Quantum Transport

5.5.2015

Superconducting Tunnel Junctions

1

Reminder: superconducting order parameter

$$\Delta(\vec{r}) = |\Delta(\vec{r})|\, e^{i\varphi(r)}$$

Josephson effect:

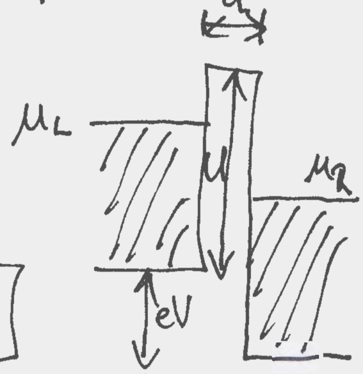
$$\boxed{I = I_c \cdot \sin(\Delta\varphi)}$$

$$\Delta\varphi = \varphi_2 - \varphi_1 \qquad \boxed{\frac{d}{dt}(\Delta\varphi) = \frac{2e}{\hbar}\cdot V}$$

This week: tunnel junctions between S, N, I, ...

Current across a tunnel junction (quasiparticles):

$$I = e\cdot \underset{\text{const.}}{C}\cdot \underset{\text{area}}{A}\cdot \tau \cdot \int_{-\infty}^{+\infty} dE \cdot N_L(E-\mu_L)\, N_R(E-\mu_R) \times \left[f_L(E) - f_R(E)\right]$$



$\tau \simeq \exp\left(-2d\sqrt{2mU}/\hbar\right)$; $f_{L/R}$: Fermi-Dirac distribution

for normal metals (N): $N_{L/R}(E) \simeq \text{const} = N_{L/R}(E_F)$

$\Rightarrow$ NIN junction: $I = \frac{1}{R_T}\cdot V$ ($\to$ linear!)

$$R_T = \frac{1}{e^2\cdot C\cdot A\cdot \tau\, N_L(E_F)\, N_R(E_F)} = \text{const.}$$

NIS junction:

$$I = \frac{1}{e \cdot R_T N^S(E_F)} \int_{-\infty}^{\infty} dE \cdot N_S(E) \left[f(E, T_R) - f(E+eV, T_R) \right]$$

with $N_S(E) = N^S(E_F) \dfrac{|E|}{\sqrt{E^2 - |\Delta|^2}} \Theta(|E| - |\Delta|)$

take $T \to 0$, differential conductance $\dfrac{dI}{dV}$

$$\frac{dI}{dV} = \frac{1}{R_T} \int_{-\infty}^{\infty} \frac{N_S(E)}{N_S(E_F)} \cdot \left[- \frac{\partial f(E+eV)}{\partial(eV)} \right] \cdot dE$$

$$\underset{T \to 0}{=} \frac{N_S(e|V|)}{N_S(E_F) \cdot R_T}$$

$\to$ current flow for $V \geq \frac{\Delta}{e}$

SIS junction:

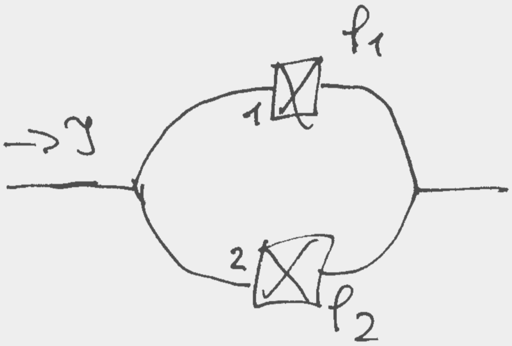
$$I = \frac{1}{e R_T N^L(E_F) N^R(E_F)} \int_{-\infty}^{+\infty} dE \cdot N_S(E) N_S(E+eV) \left[f(E, T_L) - f(E+eV, T_R) \right]$$

for $T \ll T_c$: current flow for $V \geq \frac{2\Delta}{e}$

Effects than include Josephson current:

parallel arrangements of two SIS junctions:

SQUID:



definition:

gauge invariant phase between two points A and B:

$$\phi = \varphi_B - \varphi_A - \frac{2\pi}{\Phi_s} \int_A^B \vec{A} \cdot d\vec{l} \quad , \text{with } \Phi_s = \boxed{\frac{h}{2e}}$$

integrate around the SQUID loop:

$$\Rightarrow \phi_{tot} = -\varphi_1 + \frac{2\pi}{\Phi_s} \int_1^2 \vec{A} \cdot d\vec{l} + \varphi_2 + \frac{2\pi}{\Phi_s} \int_2^1 \vec{A} \cdot d\vec{l}$$

$$= \varphi_2 - \varphi_1 + \frac{2\pi}{\Phi_s} \oint \vec{A} \cdot d\vec{l} = \varphi_2 - \varphi_1 + 2\pi \, \Phi / \Phi_s$$

SC phase should be single valued:

$$\phi_{tot} = 2\pi \cdot n \quad (n \in \mathbb{N}) \qquad \Rightarrow \varphi_1 - \varphi_2 = 2\pi \cdot \Phi / \Phi_s$$

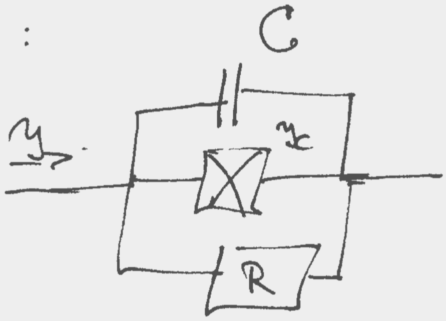
$$\Rightarrow I = I_{c1} \sin(\varphi_1) + I_{c2} \sin(\varphi_2 - 2\pi \, \Phi / \Phi_s)$$

$$= 2 I_c \cdot \left| \cos\left(\pi \, \Phi / \Phi_s\right) \right|$$

$\uparrow$

symmetric case: $I_{c1} = I_{c2} = I_c$

I–V characteristics of a Josephson junction:



RCSJ model

↙

resistively and capacitively shunted junction

$$I = I_c \cdot \sin\varphi + I_C + I_R$$

$$I_C = \dot{q} = C \cdot \dot{V} \underset{\uparrow}{=} C \cdot \frac{\hbar}{2e} \ddot{\varphi}$$

2nd Josephson equation

$$I_R = \frac{V}{R} = \frac{\hbar}{2e} \cdot \frac{1}{R} \cdot \dot{\varphi}$$

$$\Rightarrow \frac{I}{I_c} = \sin\varphi + \frac{\hbar}{2e} \frac{1}{R I_c} \dot{\varphi} + \frac{\hbar}{2e} \frac{C}{I_c} \ddot{\varphi}$$

simplifications: $t \Rightarrow$ $\tau = \omega_p \cdot t \Rightarrow \frac{\partial}{\partial t} = \omega_p \cdot$

$$\omega_p = \sqrt{\frac{2e I_c}{\hbar C}}, \quad Q = \omega_p \cdot R \cdot C$$

ω_p: Plasma frequency
Q: quality factor

$\vdots$

$$\ddot{\varphi} + \frac{1}{Q}\dot{\varphi} + \sin\varphi = \frac{I}{I_c}$$

mechanical analogy: particle, mass $\left(\frac{\hbar}{2e}\right)^2 C$

moving in potential $U(\varphi) = -E_J \cdot \cos\varphi - \frac{\hbar}{2e} \cdot I \cdot \varphi$

with friction $\left(\frac{\hbar}{2e}\right)^2 \cdot R^{-1} \cdot \dot{\varphi}$; $\left(E_J = \frac{\hbar}{2e} I_c\right)$

$U(\varphi)$: tilted washboard potential

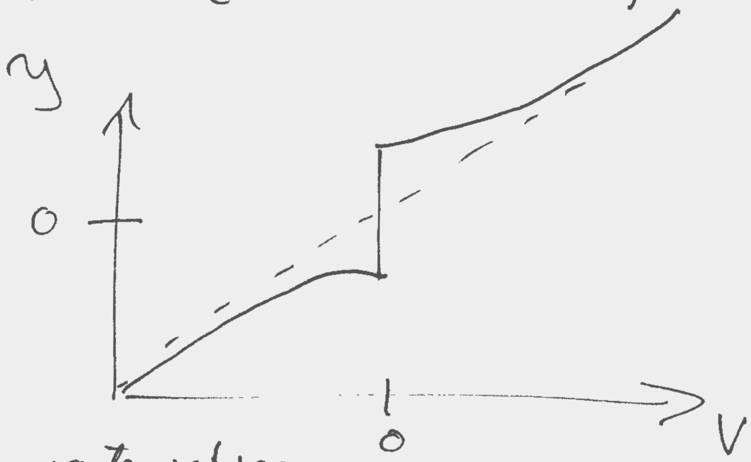
2 regimes:

$Q < \frac{1}{2}$: overdamped

$Q > \frac{1}{2}$: underdamped.

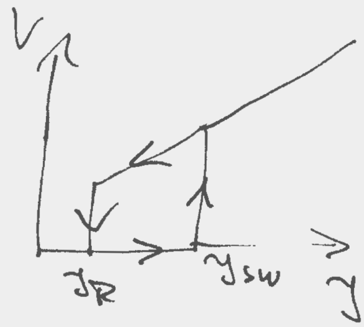
Overdamped regime: $(Q \ll 1)$:

$$\langle V \rangle = R \cdot \sqrt{\gamma^2 - \gamma_c^2} \cdot \theta(|\gamma| - \gamma_c) \cdot \mathrm{sign}(\gamma)$$



similar to V-γ characteristics of Coulomb blockaded junction with γ/V interchanged ("duality")

Underdamped regime:



hysteretic I-V curves

retrapping current $\gamma_R \simeq \dfrac{4\gamma_c}{\pi \cdot Q}$

escape processes:

thermal fluctuations: $\Gamma(\gamma) = \dfrac{\omega_A(\gamma)}{2\pi} \exp\left(-\dfrac{\Delta U(\gamma)}{k_B T}\right)$

$\omega_A(\gamma)$: attempt frequency $\simeq \omega_p$

$$\Delta U = \frac{8\sqrt{2}}{3} E_J \left(1 - \frac{\gamma}{\gamma_c}\right)^{3/2}$$

escape probability: $P_e(\tau, \gamma) = \exp -(\Gamma(\gamma)\,\tau)$

quantum fluctuations: $\left(\hbar\omega_p/k_B > T\right)$ (6)

QM tunneling through barrier $\Delta U(\gamma)$

(temperature independent escape rate)

"macroscopic quantum tunneling" (MQT)

⋮

Quantum effects in small Josephson junctions

→ two energy scales E_J, E_C (charging energy)

classical variables Q, φ → operators

$$[\hat{\varphi}, \hat{Q}] = 2\cdot e \cdot i$$

$$\hat{H} = \frac{(\hat{Q}-Q_{gate})^2}{2C} - E_J \cos\hat{\varphi} - \frac{\hbar}{2e}\gamma_b \cdot \hat{\varphi}$$

<u>analogy to particle</u>:

$\hat{H} = \frac{\hat{p}^2}{2m} - U\cdot\cos\left(\frac{\hat{x}}{a}\right) - F\cdot x$	$\hat{H} = \frac{(\hat{Q}-Q_g)^2}{2C} - E_J\cos\hat{\varphi} - \frac{\hbar}{2e}\gamma_b\hat{\varphi}$
x	φ
$p = \frac{\hbar}{i}\frac{\partial}{\partial x}$	$\frac{\hbar}{2e}\hat{Q} = -2ei\frac{\partial}{\partial\varphi}$
$v = \frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}$	$\frac{2e}{\hbar}V = \frac{\partial\varphi}{\partial t} = \left(\frac{2e}{\hbar}\right)^2\frac{1}{C}\frac{\hbar}{2e}\hat{Q}$
m	$\left(\frac{\hbar}{2e}\right)^2 C$
F	$\frac{\hbar}{2e}\gamma_b$

→ time independent Schrödinger equation

$$4E_C\left(-i\frac{\partial}{\partial\varphi}-\frac{Q_g}{e}\right)^2\Psi_n-E_J\cos(\varphi)\cdot\Psi_n=E_n\cdot\Psi_n$$

"Mathieu differential equation"

parameters: $E_C=\frac{e^2}{2C}$

$E_J=\frac{\hbar}{2e}\cdot I_c$

→ two limiting cases:

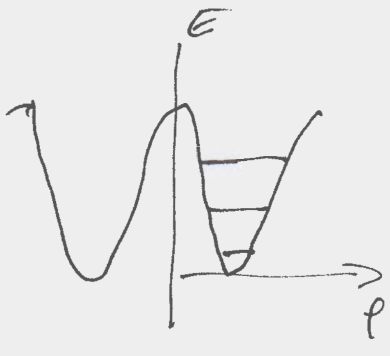
- $\underline{\underline{E_J \gg E_C}}$ "tight binding limit"

dominating term: $-E_J\cos\varphi \simeq -E_J\left(1-\frac{\varphi^2}{2}\right)$

$$\Rightarrow \quad H\Psi_n=-4E_C\,\partial_\varphi^2\Psi_n+\frac{1}{2}E_J\varphi^2\Psi_n=E\Psi_n$$

$(Q_g=2e\cdot n)$ harmonic oscillator

$$\rightarrow \quad E_n=\left(N+\tfrac{1}{2}\right)\hbar\omega_p$$



- $\underline{\underline{E_J \ll E_C}}$ "free electron limit"

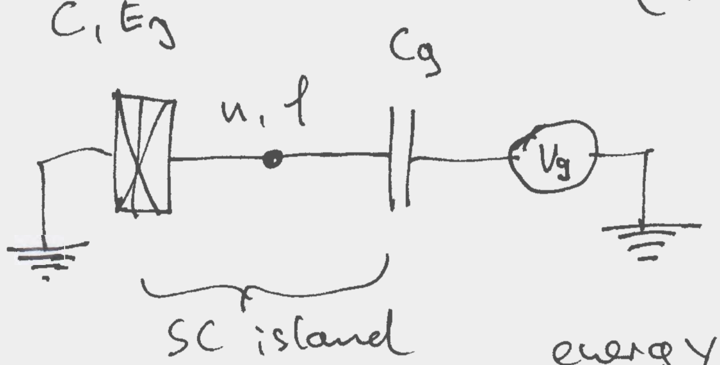
start from $E_J = 0$:

-> plane wave solutions

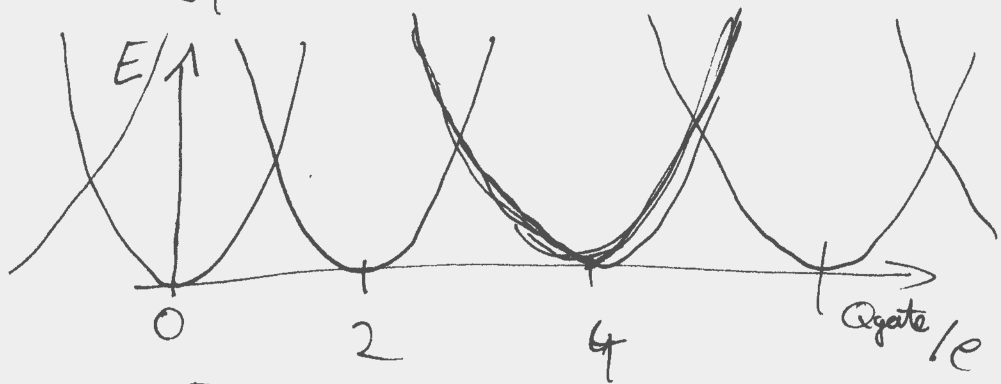
$$\Psi_Q(\varphi) = A_Q\, e^{i \frac{Q}{2e} \cdot \varphi} \; ; \; Q = 2e \cdot n + Q_{gate}$$

control of Q_{gate}: S-SET / Cooper pair box

(SET / single electron box for N)



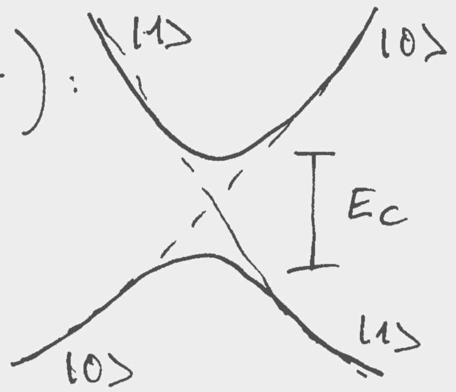
energy Eigenvalues: $E_n = \frac{Q^2}{2C}$



-> switch on small $E_J \ll E_C$:

-> anticrossing between states $|0\rangle$, $|1\rangle$

close to $Q_{gate} = 1 \cdot e$ $\left(n_{gate} = \frac{Q_{gate}}{2e} \right)$:



$$\hat{H} = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x$$

with $B_z = 4E_C(1 - 2n_g)$

$B_x = E_J$

Eigenfunctions: $|e1\rangle = \cos\frac{\eta}{2}|0\rangle + \sin\frac{\eta}{2}|1\rangle$; $\eta = \arctan\left(\frac{B_x}{B_z}\right)$

$|e2\rangle = -\sin\frac{\eta}{2}|0\rangle + \cos\frac{\eta}{2}|1\rangle$

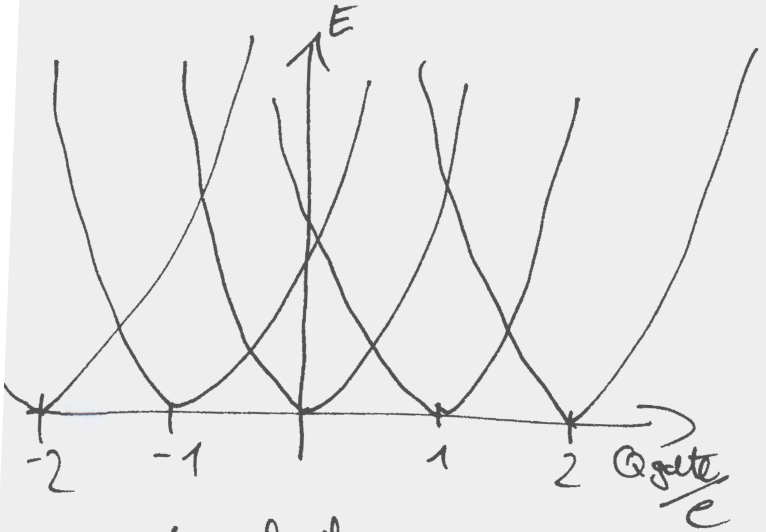
$$E_\pm = \pm \sqrt{B_x^2 + B_z^2} \Big/ 2 = \frac{E_y}{2 \sin \eta}$$

9

---

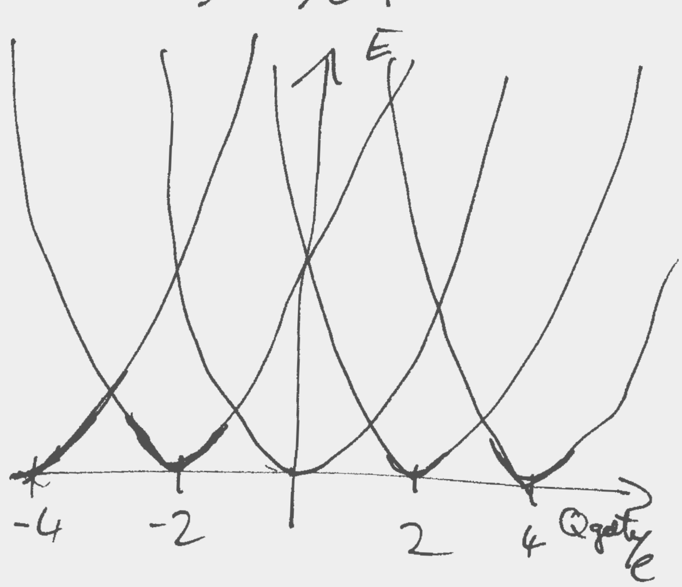
side remark: parity effects

N-SET:



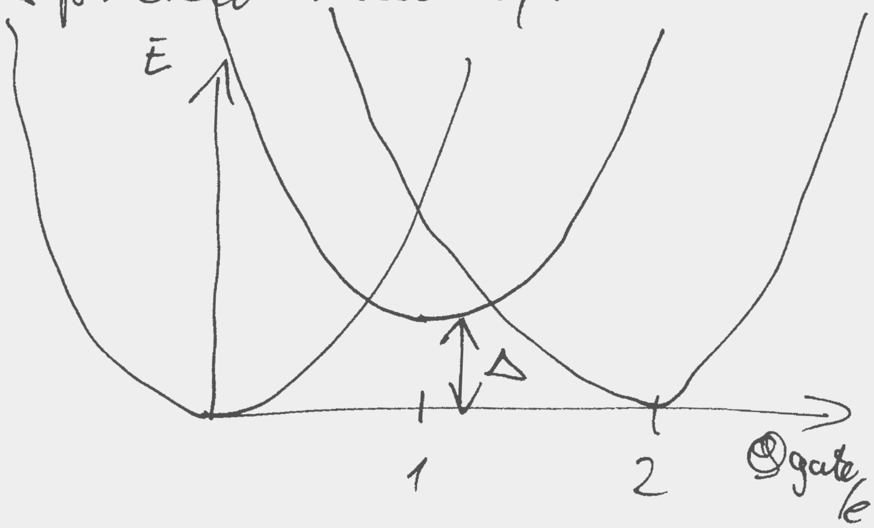
single electron tunneling

S-SET



Cooper pair tunneling

for quasiparticle tunneling in S-SET, additional energy Δ per electron necessary:



-> 2e-periodicity (even-odd effect) occurs only for $E_C > \Delta$

-> for quantum effects:

$E_C < \Delta$

(-> otherwise: "quasiparticle poisoning")

Superconducting Q-bits

requirements (di Vincenzo criteria):

- quantum two-level system
- initialization
- long coherence time
- one qbit operations
  + two qbit operations (controlled coupling of two qbits)
- separate measurement of individual qbits

- charge Qbit : S-SET / Cooper pair box

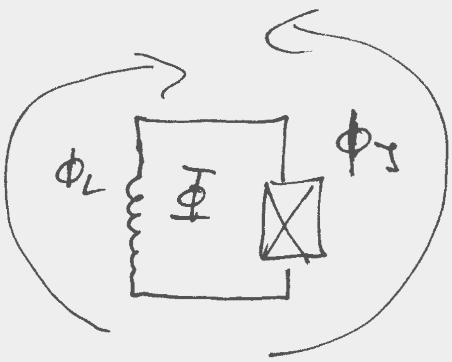
$E_C \gg E_J$, at degeneracy point

$n_g \simeq 1$

Nakamura et al., Nature 398, 786 (1999)

disadvantage:
sensitivity to charge fluctuations

- flux Qbit:



$E_J \gg E_C$

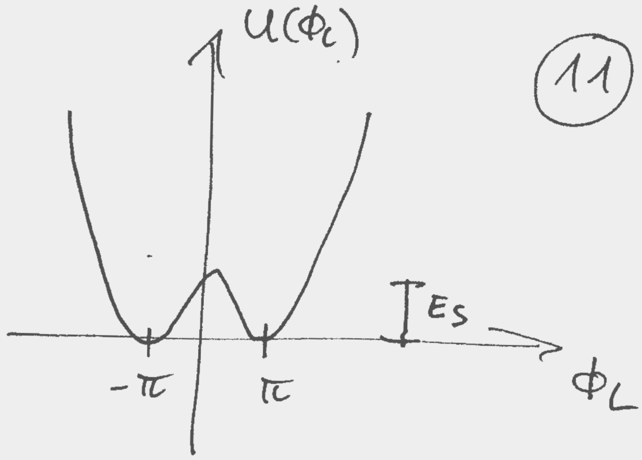
SC loop (inductance L)
with one Josephson Junction
(rf SQUID)

$$\rightarrow H_{flux} = \frac{Q^2}{2C} + \frac{(2\pi \Phi_s)^2}{2L}\phi_L^2 - E_J \cos\left(\phi_L - 2\pi \frac{\Phi}{\Phi_s}\right)$$

close to $\Phi = \Phi_s/2$: $U(\phi_L) = E_J\left[\beta \phi_L^2 + \cos(\phi_L + f)\right]$

$\Phi \simeq \frac{1}{2}\Phi_s + \frac{f}{2\pi}\Phi_s$ $\qquad \beta = 2\pi^2 \Phi_s^2 / L E_J$

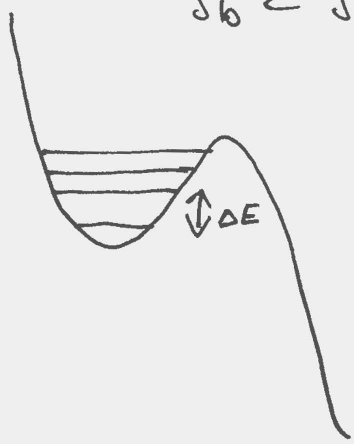
screening current going clockwise and anti-clockwise



- phase qbit

tilted washboard potential:

$I_b < I_c$:



quasi-bound states in minima of tilted washboard potential

for $I_b \ll I_c$: harmonic oscillator

finite I_b: anharmonic effects

$$(E_2 - E_1 \neq E_1 - E_0)$$

use $|0\rangle$ and $|1\rangle$ as Qbit state

- manipulation through microwave pulses or changes in I_b

- qbit measurement $\rightarrow$ escape process

disadvantage: sensitivity to flux noise

- other Qbits: quantronium, fluxonium

transmon (coupling to microwave transmission line)