PHYSICAL REVIEW B VOLUME 46, NUMBER 19 15 NOVEMBER 1992-I

Quantum transport in semiconductor-superconductor microjunctions

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(Received 8 September 1992)

A formula is derived that relates the conductance of a normal-metal–superconductor (NS) junction to the single-electron transmission eigenvalues. The formula is applied to a quantum point contact (yielding conductance quantization at multiples of $4e^2/h$), to a quantum dot (yielding a non-Lorentzian conductance resonance), and to quantum interference effects in a disordered NS junction (enhanced weak-localization and reflectionless tunneling through a potential barrier).

Electrical transport through the interface between a normal metal and a superconductor requires the conversion of normal current (carried by excitations) to supercurrent (a ground-state property). The process by which this conversion occurs is a non-charge-conserving scattering process known as Andreev reflection:[1] An electron excitation above the Fermi level in the normal metal is reflected at the normal-metal–superconductor (NS) interface as a hole excitation below the Fermi level. The missing charge of $2e$ is removed as a supercurrent. The early theoretical work[1] on the conductance of an NS junction treats the dynamics of the quasiparticle excitations *semiclassically*, as is appropriate for macroscopic junctions. Interest in mesoscopic NS junctions, where quantum interference effects play a role, is a new development,[2–4] motivated in part by a recent experiment.[5] Much of the present technological effort in this field is aimed at fabricating a direct contact between a superconducting film and the two-dimensional (2D) electron gas in a semiconductor heterostructure. Such a system would be ideal for the study of the interplay of Andreev reflection and the mesoscopic effects known to occur in semiconductor nanostructures.[6]

In this paper a quantum transport theory for conduction through an NS interface is developed, and is used to study a variety of mesoscopic effects which are expected to occur in semiconductor-superconductor microjunctions. The key result, Eq. (5), is a relation between the conductance $G_{\rm NS}$ of the NS junction and the eigenvalues of the normal-state transmission matrix product $tt^\dagger$. We give four illustrative applications of this conductance formula. One can think of many more applications, some of which are briefly mentioned in the concluding paragraph.

The model considered is illustrated in the inset of Fig. 1. It consists of a disordered normal region (possibly containing a geometrical constriction), adjacent to a superconductor (S). To obtain a well-defined scattering problem we insert ideal (impurity-free) normal leads N_1 and N_2 to the left and right of the disordered region. We assume that the only scattering in the superconductor consists of Andreev reflection at the NS interface, i.e., we consider the case that the disorder is contained entirely within the normal region. The model is directly applicable to a superconductor in the clean limit (mean free path in S large compared to the superconducting coherence length ξ).[7] The spatial separation of normal and Andreev scattering also applies to a microjunction of dimensions $W \ll \xi$, because of the separation of the length scales for normal scattering (W) and Andreev reflection (ξ). Let the NS interface be located at $x = 0$, and let the pair potential $\Delta(r)$ in the bulk of the superconductor ($x \gg \xi$) have amplitude Δ_0 and phase ϕ. For $x < 0$ one has $\Delta(r) \equiv 0$ in the assumed absence of electron-electron interactions in the normal metal. The simplest model consistent with these two boundary conditions is the step-function model $\Delta(r) = \Delta_0 e^{i\phi}\theta(x)$. Likharev[8] discusses in detail the conditions for its validity: If $W \ll \xi$, nonuniformities in Δ extend only over a distance of order W from the junction. Since nonuni-

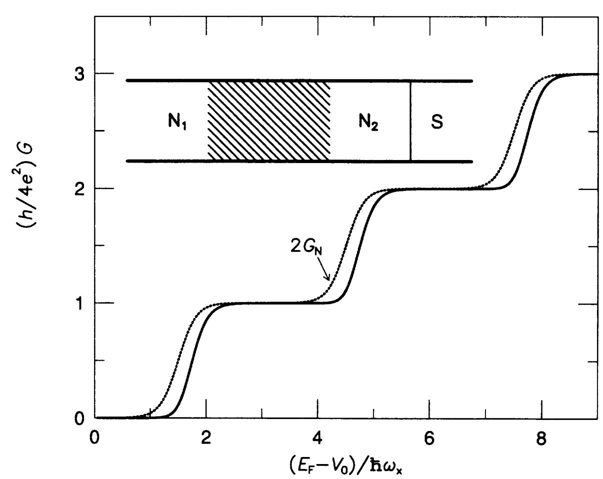

FIG. 1. Solid curve: conductance $G_{\rm NS}$ vs Fermi energy of a quantum point contact between a normal and a superconducting reservoir. The dotted curve is twice the conductance G_N for the case of two normal reservoirs (Ref. 15). The constriction is defined by the 2D saddle-point potential $V(x,y) = V_0 - \frac{1}{2}m\omega_x^2x^2 + \frac{1}{2}m\omega_y^2y^2$, with $\omega_y/\omega_x = 3$; $G_{\rm NS}$ is calculated from Eq. (5), with $T_n = [1+\exp(-2\pi\varepsilon_n/\hbar\omega_x)]^{-1}$, $\varepsilon_n \equiv E_F - V_0 - (n-\frac{1}{2})\hbar\omega_y$.

formities on length scales $\ll \xi$ do not affect the dynamics of the quasiparticles, these can be neglected and the step-function model holds. Alternatively, it holds if the resistivity of the junction region is much bigger than the resistivity of the bulk superconductor.[8]

The construction of a scattering matrix (s matrix) for the electron and hole quasiparticle excitations of the NS junction proceeds in a similar way as in Ref. 9. The s matrix s_N of the normal region (at energy ε, relative to the Fermi energy $E_{\rm F}$) has the block-diagonal form

$$s_N(\varepsilon) = \begin{pmatrix} s_0(\varepsilon) & 0 \\ 0 & s_0(-\varepsilon)^* \end{pmatrix}, \quad s_0 \equiv \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix}, \quad (1)$$

where s_0 is the unitary single-electron s matrix. The off-diagonal blocks of s_N are zero, because the normal region does not couple electrons and holes. The reflection and transmission matrices r and t are $N \times N$ matrices, with N the number of propagating modes in leads N_1 and N_2. For energies $0 < \varepsilon < \Delta_0$ there are no propagating modes in the superconductor. We can then define a $2N \times 2N$ s matrix s_A for Andreev reflection at the NS interface. In the step-function model, and neglecting terms of order Δ_0/E_F (the Andreev approximation[1]), one has

$$s_A(\varepsilon) = \exp[-i \arccos(\varepsilon/\Delta_0)] \begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix}. \quad (2)$$

Andreev reflection transforms an electron mode into a hole mode, without change of mode index, and with a mode-independent phase shift.

We can now construct the scattering matrix s of the whole system for energies $0 < \varepsilon < \Delta_0$. An electron incident in lead N_1 is reflected either as an electron (with scattering amplitudes $s_{\rm ee}$) or as a hole (with scattering amplitudes $s_{\rm he}$). Similarly, the matrices $s_{\rm hh}$ and $s_{\rm eh}$ contain the scattering amplitudes for reflection of a hole as a hole or as an electron. For the linear-response conductance $G_{\rm NS}$ of the NS junction at zero temperature we only need the s matrix at the Fermi level, i.e., at $\varepsilon = 0$. We limit ourselves to this case and omit the argument ε in what follows. We apply the general formula[2,10,11]

$$G_{\rm NS} = \frac{2e^2}{h} {\rm Tr}\,(1 - s_{\rm ee}s_{\rm ee}^\dagger + s_{\rm he}s_{\rm he}^\dagger) = \frac{4e^2}{h} {\rm Tr}\, s_{\rm he}s_{\rm he}^\dagger. \quad (3)$$

The second equality follows from unitarity of s. We express s in terms of s_N and s_A, and find

$$G_{\rm NS} = \frac{4e^2}{h} {\rm Tr}\, t_{12}^\dagger t_{12} (1 + r_{22}^* r_{22})^{-1} t_{21}^* t_{21}^{\rm T} (1 + r_{22}^\dagger r_{22}^{\rm T})^{-1}. \quad (4)$$

In what follows we will consider a junction in zero magnetic field. Then s_0 is a symmetric matrix, $s_0 = s_0^T$. Equation (4) simplifies to

$$G_{\rm NS} = \frac{4e^2}{h} {\rm Tr} \left(\frac{t_{12}t_{12}^\dagger}{2 - t_{12}t_{12}^\dagger} \right)^2 \equiv \frac{4e^2}{h} \sum_{n=1}^{N} \frac{T_n^2}{(2 - T_n)^2}, \quad (5)$$

where T_n $(n = 1, 2, \ldots, N)$ are the eigenvalues of the Hermitian matrix $t_{12}t_{12}^\dagger$. Equation (5) holds for an arbitrary transmission matrix, i.e., for arbitrary disorder potential. It is the *multichannel* generalization of a formula first obtained by Blonder, Tinkham, and Klapwijk[10] (and subsequently by others[12,13]) for the *single-channel* case. A formula of similar generality for the normal-metal conductance G_N is the multichannel Landauer formula

$$G_N = \frac{2e^2}{h} {\rm Tr}\, t_{12}t_{12}^\dagger \equiv \frac{2e^2}{h} \sum_{n=1}^{N} T_n. \quad (6)$$

To illustrate the power and generality of Eq. (5), we now apply it to a variety of NS junctions.

(1) Quantum point contact. Consider first the case that the junction consists of a ballistic constriction with a normal-state conductance quantized at $G_N = 2N_0e^2/h$. The quantization occurs because the transmission eigenvalues are equal to either zero or one.[6] (This does *not* imply that the transport through the constriction is adiabatic.) We thus conclude from Eq. (5) that the conductance of the NS junction is quantized in units of $4e^2/h$: $G_{\rm NS} = 4N_0e^2/h$. For a qualitative discussion of this *doubling* of the conductance step height we refer to Ref. 14. In the classical limit $N_0 \to \infty$ we recover the well-known result $G_{\rm NS} = 2G_N$ for a *classical* ballistic point contact.[10,12,13] In the quantum regime, however, the simple factor-of-2 enhancement only holds for the conductance plateaus, and not to the transition region between the plateaus. To illustrate this, we compare in Fig. 1 the conductances $G_{\rm NS}$ and $2G_N$ for Büttiker's model[15] of a saddle-point constriction in a 2D electron gas. Appreciable differences appear in the transition region, where $G_{\rm NS} \leq 2G_N$. This is actually a rigorous inequality within the present model, which follows directly from Eqs. (5) and (6) for *arbitrary* transmission matrix.

(2) Quantum dot. Consider next a small confined region, which is weakly coupled by tunnel barriers to two electron reservoirs. We assume that transport through this "quantum dot" occurs via resonant tunneling through a single bound state. Let $\varepsilon_{\rm res}$ be the energy of the resonant level, relative to the Fermi level in the reservoirs, and let $\Gamma_1/\hbar$ and $\Gamma_2/\hbar$ be the tunnel rates through the two barriers. The normal-state conductance G_N has the Breit-Wigner form

$$G_N = \frac{2e^2}{h} \frac{\Gamma_1\Gamma_2}{\varepsilon_{\rm res}^2 + \Gamma^2/4} \equiv \frac{2e^2}{h} T_{\rm BW}, \quad (7)$$

with $\Gamma \equiv \Gamma_1 + \Gamma_2$. The transmission matrix which yields this conductance has elements[16]

$$t_{12}(\varepsilon) = U_1\tau(\varepsilon)U_2, \; \tau(\varepsilon)_{nm} \equiv \frac{\sqrt{\Gamma_{1n}\Gamma_{2m}}}{\varepsilon - \varepsilon_{\rm res} + {\rm i}\Gamma/2}, \quad (8)$$

where $\sum_n \Gamma_{1n} \equiv \Gamma_1$, $\sum_n \Gamma_{2n} \equiv \Gamma_2$, and U_1, U_2 are unitary matrices. The matrix $t_{12}t_{12}^\dagger$ (at $\varepsilon = 0$) has eigenvalues $T_n = T_{\rm BW}\delta_{n1}$. Substitution into Eq. (5) yields

$$G_{\rm NS} = \frac{4e^2}{h} \left(\frac{2\Gamma_1\Gamma_2}{4\varepsilon_{\rm res}^2 + \Gamma_1^2 + \Gamma_2^2} \right)^2. \quad (9)$$

The conductance on resonance ($\varepsilon_{\rm res} = 0$) is maximal if $\Gamma_1 = \Gamma_2$, and is then equal to $4e^2/h$ — *twice* the normal-state value. Note that the line shape (9) differs substantially from the Lorentzian line shape (7) of the Breit-Wigner formula, decaying as $\varepsilon_{\rm res}^{-4}$ rather than $\varepsilon_{\rm res}^{-2}$.

(3) Disordered junction. Consider a point contact or microbridge between a normal and superconducting reservoir, of length L much greater than the mean free path l for elastic impurity scattering. We calculate the average $\langle G_{\rm NS}\rangle_L$, averaged over an ensemble of impurity configurations. The transmission eigenvalue T_n can be parametrized in terms of a channel-dependent localization length ζ_n: $T_n \equiv \cosh^{-2}(L/\zeta_n)$. Using a trigonometric identity, the ensemble-average of Eq. (5) becomes

$$\langle G_{\rm NS}\rangle_L = \frac{4e^2}{h}\int_0^\infty d\zeta\, p_L(\zeta)\cosh^{-2}(2L/\zeta), \tag{10}$$

where $p_L(\zeta) \equiv \langle \sum_n \delta(\zeta-\zeta_n)\rangle_L$ is the density of localization lengths. In the same parametrization, one has

$$\langle G_{\rm N}\rangle_L = \frac{2e^2}{h}\int_0^\infty d\zeta\, p_L(\zeta)\cosh^{-2}(L/\zeta). \tag{11}$$

In the regime $l \ll L \ll Nl$ the L dependence of the density $p_L(\zeta)$ can be rigorously disregarded [$p_L(\zeta)$ corresponds to a uniform distribution of $1/\zeta$, with average spacing $\propto 1/Nl$ independent of L (Ref. 17)]. The whole L dependence of the integrands then lies in the argument of the hyperbolic cosine, so that

$$\langle G_{\rm NS}\rangle_L = 2\langle G_N\rangle_{2L}. \tag{12}$$

This derivation[18] formalizes the intuitive notion that Andreev reflection at an NS interface effectively doubles the length of the normal-metal conductor.[2] Since $\langle G_N\rangle \propto Nl/L$, it follows from Eq. (12) that $\langle G_{\rm NS}\rangle_L = \langle G_N\rangle_L$. We conclude that — although $G_{\rm NS}$ according to Eq. (5) is of *second* order in the transmission eigenvalues T_n — the ensemble average $\langle G_{\rm NS}\rangle$ is of *first* order in l/L. The resolution of this paradox is that the T's are not distributed uniformly, but are either exponentially small (closed channels) or of order unity (open channels).[19,20] Hence the average of T_n^2 is of the same order as the average of T_n.

Previous work[1,21] had obtained the equality of $\langle G_{\rm NS}\rangle$ and $\langle G_N\rangle$ from *semiclassical* equations of motion, with disregard of quantum interference effects, as was appropriate for macroscopic systems which are large compared to the normal-metal phase-coherence length l_ϕ. The present derivation, in contrast, applies to the "mesoscopic" regime $L < l_\phi$, in which transport is fully phase coherent. Recently, Takane and Ebisawa[2] have studied the conductance of a disordered phase-coherent NS junction by numerical simulation. They found $\langle G_{\rm NS}\rangle = \langle G_N\rangle$ within numerical accuracy for $l \ll L \ll Nl$.

If the condition $L \ll Nl$ is relaxed, differences between $\langle G_{\rm NS}\rangle$ and $\langle G_N\rangle$ appear. To lowest order in L/Nl, the difference is a manifestation of the *weak-localization* effect, as we now discuss. In the "open-channel approximation,"[17] the integrals over ζ are restricted to the range $\zeta > L$ of localization lengths greater than the length of the conductor. In this range the density $p_L(\zeta)$ remains independent of L,[22] so that Eq. (12) still applies approximately. Consider now the geometry $W \ll L$ relevant for a microbridge. Weak-localization theory tells us that $\langle G_N\rangle = g_0(e^2/h)(Nl/L) - g_1(e^2/h)$, where g_0 and g_1 are positive constants of order unity. Equation (12) then implies that $\langle G_{\rm NS}\rangle = g_0(e^2/h)(Nl/L) - 2g_1(e^2/h)$, which is smaller than $\langle G_N\rangle$ by an amount g_1e^2/h — as a consequence of the *enhanced weak localization* in the NS junction.

(4) Tunnel barrier. As a final application of Eq. (5), we consider the effect of a tunnel barrier at the NS interface on the conductance of a phase-coherent disordered junction. Let s_d be the s matrix of the disordered region and s_b that of the barrier. The s matrix s_0 of the entire normal region (disordered segment plus barrier) can be constructed from s_d and s_b. We need the transmission submatrix t_{12} of s_0, which is given by

$$t_{12} = t_{12}^d(1 - r_{11}^b r_{22}^d)^{-1} t_{12}^b. \tag{13}$$

We consider for simplicity the case of a tunnel barrier with mode-independent transmission probability Γ (a scalar). For the s matrix of the disordered region, we employ the decomposition[23]

$$s_d \equiv \begin{pmatrix} V & 0\\ 0 & U\end{pmatrix}\begin{pmatrix} -i\sqrt{\mathcal{R}} & \sqrt{\mathcal{T}}\\ \sqrt{\mathcal{T}} & -i\sqrt{\mathcal{R}}\end{pmatrix}\begin{pmatrix} V^T & 0\\ 0 & U^T\end{pmatrix}. \tag{14}$$

Here U and V are $N\times N$ unitary matrices, while $\mathcal{T}$ and $\mathcal{R} \equiv 1-\mathcal{T}$ are $N\times N$ diagonal matrices with real positive elements. Combining Eqs. (5), (13), and (14), we find for the conductance $G_{\rm NBS}$ of an NS junction containing a tunnel barrier the expression

$$G_{\rm NBS} = \frac{4e^2}{h}\left(\frac{\Gamma}{2-\Gamma}\right)^2 {\rm Tr}\,(\Xi\Omega^{-1})^2, \tag{15}$$

$$\Omega \equiv 1 + \frac{(1-\Gamma)^{1/2}}{2-\Gamma}\Big(M_-^{1/2}U^\dagger U^* M_+^{1/2} + M_+^{1/2}U^{\rm T} U M_-^{1/2}\Big),$$

where $\Xi \equiv \mathcal{T}/(2-\mathcal{T})$ and $M_\pm \equiv 1\pm\Xi$.

To proceed we adopt the isotropy assumption of random-matrix theory,[17] valid for $l, W \ll L$. In this limit the matrix U is distributed uniformly over the unitary group $\mathcal{U}(N)$, independently of the distribution of transmission eigenvalues. We denote by $\langle f\rangle_{\mathcal{U}}$ the average of a function $f(U)$ over the unitary group.[24] The average $\langle G_{\rm NBS}\rangle_{\mathcal{U}}$ can be calculated exactly in the limit $N\to\infty$, $l/L\to 0$ at fixed Nl/L and fixed Γ. In this limit we may take $M_\pm \to 1$ and factorize the average $\langle(\Xi\Omega^{-1})^2\rangle_{\mathcal{U}} \to (\Xi\langle\Omega^{-1}\rangle_{\mathcal{U}})^2$. The remaining average is easily evaluated, as it is independent of N,

$$\langle\Omega^{-1}\rangle_{\mathcal{U}} = \int_0^{2\pi}\frac{d\alpha}{2\pi}\left(1 + 2\frac{(1-\Gamma)^{1/2}}{2-\Gamma}\cos 2\alpha\right)^{-1} = \frac{2-\Gamma}{\Gamma}.$$

Substituting into Eq. (15) we see that the terms containing Γ cancel, and we are left with

$$\langle G_{\rm NBS}\rangle_{\mathcal{U}} = \frac{4e^2}{h}{\rm Tr}\,\Xi^2 \equiv \frac{4e^2}{h}{\rm Tr}\left(\frac{t_{12}^d t_{12}^{d\dagger}}{2 - t_{12}^d t_{12}^{d\dagger}}\right)^2. \tag{16}$$

This is just the expression for the conductance *without* the tunnel barrier [Eq. (5) with t_{12} substituted by the transmission matrix t_{12}^d of the disordered region alone]. We conclude that, on average, the reduction in current because of reflection at the barrier is just compensated by the current increase from Andreev reflection. The net result is as if tunneling through the barrier is *reflectionless*. van Wees *et al.*,[4] in an insightful article, have proposed such an effect if the disorder potential is so smooth that it does not randomly scatter the electrons but deflects them deterministically. In that case the Andreev-reflected hole simply retraces the path of the incident electron, which was crucial for their trajectory argument. Here we have found that this (unrealistic) condition is not required to obtain a complete suppression of the barrier resistance in the limit $l/L \ll \Gamma$. As discussed in Ref. 4, the excess conductance observed by Kastalsky *et al.*[5] may well be due to this quantum interference effect.

These are four examples of applications of Eq. (5) to the ballistic, resonant-tunneling, and diffusive transport regimes. We briefly mention a few other promising applications. A magnetic field breaks the time-reversal symmetry (TRS) of the scattering processes. As a consequence, s_0 is no longer a symmetric matrix, and Eq. (5) does not apply. Equation (4) remains valid, however, and can serve as a starting point for a study of quantum interference effects in an NS junction in the absence of TRS. In this paper we have focused on the ensemble-averaged conductance. Equations (4) and (5) are not restricted to this and can describe the sample-specific fluctuations of $G_{\rm NS}$ as well. Since Eq. (5) is a *linear statistic* on the transmission eigenvalues [i.e., a function of the form $\sum_n f(T_n)$], it follows from general considerations[19] that the fluctuations in $G_{\rm NS}$ in the presence of TRS are *universally* of order e^2/h — in agreement with a recent independent investigation.[2] For G_N, broken TRS does not affect the universality of the fluctuations, but merely reduces the variance by a universal factor of 2.[17] We expect no such simple behavior for $G_{\rm NS}$, since Eq. (4) is not a linear statistic on T_n. That is a crucial distinction with the Landauer formula, which remains a linear statistic regardless of whether TRS is broken or not.

Note added in proof. In collaboration with S. Feng, we have extended the calculation to include a magnetic field. The result (16) for $l/L \ll \Gamma$ turns out to be insensitive to a magnetic field. For a weakly reflecting barrier we find $\langle G_{\rm NBS}\rangle = G_0[1-\beta(1-\Gamma)l/L] + \mathcal{O}(1-\Gamma)^2$, where $G_0 = (2e^2/h)Nl/L$ is the conductance of the disordered normal region, and β equals 1 in the absence and 2 in the presence of a magnetic field.

I have benefited from discussions with A. F. Andreev, H. van Houten, and M. J. M. de Jong. Support by the Dutch Science Foundation NWO/FOM is gratefully acknowledged.

[1] A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)]; **51**, 1510 (1966) [**24**, 1019 (1967)].

[2] Y. Takane and H. Ebisawa, J. Phys. Soc. Jpn. **60**, 3130 (1991); **61**, 1685 (1992); **61**, 2858 (1992).

[3] H. Nakano and H. Takayanagi, Solid State Commun. **80**, 997 (1991); S. Takagi, *ibid.* **81**, 579 (1992).

[4] B. J. van Wees, P. de Vries, P. Magnée, and T. M. Klapwijk, Phys. Rev. Lett. **69**, 510 (1992).

[5] A. Kastalsky, A. W. Kleinsasser, L. H. Greene, R. Bhat, F. P. Milliken, and J. P. Harbison, Phys. Rev. Lett. **67**, 3026 (1991).

[6] C. W. J. Beenakker and H. van Houten, Solid State Phys. **44**, 1 (1991).

[7] The contribution of scattering inside the superconductor to the resistance of an NS junction has been studied extensively: A. B. Pippard, J. G. Shepherd, and D. A. Tindall, Proc. R. Soc. London Ser. A **324**, 17 (1971); A. Schmid and G. Schön, J. Low Temp. Phys. **20**, 207 (1975); T. Y. Hsiang and J. Clarke, Phys. Rev. B **21**, 945 (1980).

[8] K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).

[9] C. W. J. Beenakker, Phys. Rev. Lett. **67**, 3836 (1991); **68**, 1442(E) (1992).

[10] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).

[11] C. J. Lambert, J. Phys. Condens. Matter **3**, 6579 (1991).

[12] A. L. Shelankov, Fiz. Tverd. Tela **26**, 1615 (1984) [Sov. Phys. Solid State **26**, 981 (1984)].

[13] A. V. Zaĭtsev, Zh. Eksp. Teor. Fiz. **78**, 221 (1980) [Sov. Phys. JETP **51**, 111 (1980)]; **79**, 2016(E) (1980) [**52**, 1018(E) (1980)]; **86**, 1742 (1984) [**59**, 1015 (1984)].

[14] H. van Houten and C. W. J. Beenakker, Physica B **175**, 187 (1991).

[15] M. Büttiker, Phys. Rev. B **41**, 7906 (1990).

[16] M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988).

[17] A. D. Stone, P. A. Mello, K. A. Muttalib, and J.-L. Pichard, in *Mesoscopic Phenomena in Solids*, edited by B. L. Al'tshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991).

[18] This derivation of Eq. (12) was obtained in a collaboration with M. J. M. de Jong.

[19] Y. Imry, Europhys. Lett. **1**, 249 (1986).

[20] J. B. Pendry, A. MacKinnon, and P. J. Roberts, Proc. R. Soc. London Ser. A **437**, 67 (1992).

[21] S. N. Artemenko, A. F. Volkov, and A. V. Zaĭtsev, Solid State Commun. **30**, 771 (1979).

[22] In the language of Ref. 17, this follows from the fact that the joint probability density of the variables $\nu_n \equiv 2L/\zeta_n$ has the form $P(\{\nu_n\}) \propto \exp[-H_{\rm open}(\{\nu_n\})]$, where $H_{\rm open}$ is invariant under the scaling $L \to xL$, $\nu_n \to x\nu_n$, for arbitrary $x > 0$.

[23] Th. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992).

[24] P. A. Mello, J. Phys. A **23**, 4061 (1990).