Quantum Transport FS 2015
Superconducting Tunnel Junctions - Exercise 2 - 5.5.2015

Problem 1 - tunnel junction made from aluminum

Superconducting tunnel junction can be prepared by the successive evaporation of two aluminum layers, with an intermediate oxidation step, that creates a thin insulating Al_2O_3 layer on top of the first layer and acts as a tunnel barrier between the two superconductors. Explain how you would design an aluminum tunnel junction (with respect to area A and insulator thickness d) depending on which ratio of E_C/E_J you want to realize.
Hint: The charging energy E_C is determined by the area A and the oxide thickness d (dielectric contant of AL_2O_3: 9.34). You can estimate the Josephson energy E_J from the Ambegaokar-Baratoff relation. The number of channels can be estimates as A/λ_F^2, the transparency per channel $\tau \approx \exp(-2d\sqrt{2mU}/\hbar)$ with the barrier heigth $U = 1.2eV$ for Al_2O_3.

Problem 2 - overdamped Josephson Junction

Consider the RCSJ model in the strongly overdamped case ($Q \ll 1$):
Calculate the I-V charcteristics for this situation. Start from the usual differential equation

$$\ddot{\phi} + \dot{\phi}/Q + \sin\phi = I/I_c$$

Neglect the first term, and solve the remaining equation by separation of the variables ϕ and τ. Averaging over the phase can be done by integration from 0 to 2π.