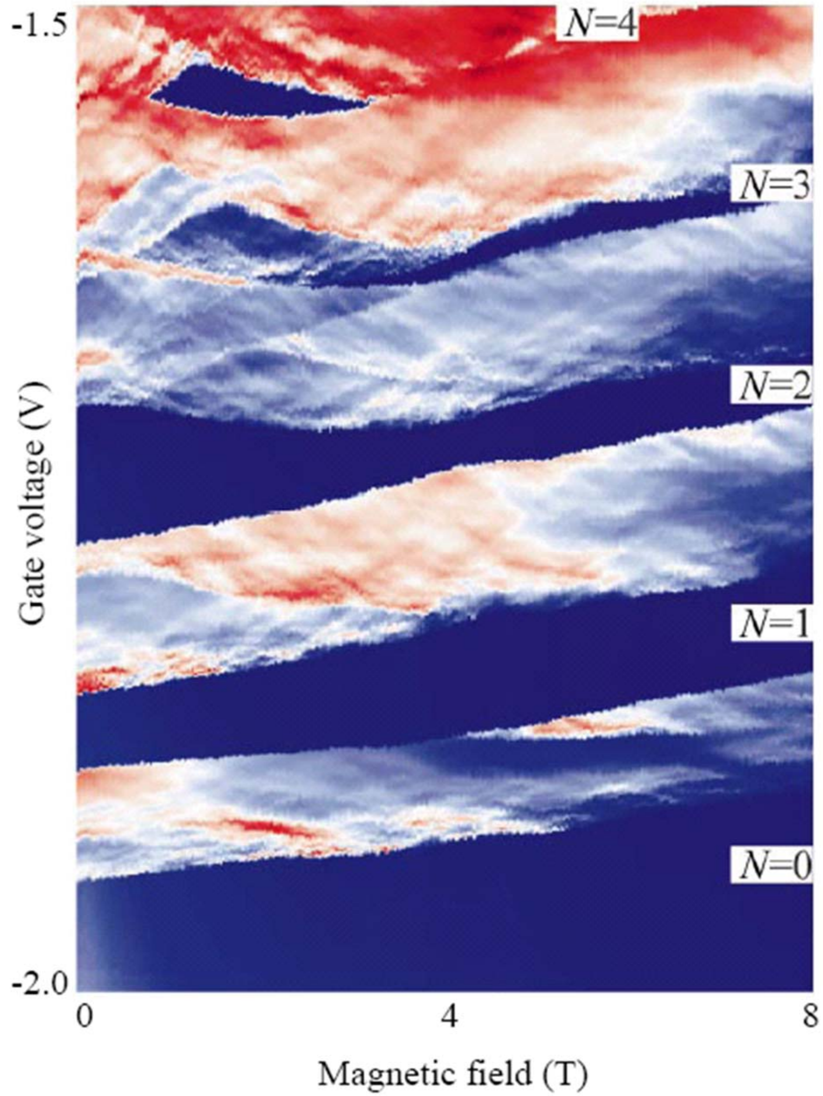
Zero to One Electron Transition



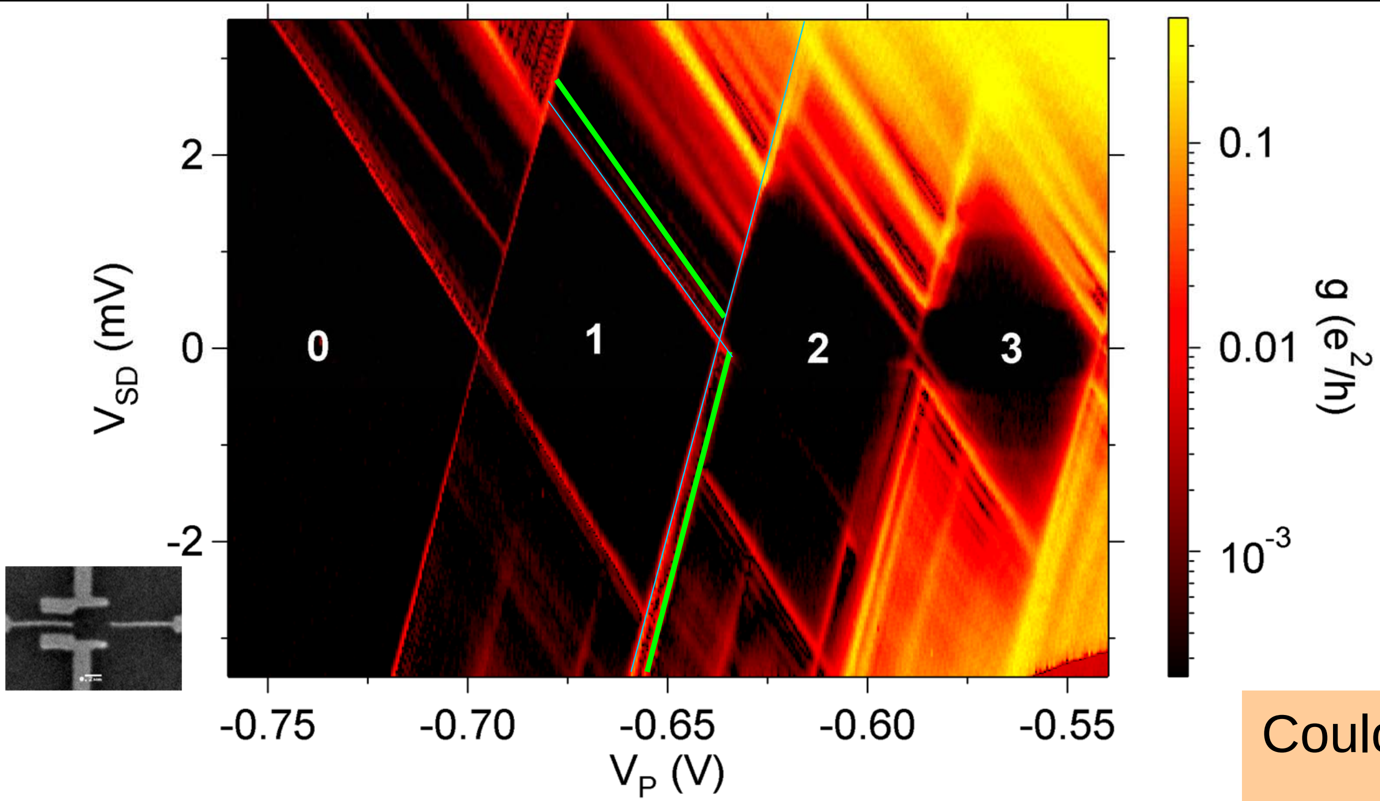


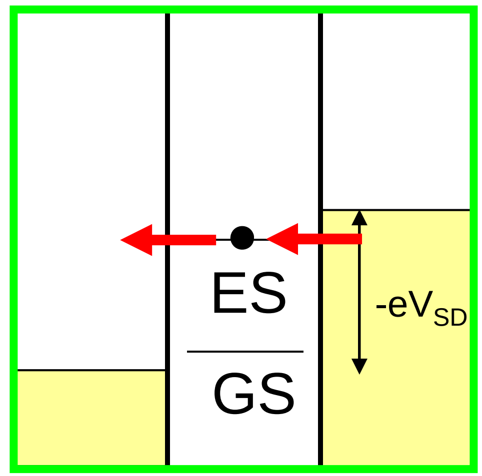
Higher Transitions

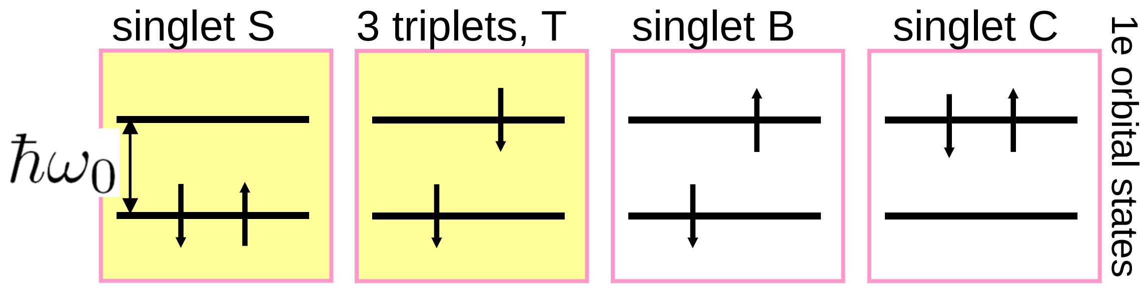




Two Electron States







GS	$\hbar\omega_0$	$\hbar\omega_0$	$2\hbar\omega_0$

noninteracting

Coulomb interactions

$E_{\text{singlet B, C}} >> E_{S, T}$

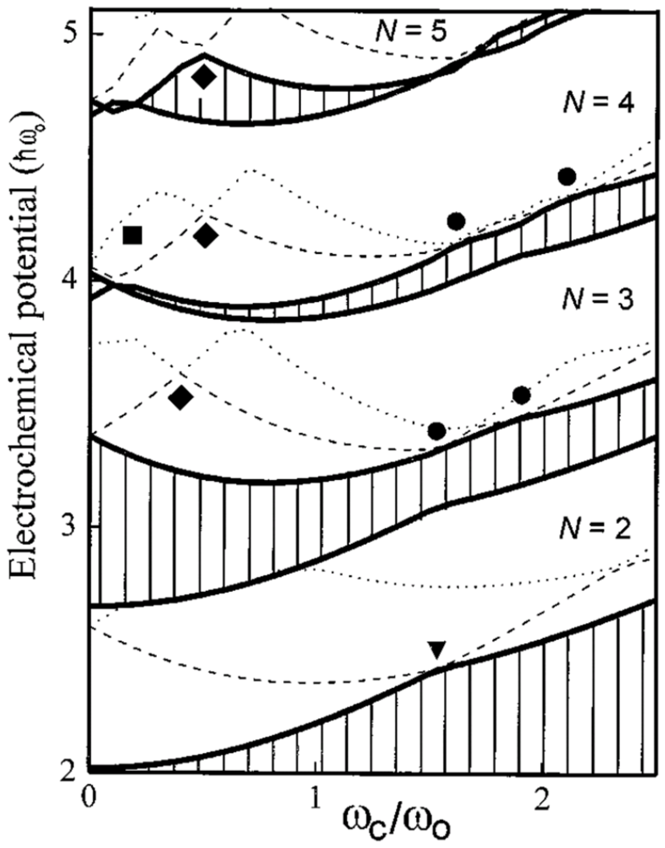
$J = E_T - E_S \sim 0.15\text{meV}$

note:

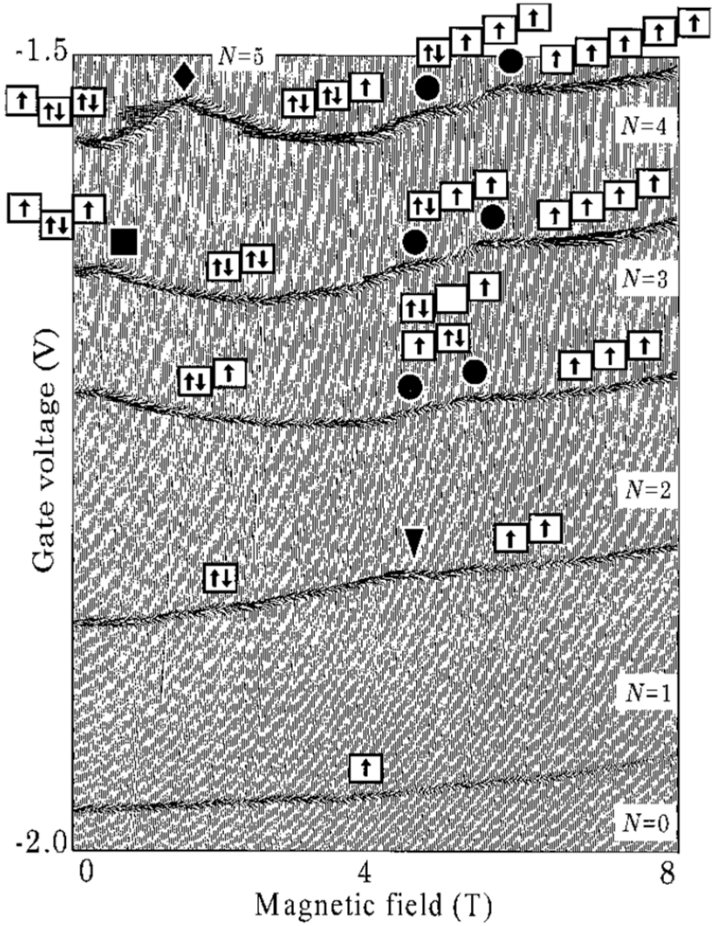
$J << \hbar\omega_0 = 1\,\text{meV}$

$E_T > E_S$ for N=2, B=0

Magnetic Field Transitions



exact calculation



experiment
peak positions vs B

"atomic physics" like experiments not accessible in real atoms!!

Quantum Dots Part 2

1. Quantum Dot Basics

2. Few Electron Dots

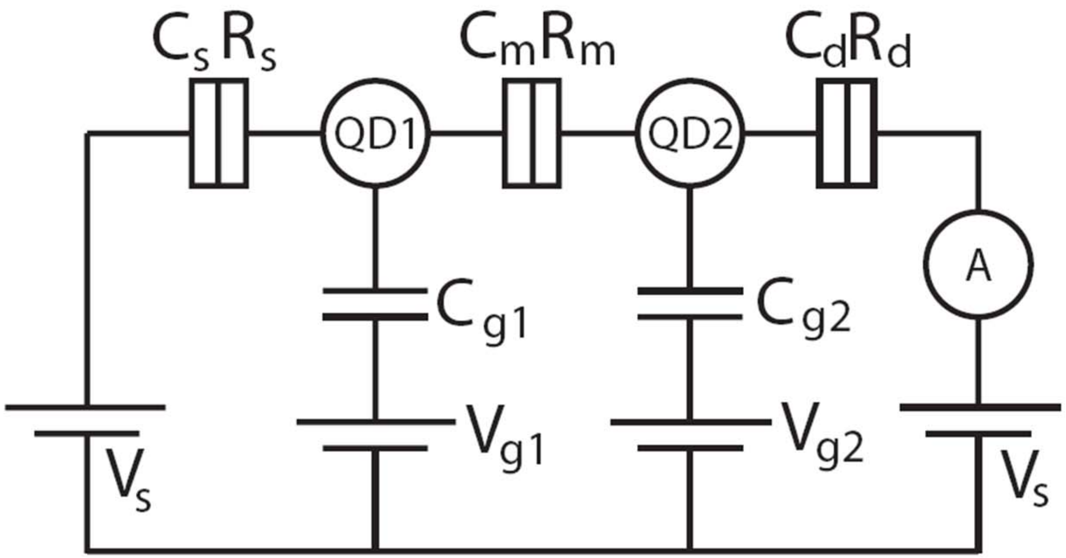
3. Double Quantum Dots and Spin Blockade

4. Kondo Effect

5. Charge Sensing and Spin Relaxation

van der Wiel et al., RMP75, 1 (2003)
A. C. Johnson, Ph. D. Thesis (2005)

Double Quantum Dots

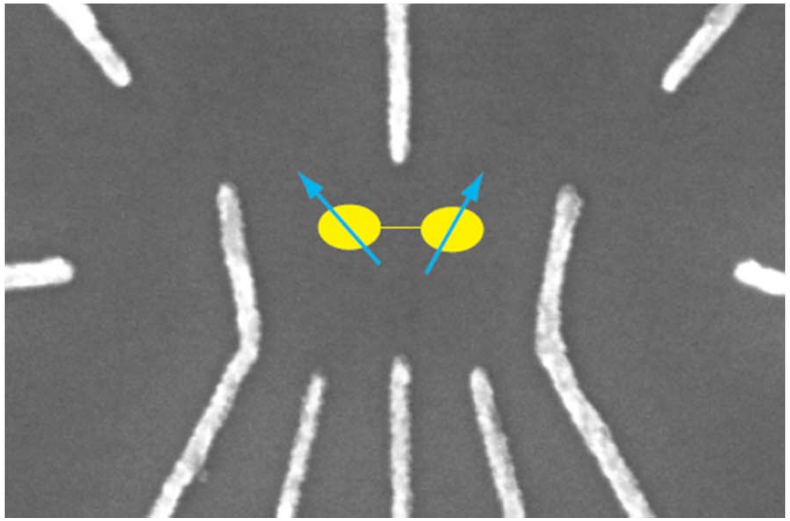


mutual charging energy

$$E_m = \frac{e^2}{C_m}\left(\frac{C_1 C_2}{C_m^2} - 1\right)^{-1}$$

interdot tunneling t

$$G_m = 4\pi \frac{e^2}{h}\left(\frac{t}{\Delta}\right)^2$$

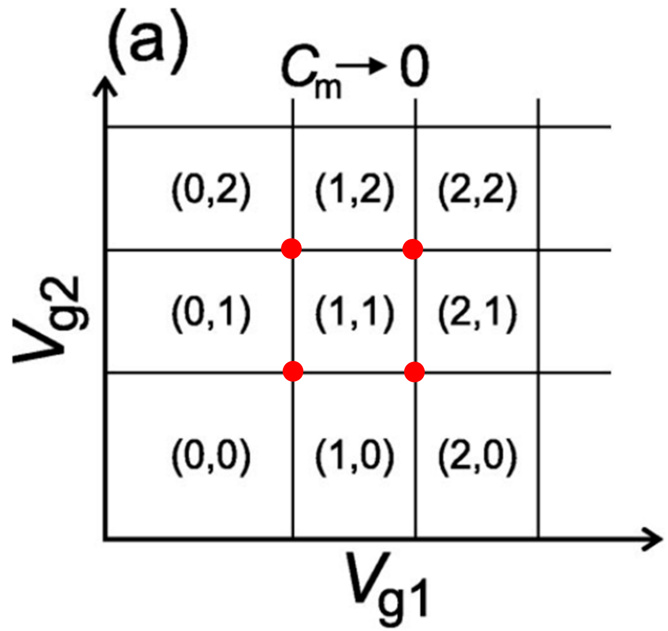


$t < \Delta$ well localized electrons

individual charging energies

$$E_{c1(2)} = \frac{e^2}{C_{1(2)}}\left(1 - \frac{C_m^2}{C_1 C_2}\right)^{-1}$$

Double Quantum Dots: Quadruple Points



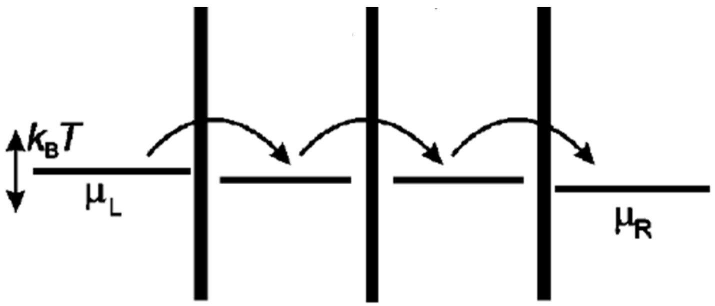
$$E_m = \frac{e^2}{C_m}\left(\frac{C_1 C_2}{C_m^2} - 1\right)^{-1} \longrightarrow 0$$

costs zero energy to add a 2nd electron
to other dot if one electron is already present

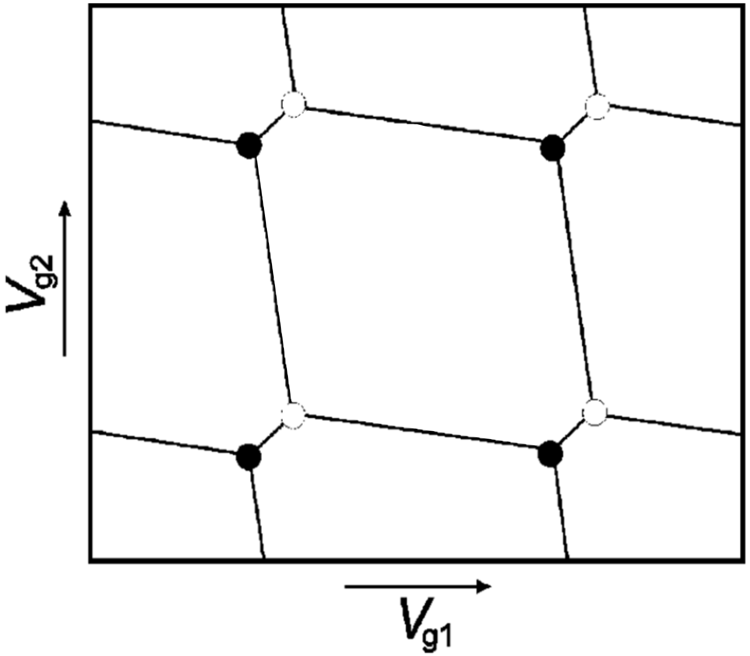
$$E_{C1(2)} = \frac{e^2}{C_{1(2)}}$$ individual charging energies

assume well localized electrons (weak tunneling,
but large enough to measure a current)

● quadruple points
degeneracy of four charge states



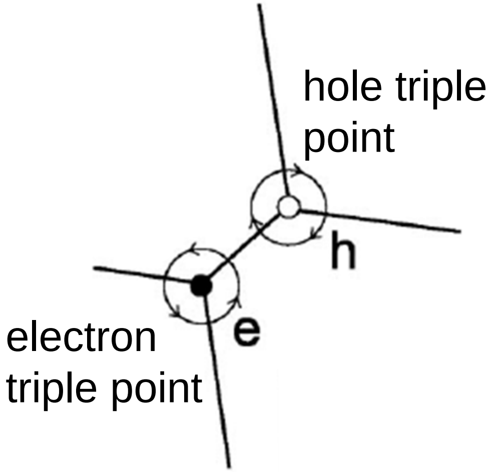
Double Quantum Dots: Triple Points and Honeycombs

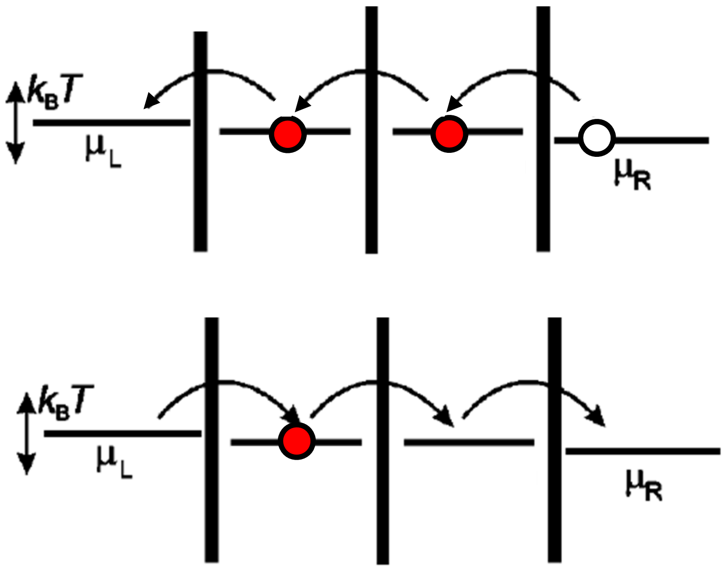


$$0 < C_m < C_{1,2} \quad 0 < E_m < E_{C_1,C_2}$$

(1,1) – (0,0) degeneracy lifted

quadruple points split into two triple points

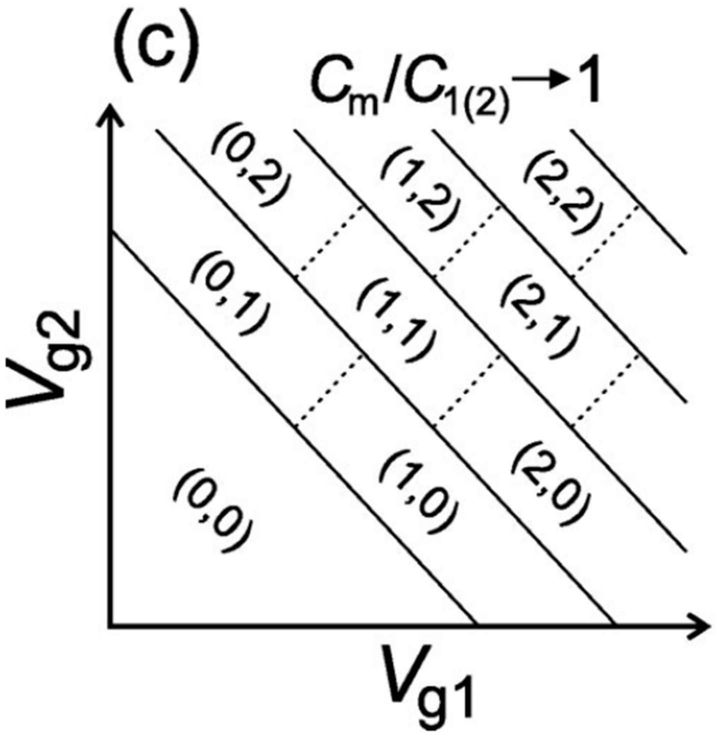




hole like process

electron like process

Double Quantum Dots: Single Dot Limit

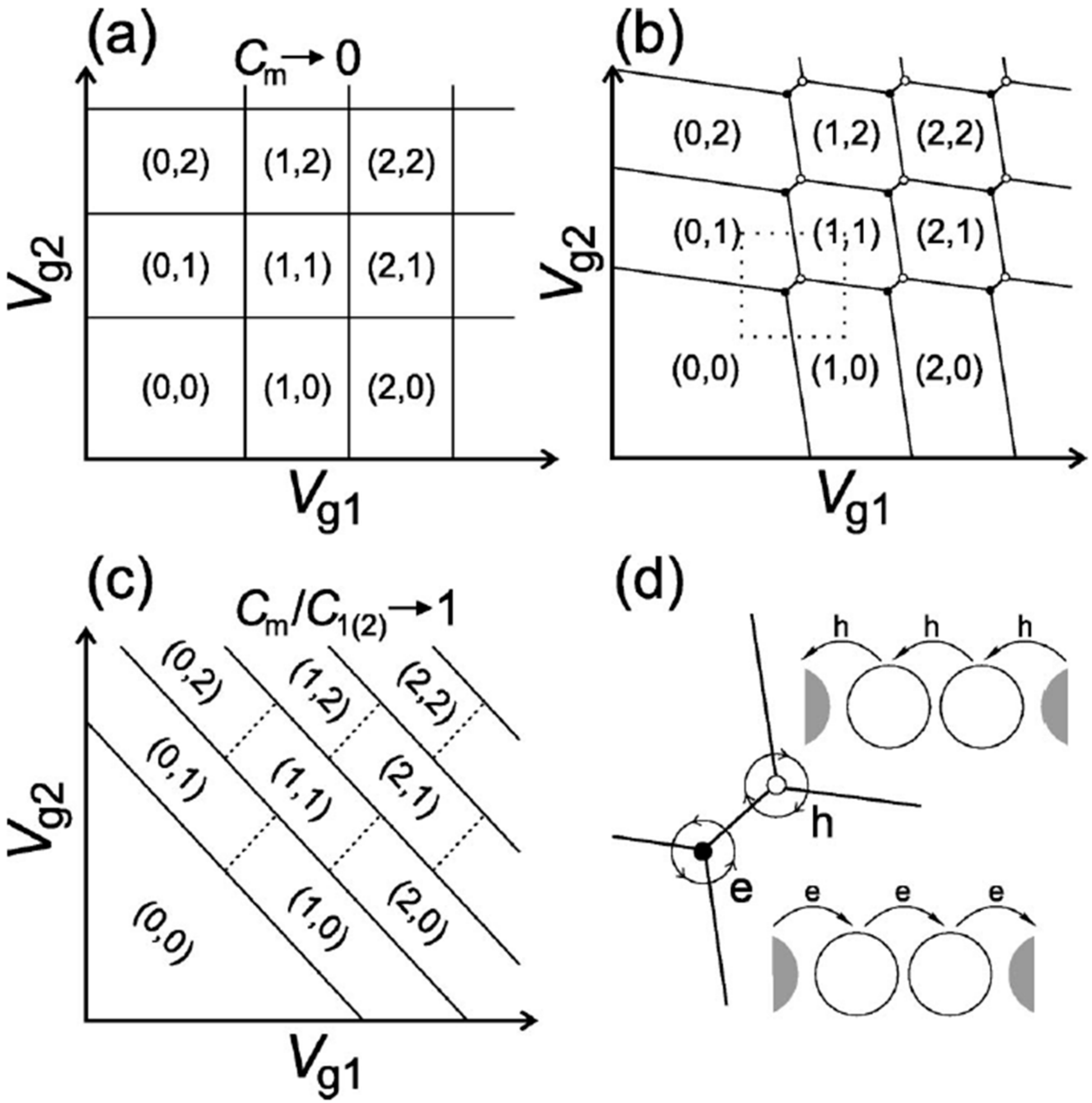


$$0 < C_m \sim C_{1,2}$$

$$E_m \sim E_{C_1,C_2}$$

double dot behaves like a
single dot with two plunger gates

Double Quantum Dots

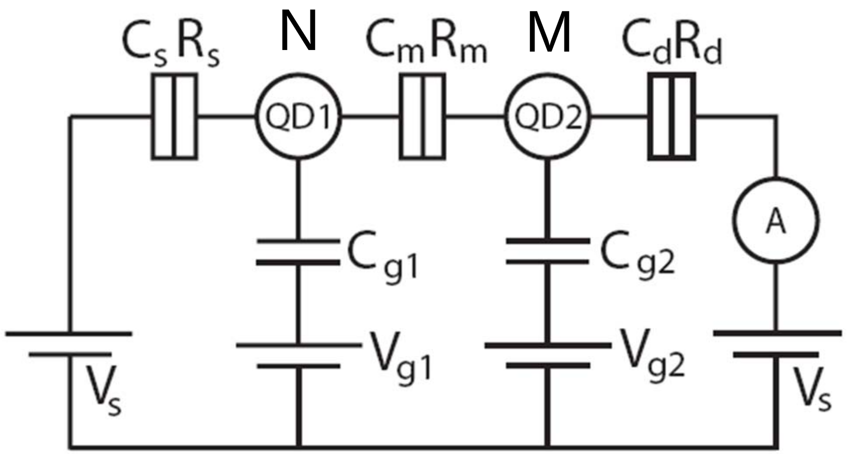


Double Dot Hamltonian

individual charging — electrostatic — quantum confinement

$$\begin{aligned} H_{DQD} &= \frac{E_{c1}}{2}N(N-1) - \frac{NE_{c1}+ME_m}{e}(C_{g1}V_{g1}+C_sV_s) + \sum_{i,\sigma} N_{i\sigma}\epsilon_{i\sigma} \\ &+ \frac{E_{c2}}{2}M(M-1) - \frac{ME_{c2}+NE_m}{e}(C_{g2}V_{g2}+C_dV_d) + \sum_{j,\sigma} M_{j\sigma}\epsilon_{j\sigma} \\ &+ E_m NM + \sum_{i,j,\sigma} t_{ij\sigma}(c^\dagger_{i\sigma}c_{j\sigma}+h.c.). \quad \text{+ lead tunneling} \qquad (3.11) \end{aligned}$$

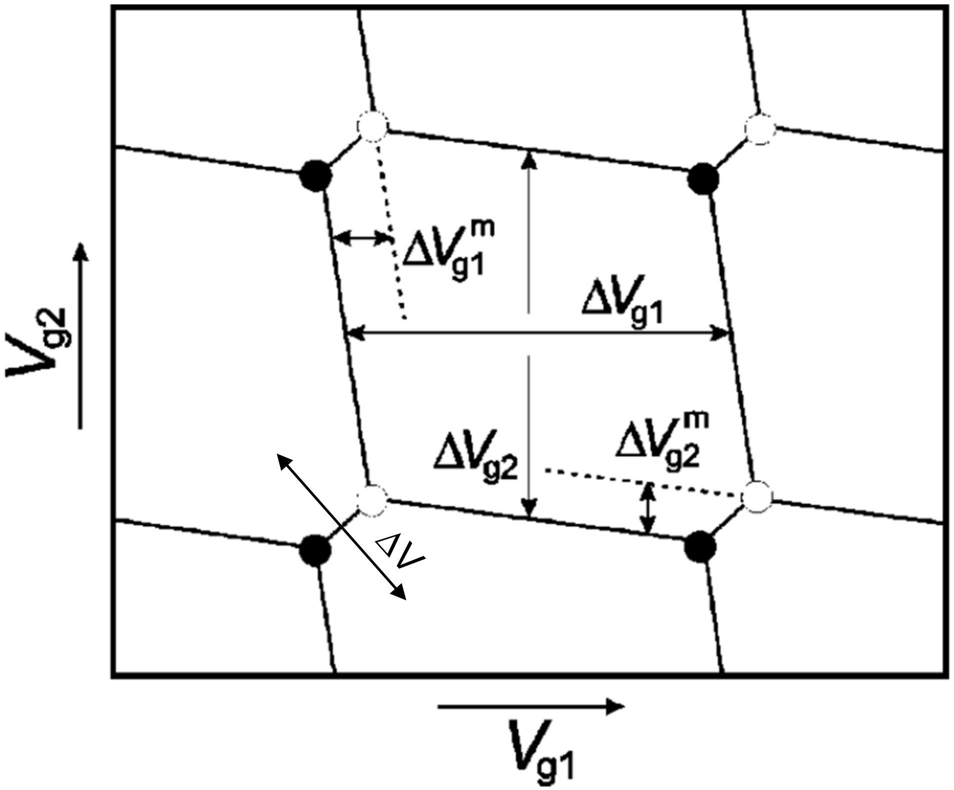
mutual charging — inter-dot tunneling



electrons well localized

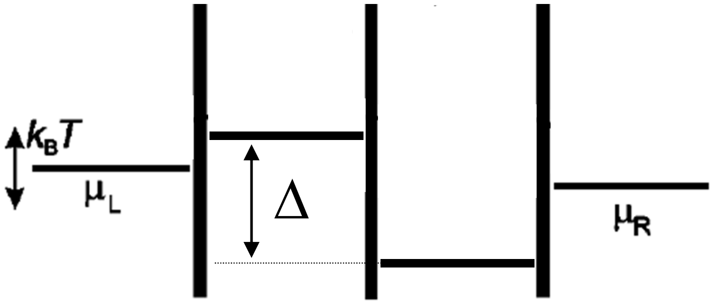
$G_m < e^2/h$

Double Dot Capacitances in the Honeycombs



$$\Delta V_{g1} = \frac{|e|}{C_{g1}}$$

$$\Delta V_{g2} = \frac{|e|}{C_{g2}}$$



$$\Delta V_{g1}^m = \frac{|e| C_m}{C_{g1} C_2} = \Delta V_{g1} \frac{C_m}{C_2}$$

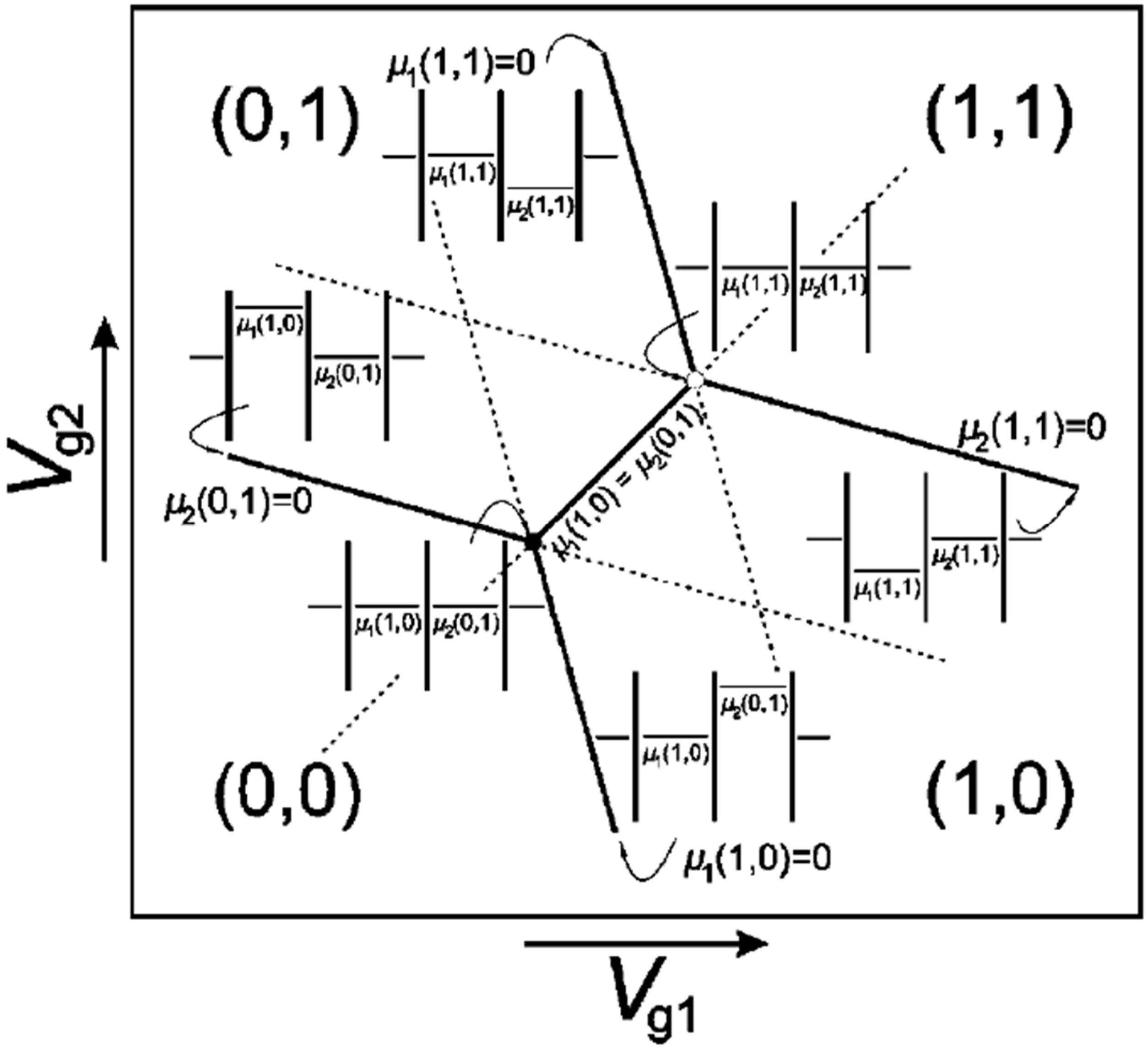
$$\Delta V_{g2}^m = \frac{|e| C_m}{C_{g2} C_1} = \Delta V_{g2} \frac{C_m}{C_1}$$

ΔV : detuning
controls energy difference Δ
between the dot levels
keeping constant the
total dot occupation N + M

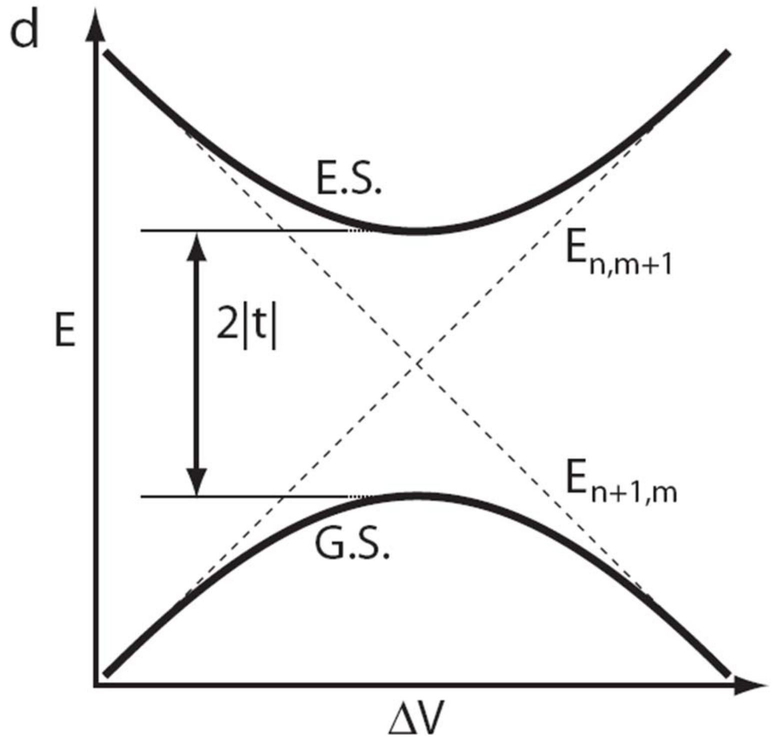
Double Dot Transport

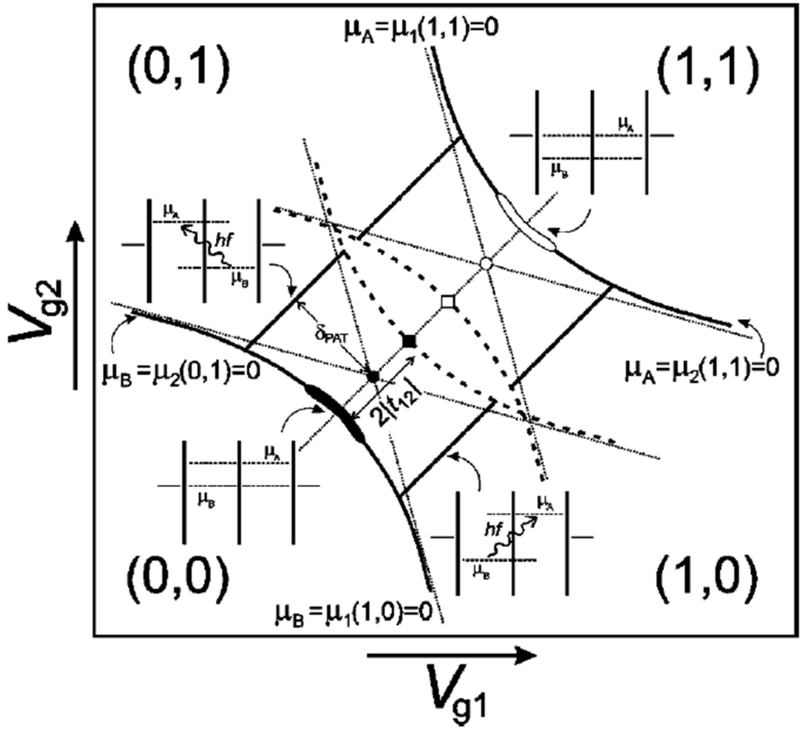
triple points:
sequential tunneling

honey comb lines:
cotunneling



Interdot Tunneling: Anticrossing





$$\mathbf{H}_0|\phi_1\rangle = E_1|\phi_1\rangle$$

$$\mathbf{H}_0|\phi_2\rangle = E_2|\phi_2\rangle$$

$$\mathbf{T} = \begin{pmatrix} 0 & t_{12} \\ t_{21} & 0 \end{pmatrix}, \quad t_{12} = t_{21}^*, \quad t_{21} = |t_{21}|e^{i\varphi}$$

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{T}$$

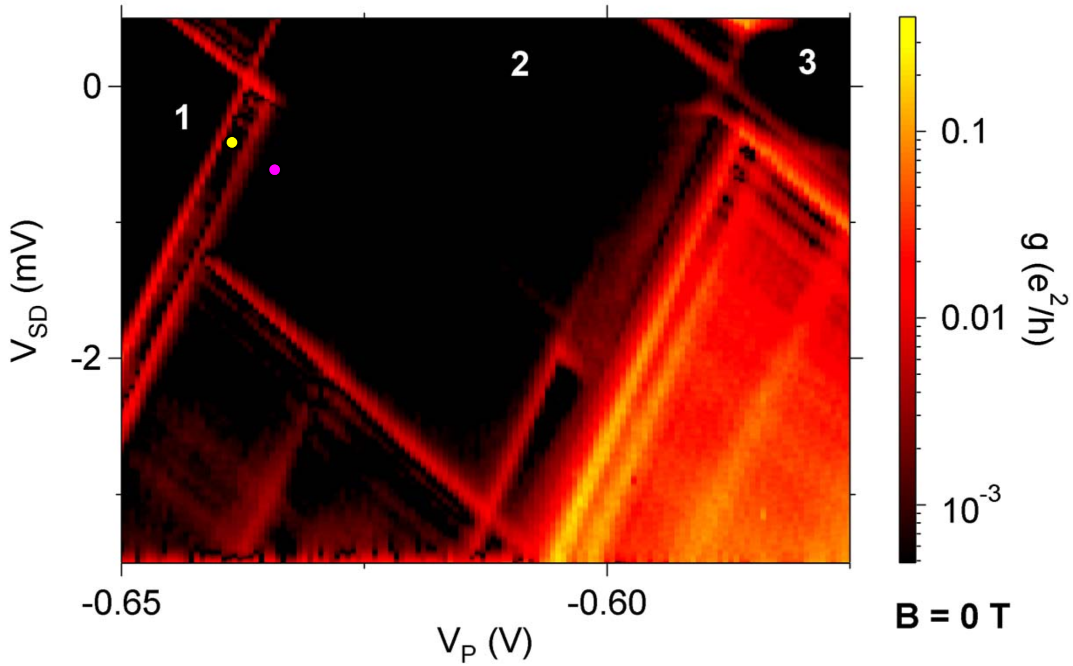
$$\mathbf{H}|\psi_B\rangle = E_B|\psi_B\rangle$$

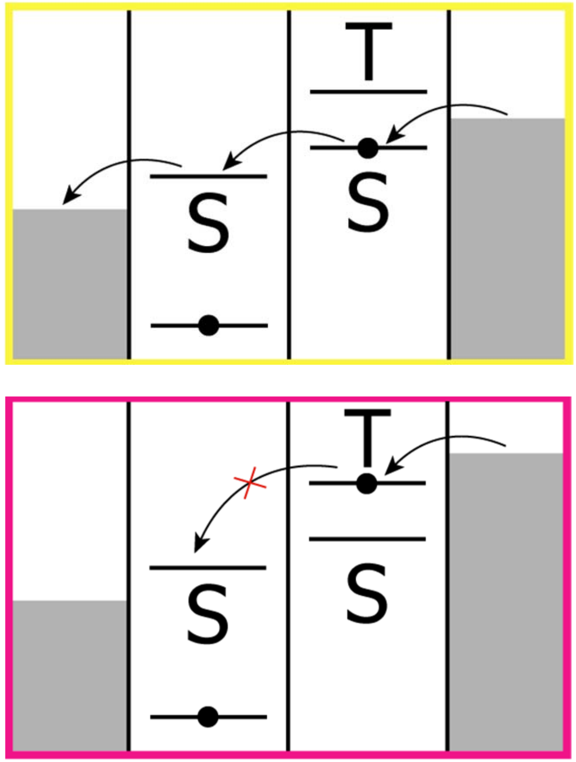
$$\mathbf{H}|\psi_A\rangle = E_A|\psi_A\rangle$$

$$E_B = E_M - \sqrt{\frac{1}{4}(\Delta E)^2 + |t_{12}|^2}$$

$$E_A = E_M + \sqrt{\frac{1}{4}(\Delta E)^2 + |t_{12}|^2}$$

Spin-Blockade





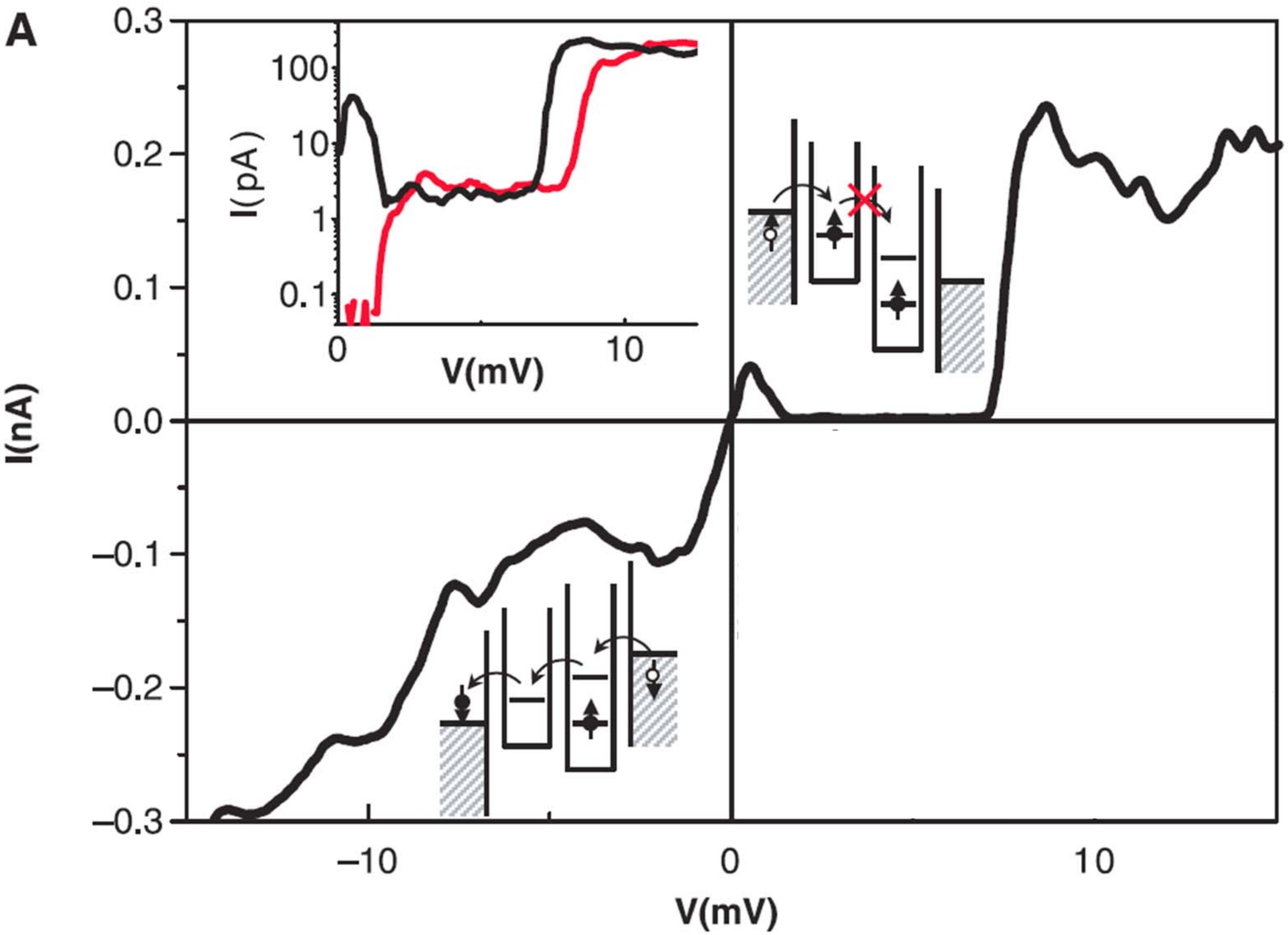
asymmetry in confinement potential
can trap one electron

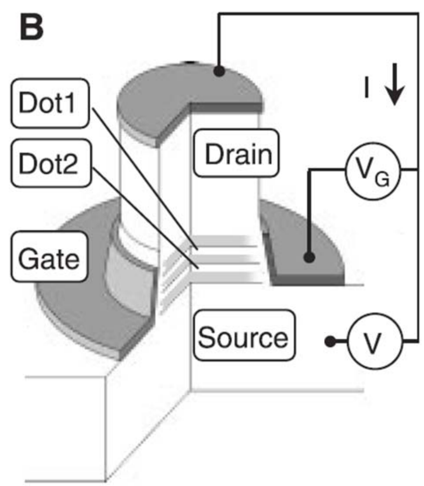
Ono et al., Science **297**, 1315 (2002)
Johnson et al., PRB**72**, 165308 (2005)

Current Rectification by Pauli Exclusion in a Weakly Coupled Double Quantum Dot System

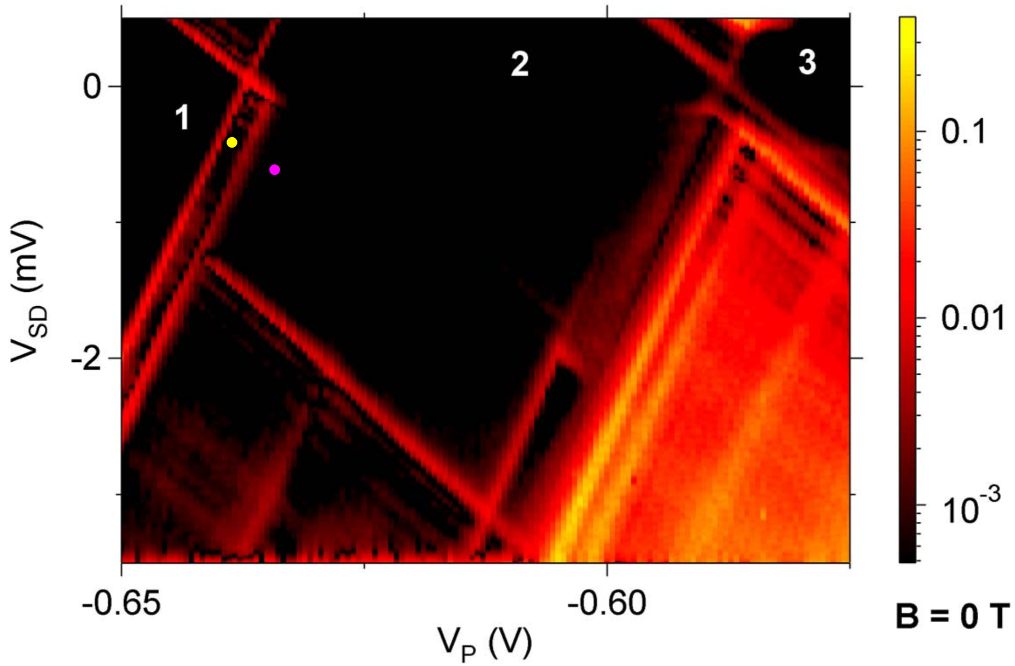
K. Ono,[1] D. G. Austing,[2,3] Y. Tokura,[2] S. Tarucha[1,2,4]*

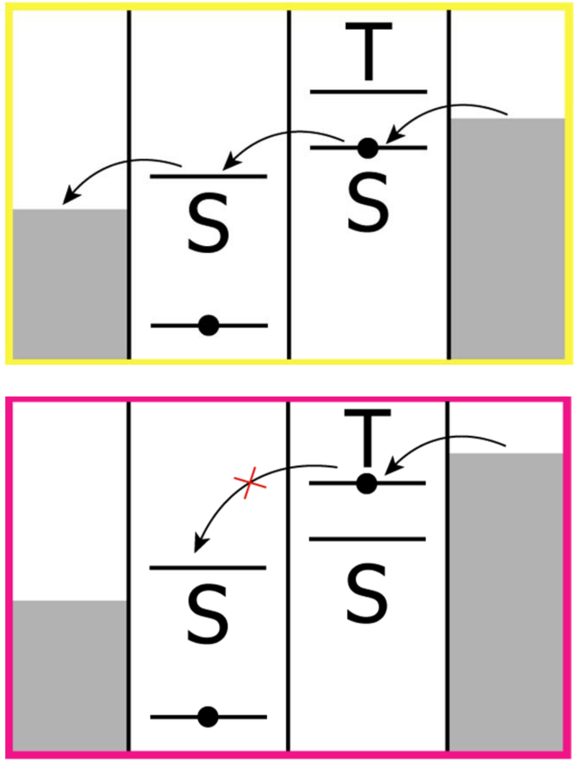
SCIENCE VOL 297 23 AUGUST 2002 1313





Spin-Blockade





asymmetry in confinement potential
can trap one electron

blockade lifted when
- only singlet available
- one electron excited state availabe
- three electron transport possible
- asymmetry lifted by gate voltage change
excellent stability of spin

Ono et al., Science **297**, 1315 (2002)
Johnson et al., PRB**72**, 165308 (2005)

Quantum Dots Part 2

1. Quantum Dot Basics

2. Few Electron Dots

3. Double Quantum Dots and Pauli Spin Blockade

4. Kondo Effect in Quantum Dots (skipped, no time)

5. Charge Sensing and Spin Relaxation

Goldhaber-Gordon et al., Nature **391**, 156 (1998)
Cronenwett et al., Science **281**, 540 (1998)
S. Cronenwett, Ph. D. Thesis (2001)

Quantum Dots Part 2

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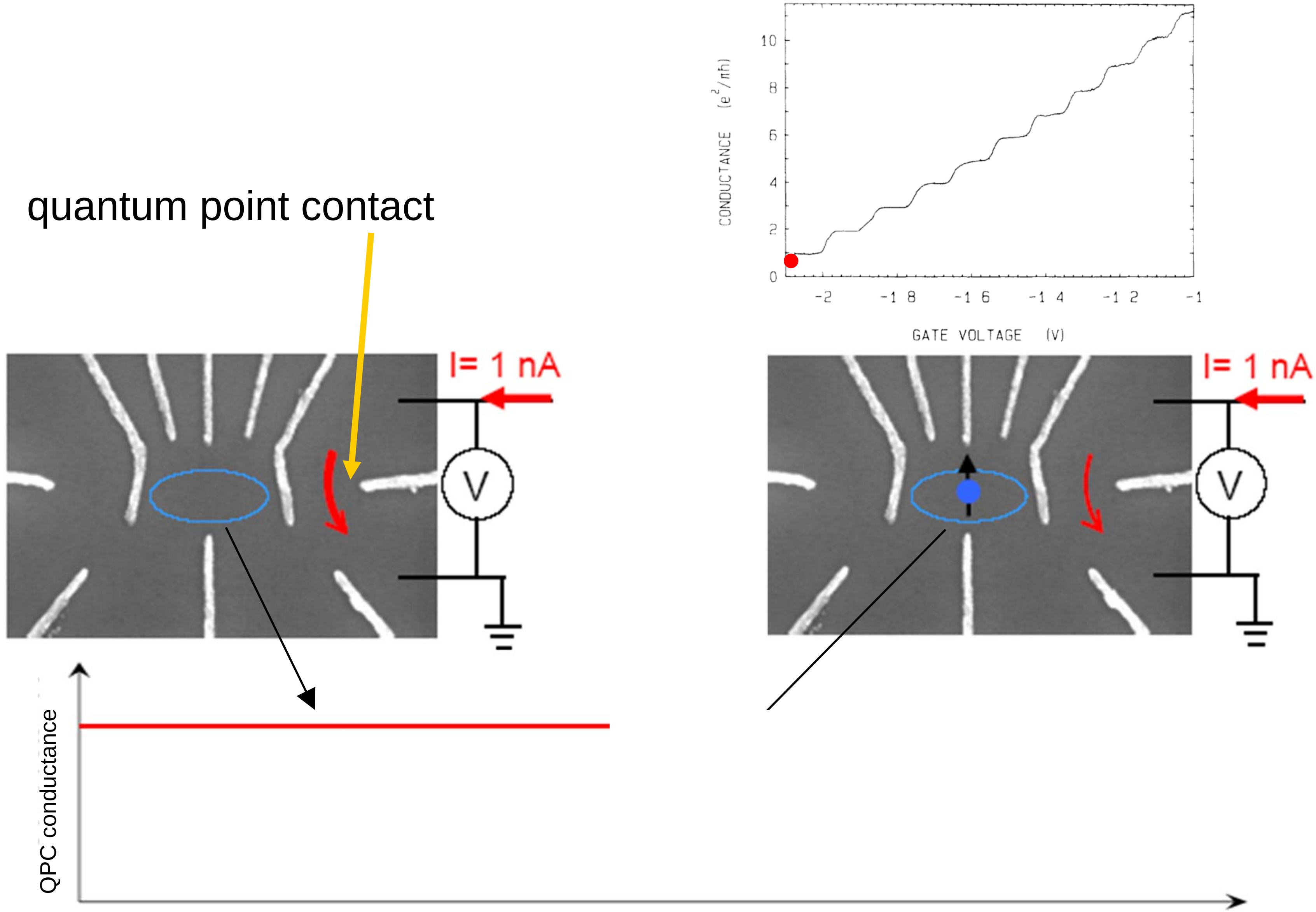
4. Kondo Effect in Quantum Dots (skipped, no time)

5. Charge Sensing and Spin Relaxation

Hanson et al., Rev. Mod. Phys. 2004
Amasha et al. PRL100, 046803 (2008).

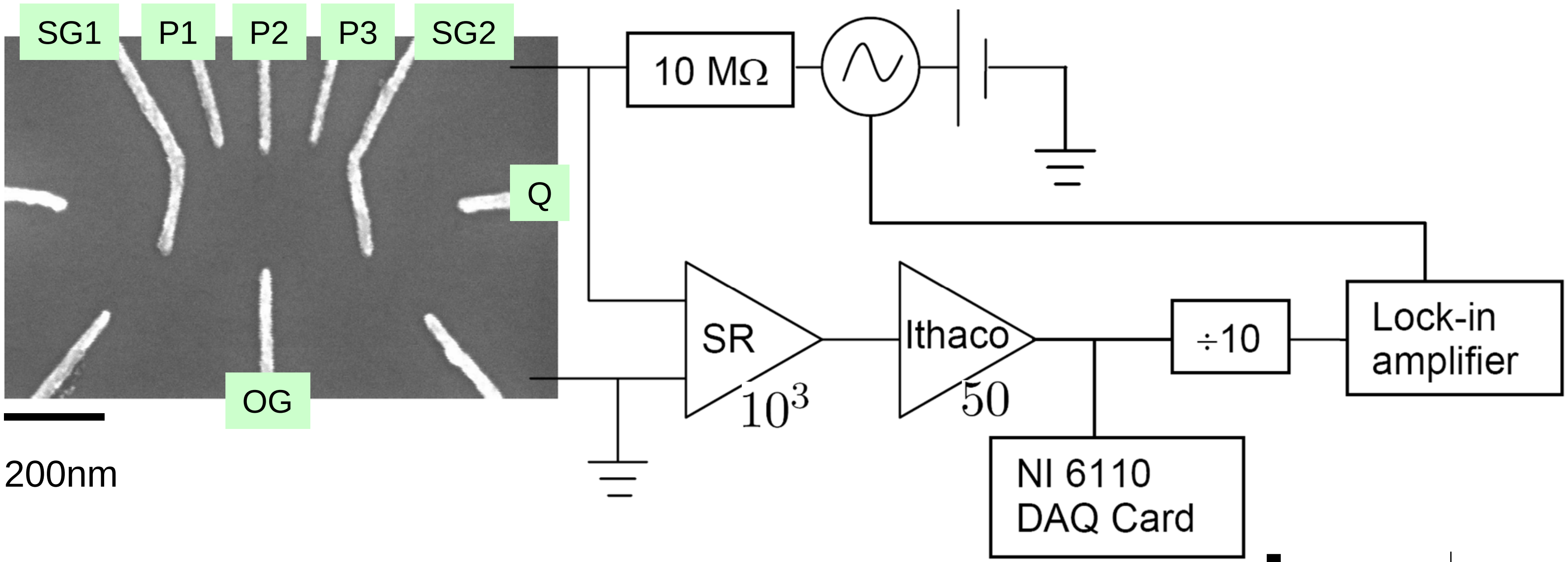
rev 150428, dmz

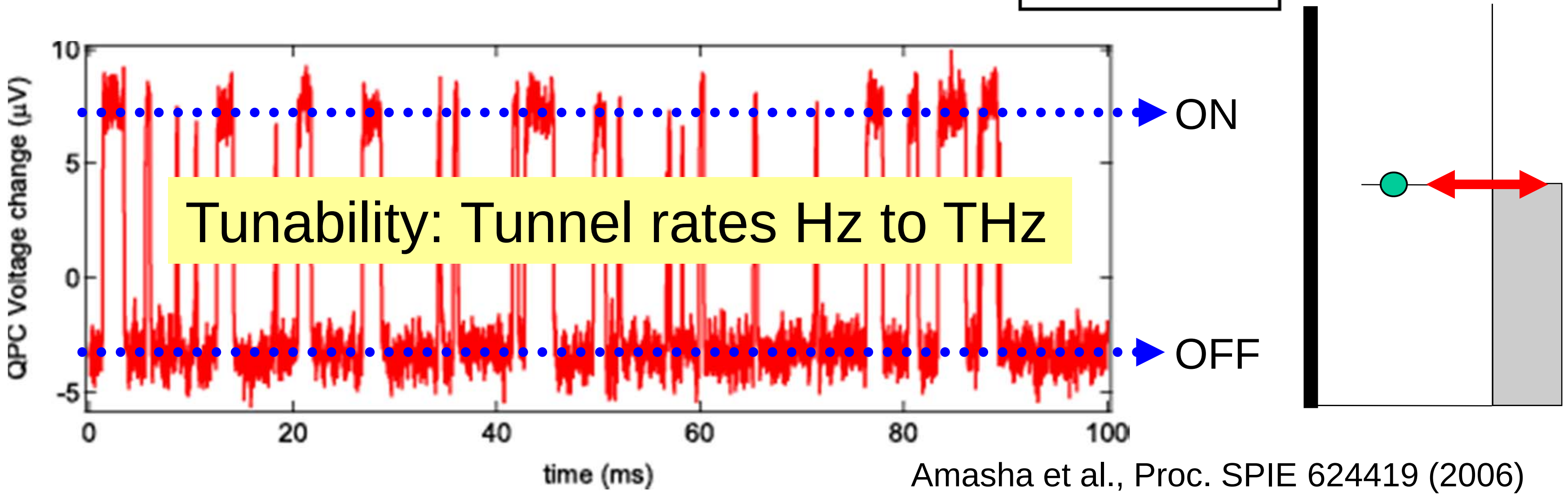
Sensing a Single Electron Charge *in situ*



M. Field et al., PRL 1993

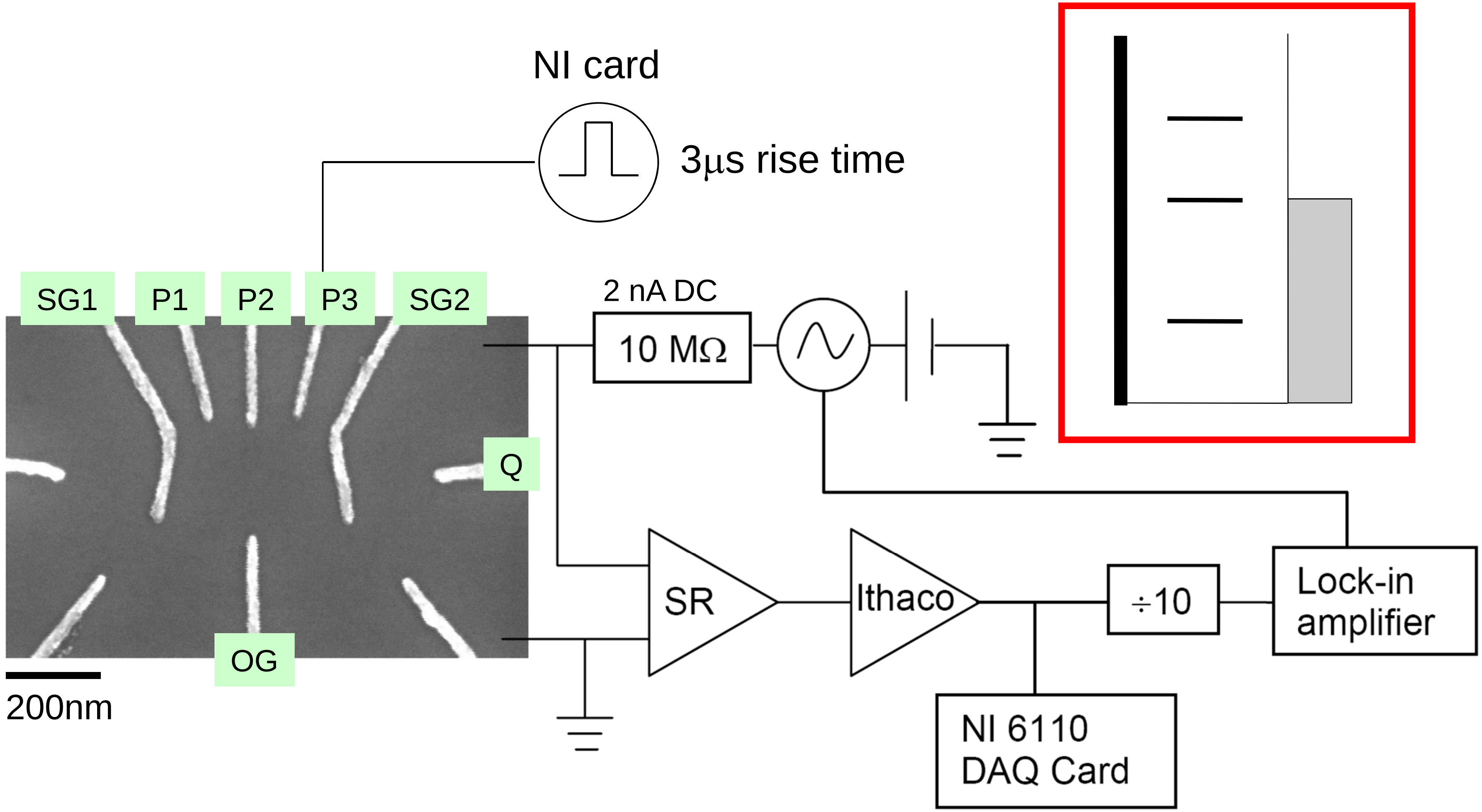
Real Time Charge Readout



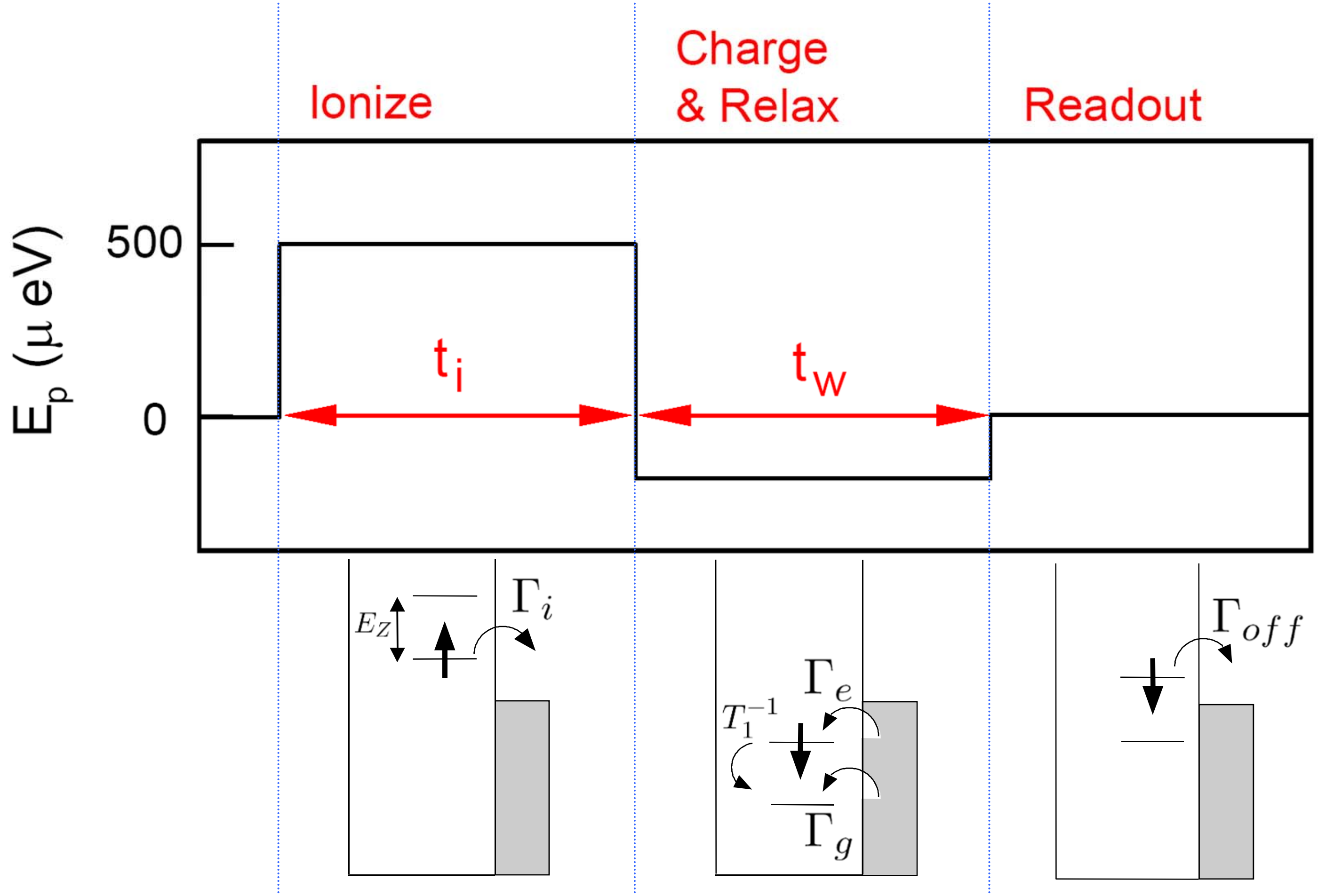


Amasha et al., Proc. SPIE 624419 (2006)

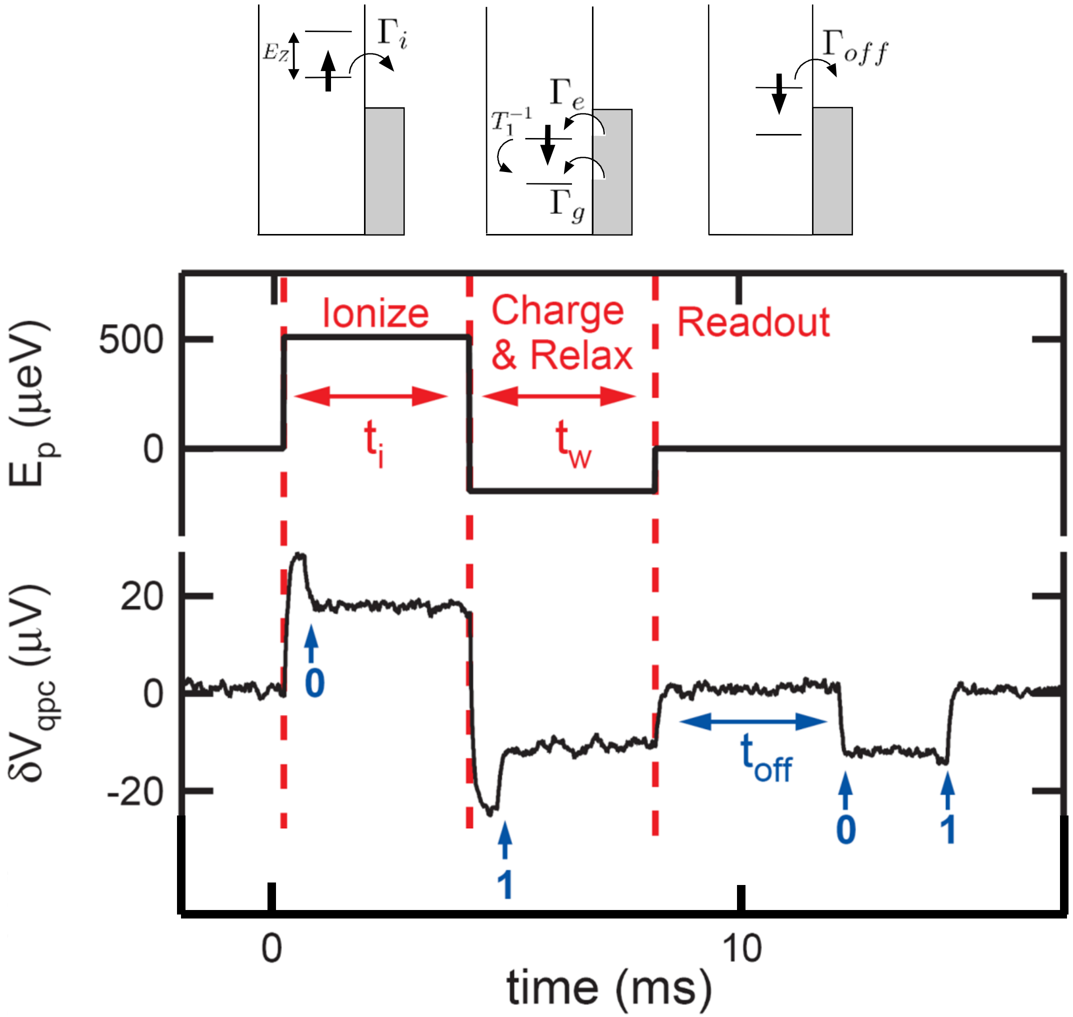
Pulsed Gate Technique



Single Electron Spin Relaxation time: measurement

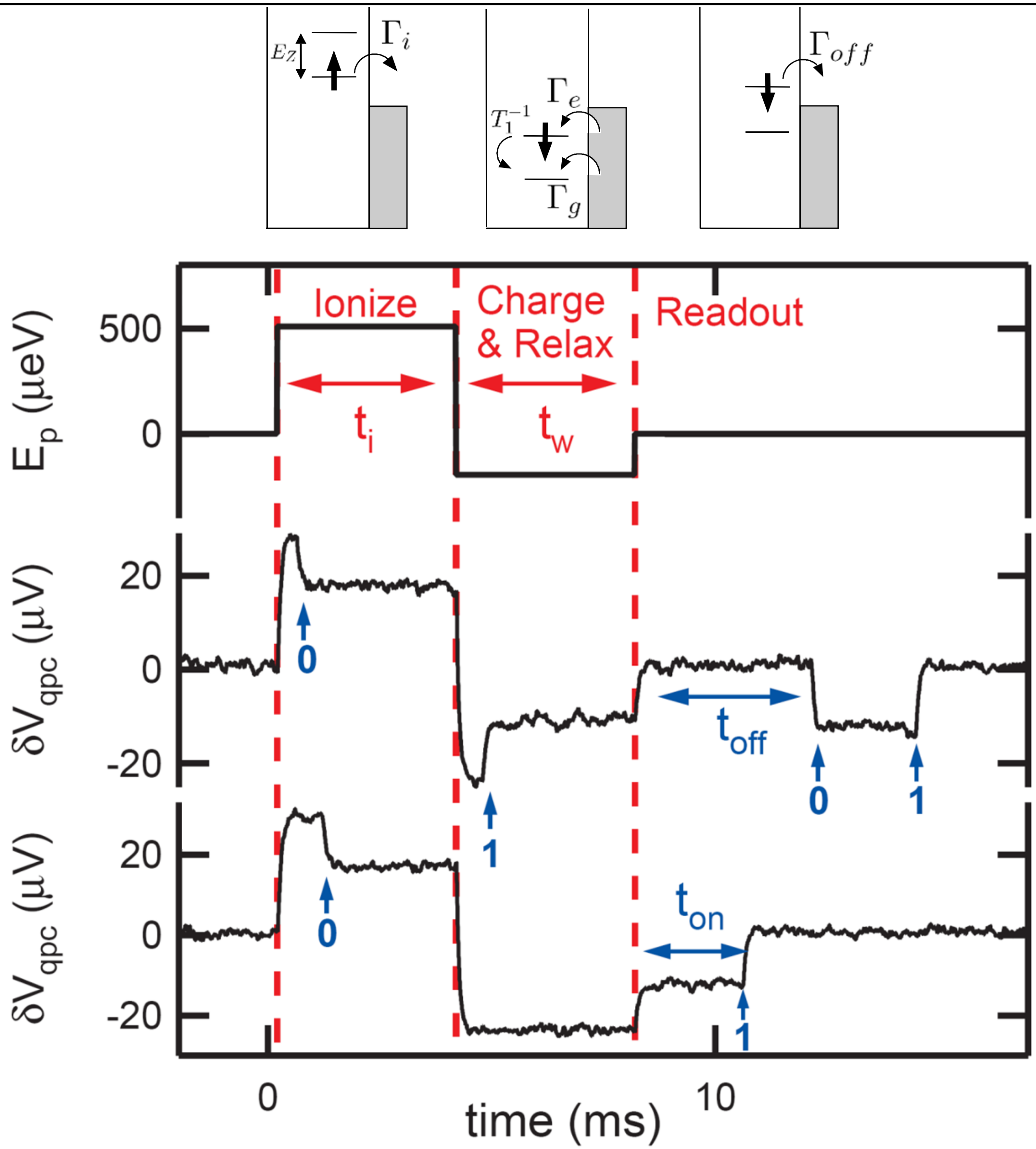


Pulse Sequence



tunnel off event
(electron tunneled onto dot)

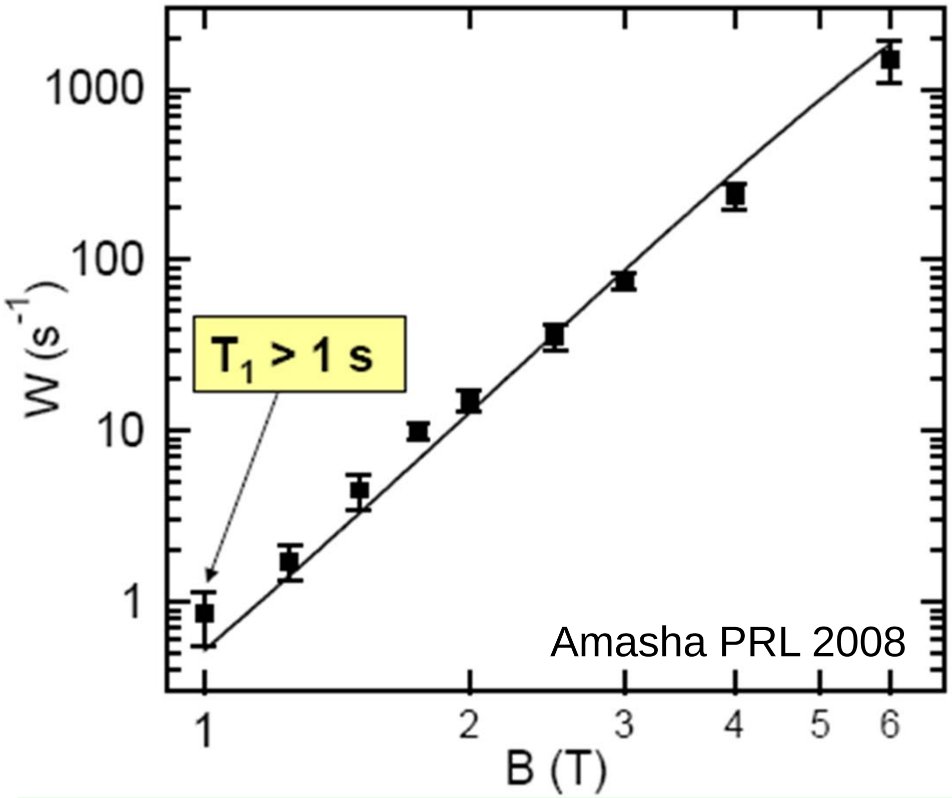
Tunnel-Off and Tunnel-On Events



tunnel off event
(electron tunneled onto dot)

tunnel on event
(ionization event,
no elec. tunneled onto dot)

Spin Relaxation Measurement



three orders of magnitude of T_1

measure down to fields where Zeeman splitting ~ T

T_1 ~ 1 s .... even useful as classical memory!

$$T_1^{-1} = A \frac{B^5}{\omega^4 \lambda_\pm^2}$$

theory: **Golovach *et al.*, PRL 2004**
mechanism: piezoelectric phonons + ***spin-orbit coupling***

parameters:
-|g|= 0.38 (inelastic cotunneling)
- orbital confinement ω_x, ω_y
- GaAs phonon material parameters (lit.)
- λ_{SO} = 1.7 μm (only fit parameter)

How do Electron Spins Couple to the Environment?

- **spin orbit interaction**

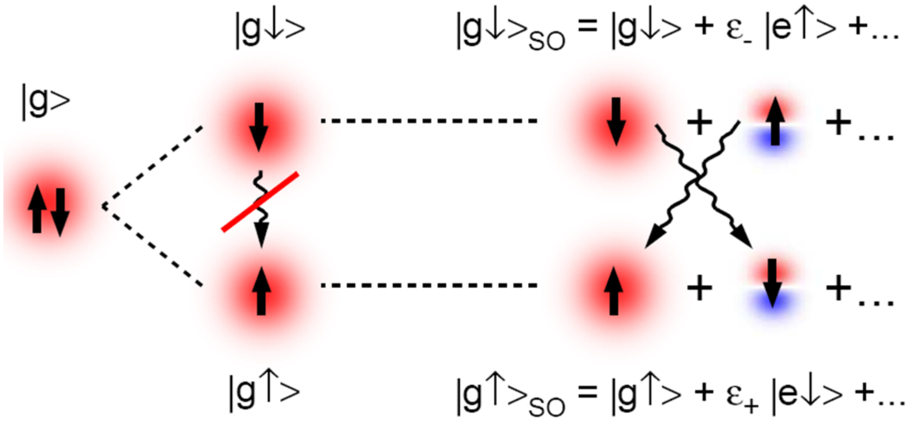
  mixes electron spin levels

  relaxation: phonon emission

  no decoherence due to SO

  $T_2 = 2\,T_1$ if only spin orbit coupling

  (Golovach, Khaetskii, Loss PRL)



- **hyperfine contact interaction**

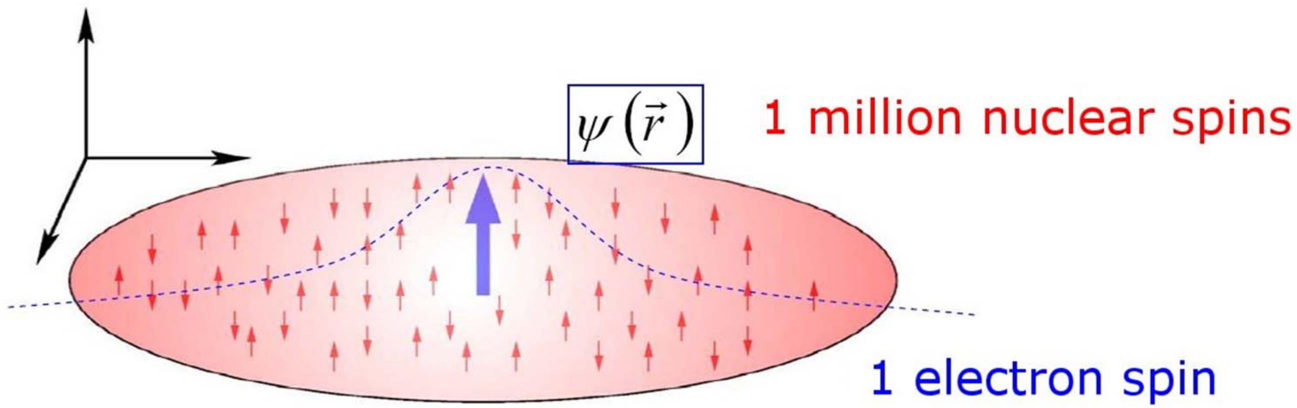
  couples to host nuclear spins

  dominant decoherence mechanism

  Overhauser field

  Knight shift

^{69}Ga (60%), ^{71}Ga (40%): spin 3/2
^{75}As: spin 3/2
^{27}Al: spin 5/2



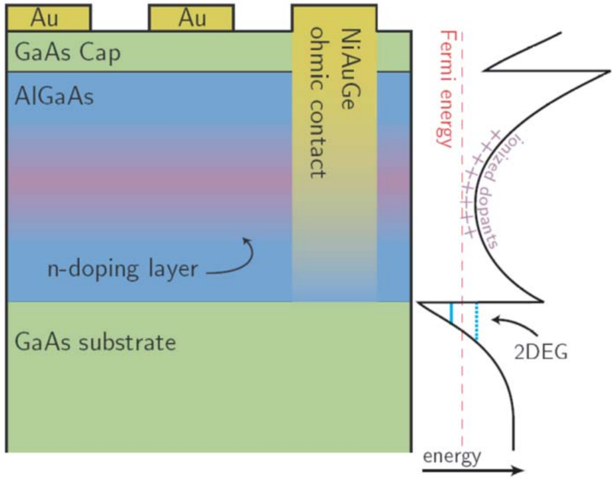
Spin Orbit Coupling

electric fields in material
electrons move → Lorentz transform. rest frame: magnetic field

Rashba term

triangular well at 2D interface

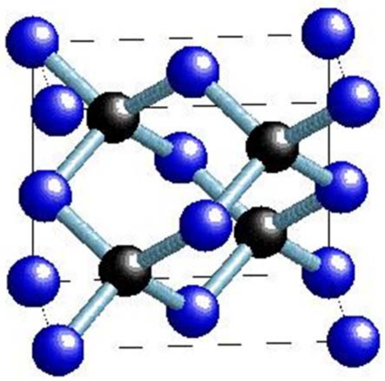
$$H_R = \alpha(p_x\sigma_y - p_y\sigma_x)$$



Dresselhaus term

GaAs: Zinc blende, no inversion symm.

$$H_D = \beta(p_y\sigma_y - p_x\sigma_x)$$


Conventional cell

also: cubic p term (neglected)

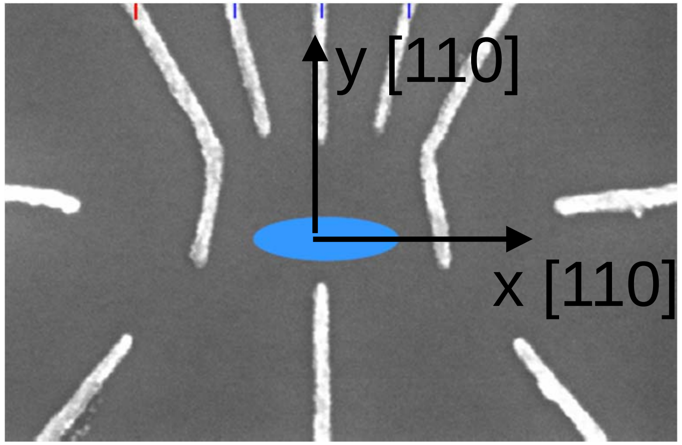
Spin Relaxation mediated by Spin Orbit Coupling

$$T_1^{-1} = A \frac{B^5}{\lambda_\pm^2 \omega^4}$$

magnetic field (Zeeman splitting, phonon density) → B^5 dependence as observed

confinement strength → ? ω_x? ω_y?

spin-orbit coupling strength → consistent with independent SO measurements



Spin-Orbit Hamiltonian

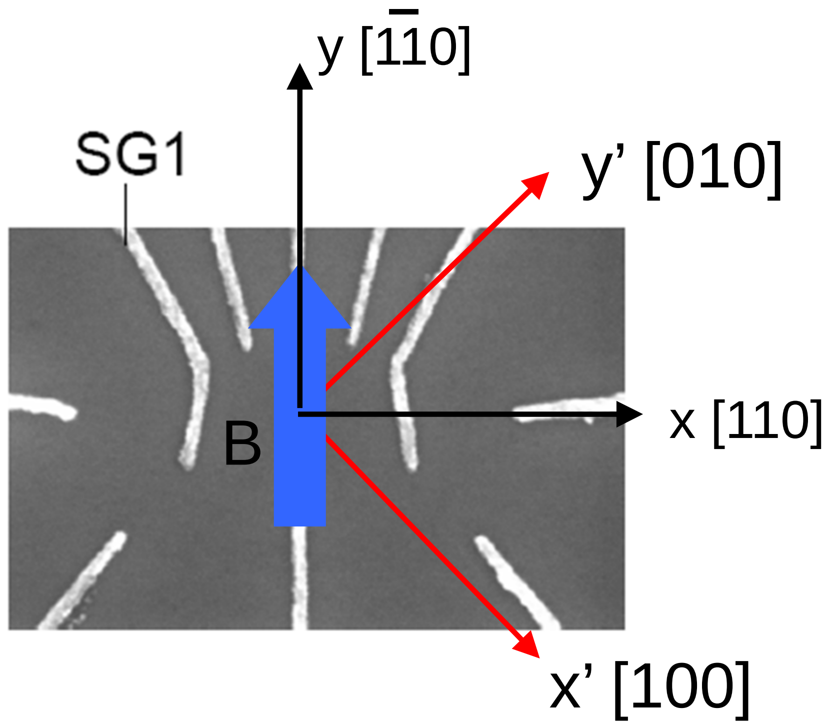
$$H_{so} = \alpha(p_x\sigma_y - p_y\sigma_x) + \beta(p_y\sigma_y - p_x\sigma_x)$$

45º rotation

$$H_{SO} = (\beta-\alpha)\, p_y\, \sigma_x + (\beta+\alpha)\, p_x\, \sigma_y$$

only σ_x mixes spin states along y || B

only confinement along y relevant!! as consistent with data



Quantum Dots Part 2

1. Quantum Dot Basics

2. Few Electron Dots

3. Double Quantum Dots and Pauli Spin Blockade

4. Kondo Effect in Quantum Dots (skipped, no time)

5. Charge Sensing and Spin Relaxation