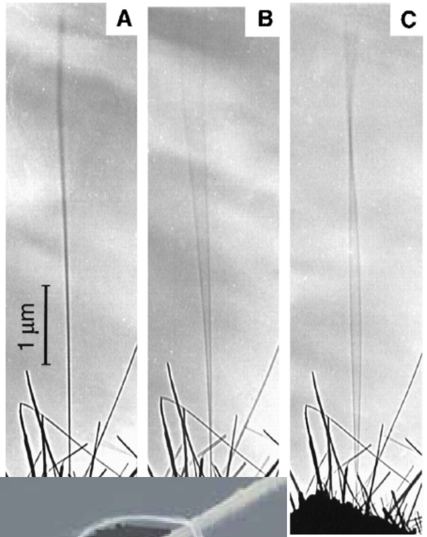
Nanoelectromechanical Systems

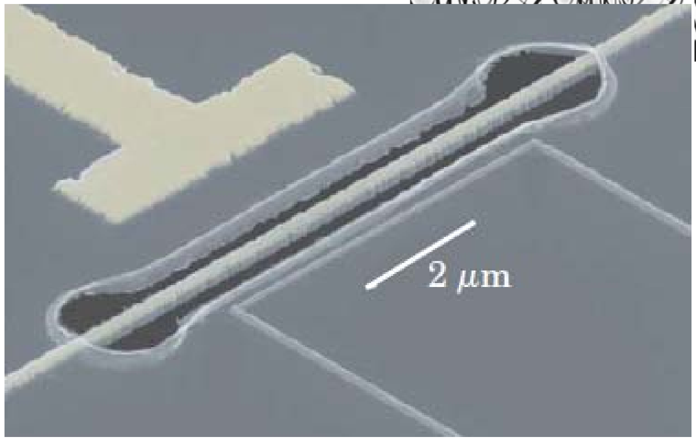
A much too short introduction

Overview

Intro
- up to now: electronic quantum / nanoscale systems, e.g. Quantum dots (QDs)
- coupling electronic to other quantum systems: examples
- nanoelectromechanical systems: examples and motivation



Basics
- some basics of continuum mechanics
- oscillating beams
- from modes to harmonic oscillators
- coupling mechanisms: actuation and sensing
- [(quantum) back-action]



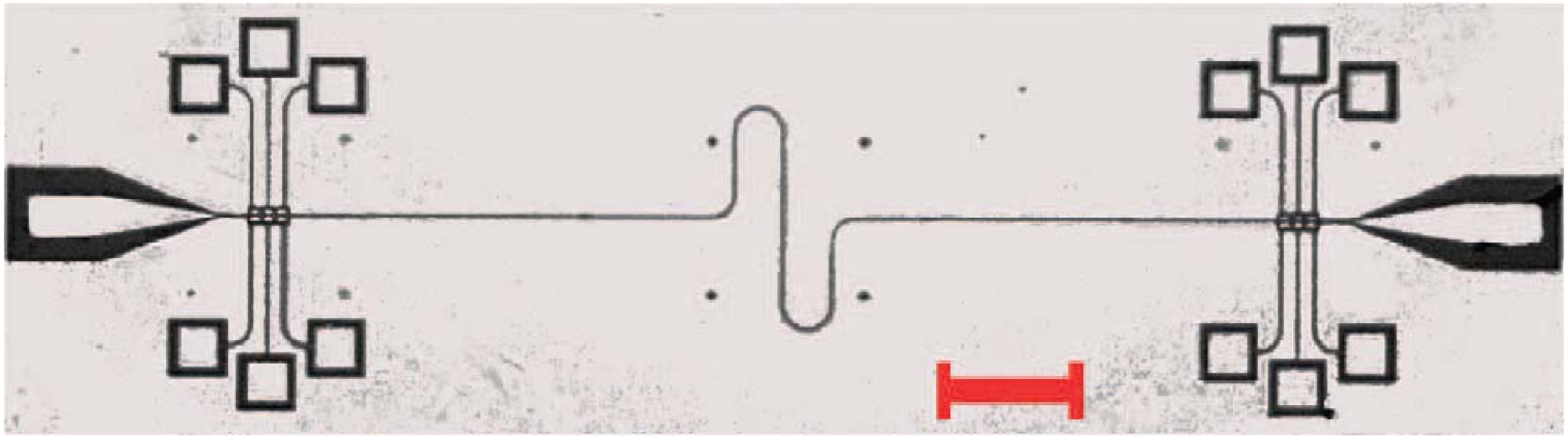
Fundamental experiments and applications
- ground state cooling (Teufel et al.)
- enigineering of coupling to a QD (Ilani et al.)
- [Franck- Condon blockade]
- [Phonon-assisted Andreev tunneling]
- ...

Coupling electronic to other quantum systems

Coupling to Phonons / vibrations:
- electrical resistance due to phonon scattering (week 2)
- many more... (topic of this part)

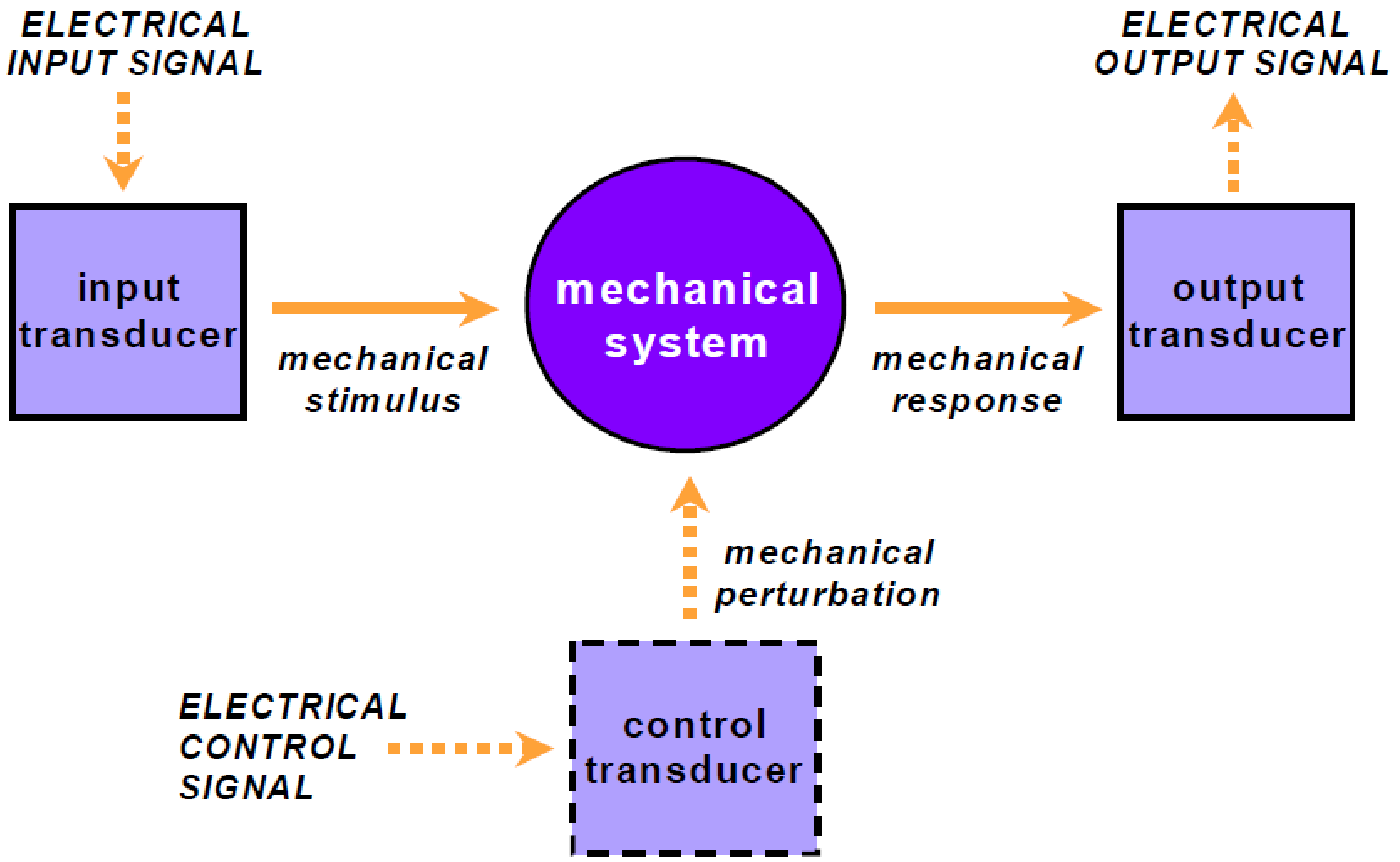
Coupling to cold atoms / ions / Rydberg atoms:

Coupling to Photons:
- rf cavity photons
- quantum dot LEDs
- electrically driven quantum cascade, QD, double-QD and other lasers (e.g. Kolloquium!)



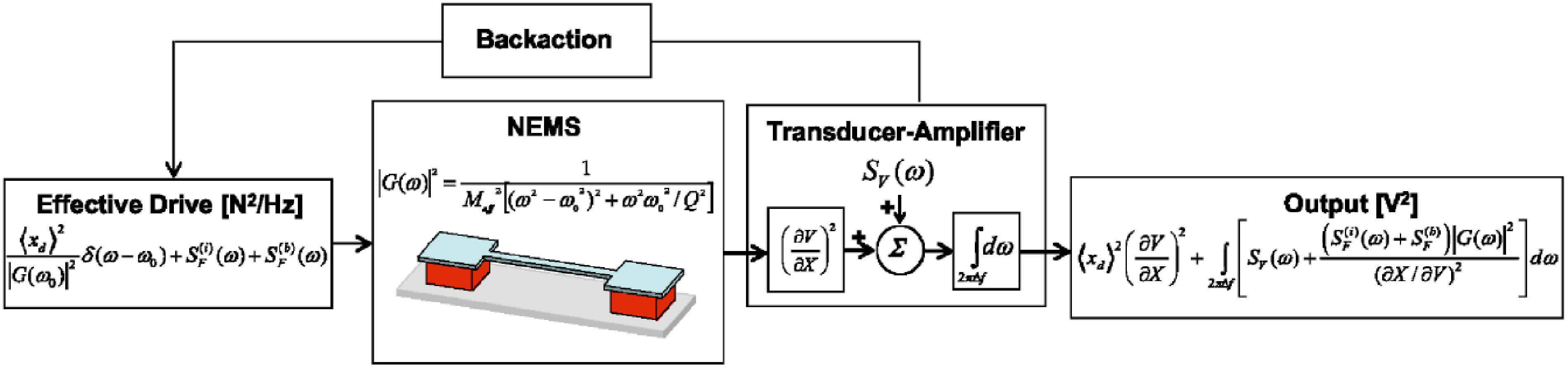
Photon-mediated interaction between distant quantum dots
Delbecq *et al.*, Nature Comm. **4**, 1400 (2013)

Nanoelectromechanical Systems: intro



Ekinci and Roukes, Rev. Sci. Intr. **76**, 061101 (2005)

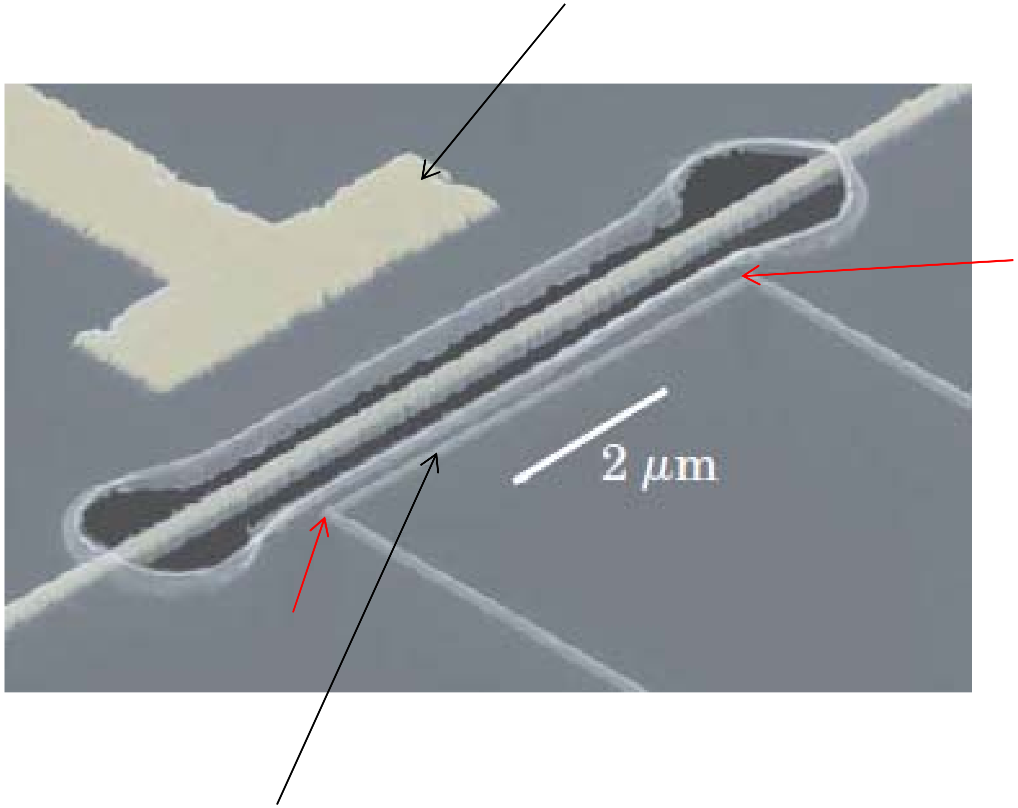
Nanoelectromechanical Systems: intro



Ekinci and Roukes, Rev. Sci. Intr. **76**, 061101 (2005)

Nanoelectromechanical Systems: examples

Approaching the Quantum Limit of a Nanomechanical Resonator
M. D. LaHaye, *et al.*
Science **304**, 74 (2004);



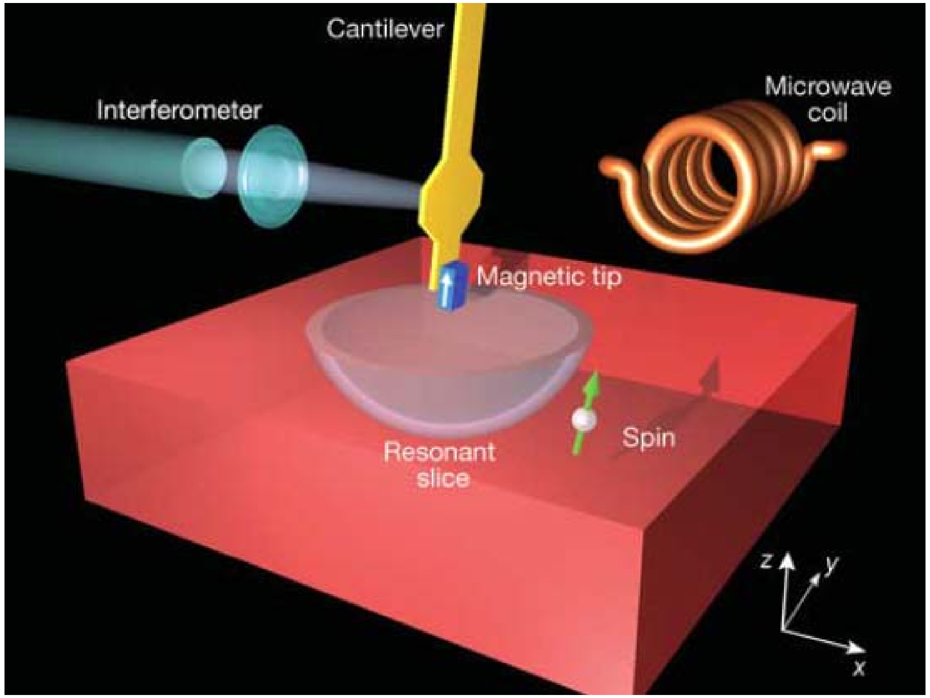
Nanoelectromechanical Systems: examples

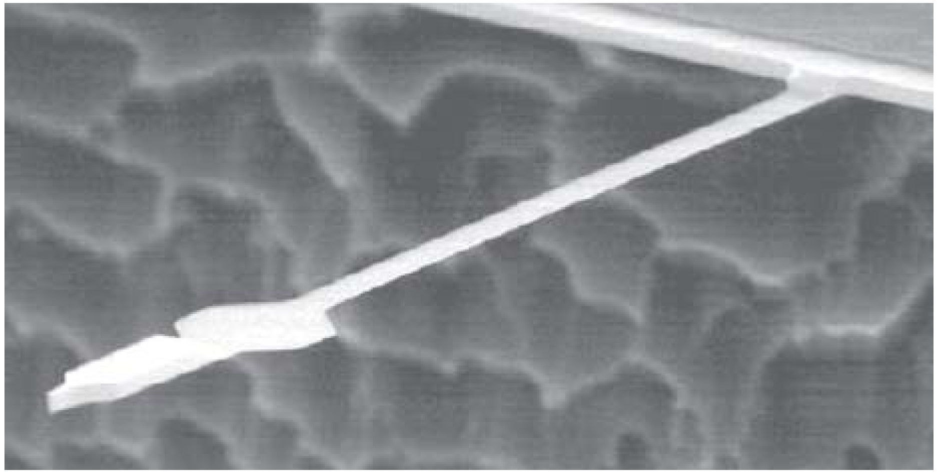
Single spin detection by magnetic resonance force microscopy

D. Rugar, R. Budakian, H. J. Mamin & B. W. Chui

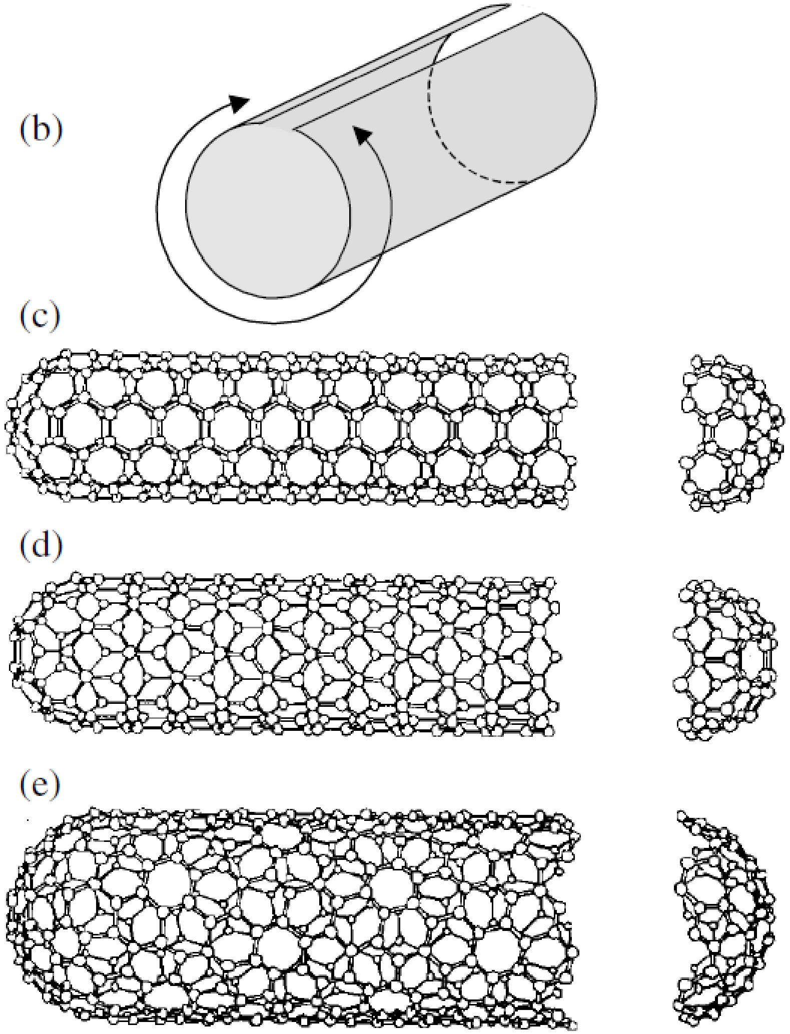
NATURE | VOL 430 | 15 JULY 2004

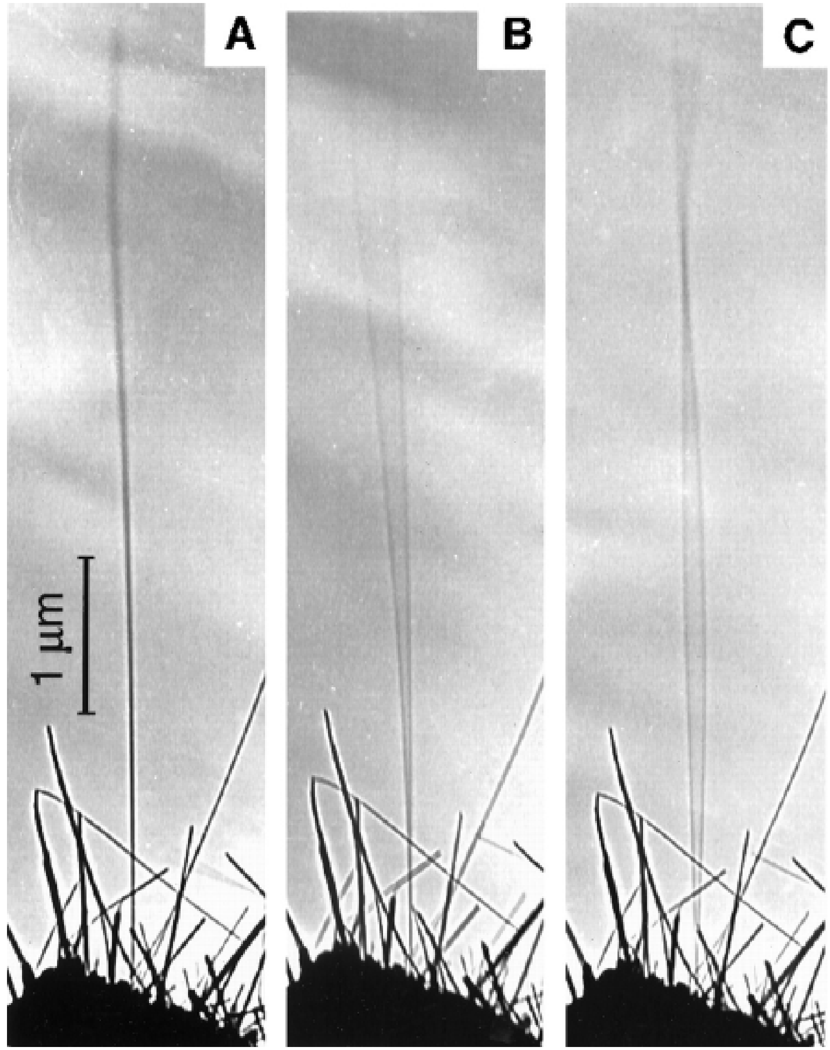
Magnetic cantilever





CNTs

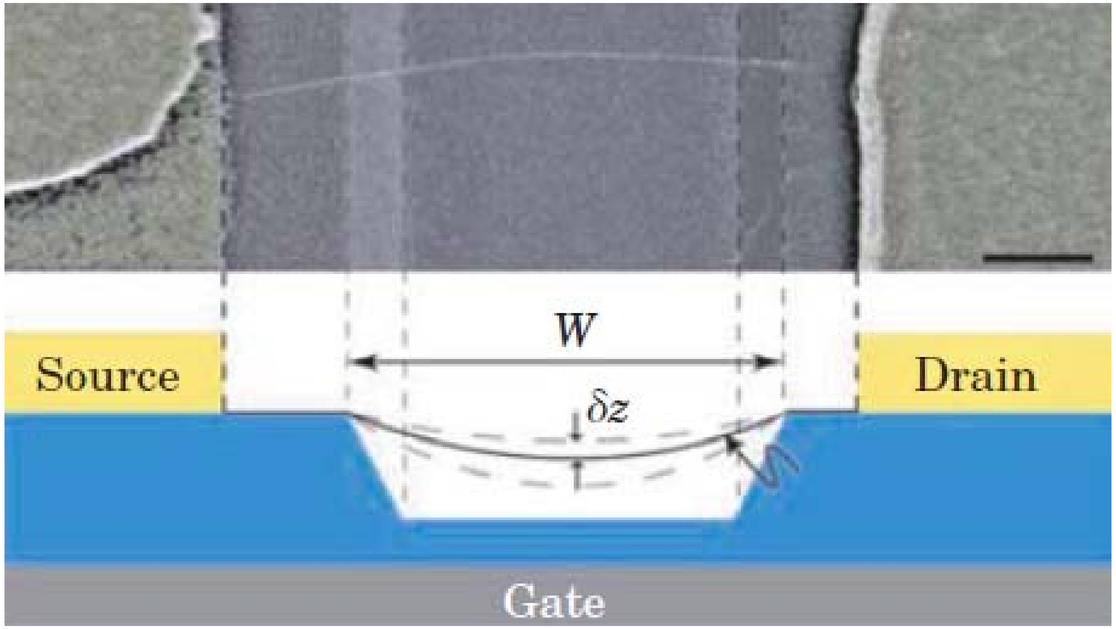




TEM images of CNTs at different gate voltages
Ponchoral *et al.*, *Science 283, 1513 (1999)*

Nanoelectromechanical Systems: examples

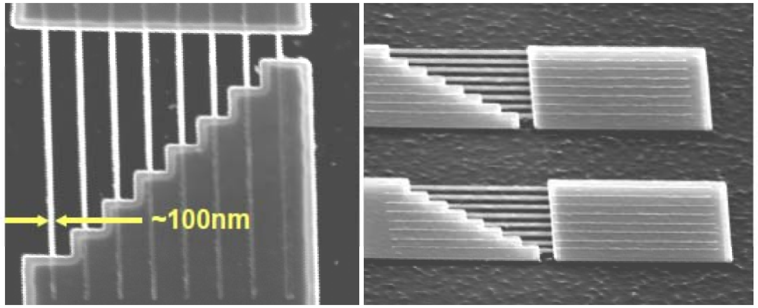
A tunable carbon nanotube electromechanical oscillator



Sazonova *et al.,* Nature **431**, 284 (2004)

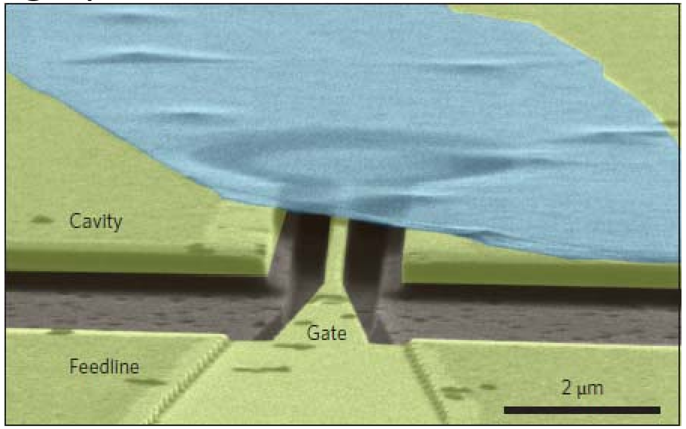
Nanoelectromechanical Systems: examples

SiC beams



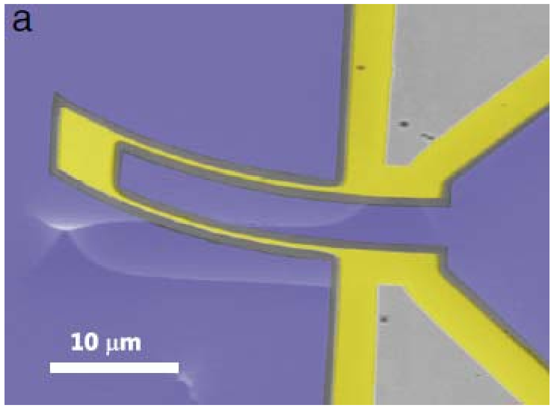
Solid-State Sensor and Actuator Workshop, arXiv:cond-mat/0008187 (2000)

graphene drum



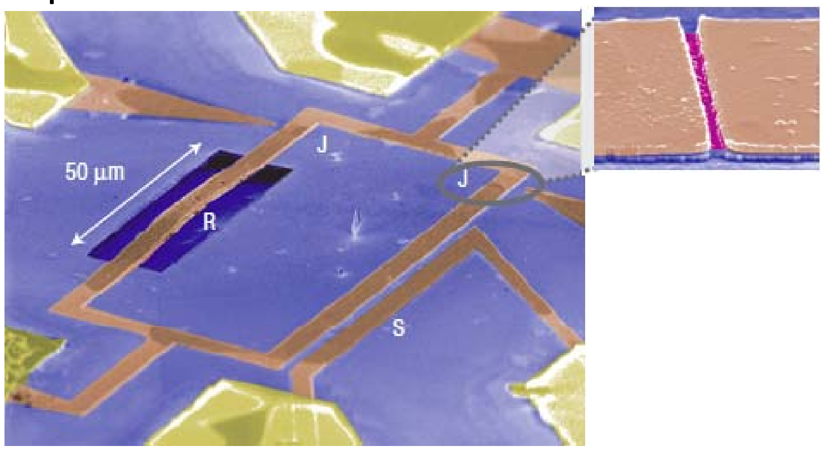
Singh et al., Nature Nanotech. 9, 820 (2014)

Piezo-resistive cantilever, piezoelectric tuning forks, ...



Poot et al., Physics Reports 511, 273 (2012)

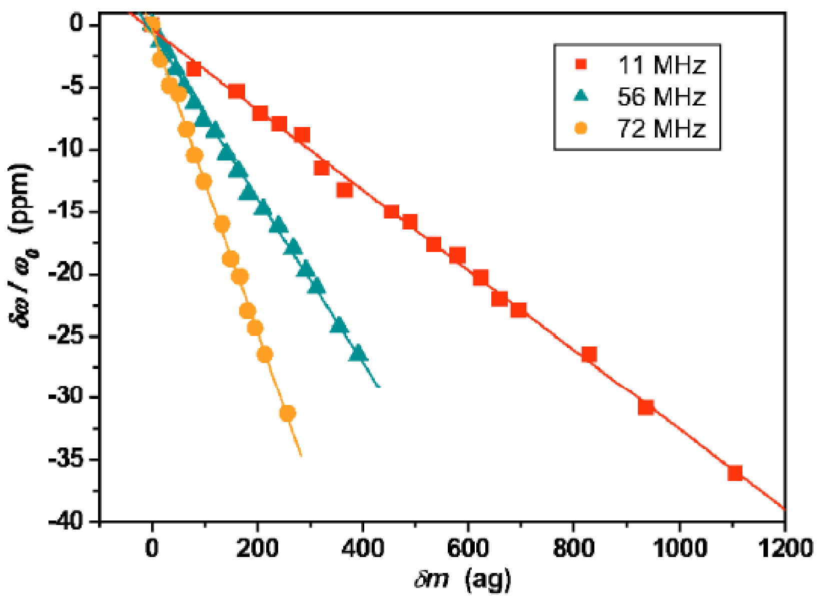
SQUID with variable area/flux couples motion to SQUID

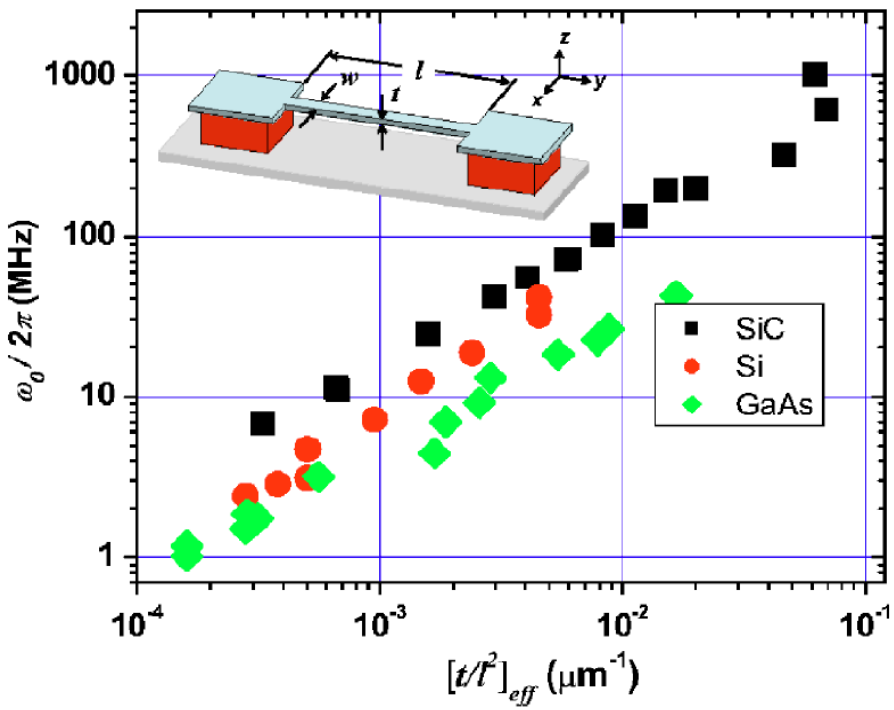


Etaki et al., Nature Nanotech. 4, 785 (2008)

Nanoelectromechanical Systems: application

mass detection





Today: mass of a proton ~10^{-27} kg!

Ekinci et al., Rev. Sci. Intr. **76**, 061101 (2005)

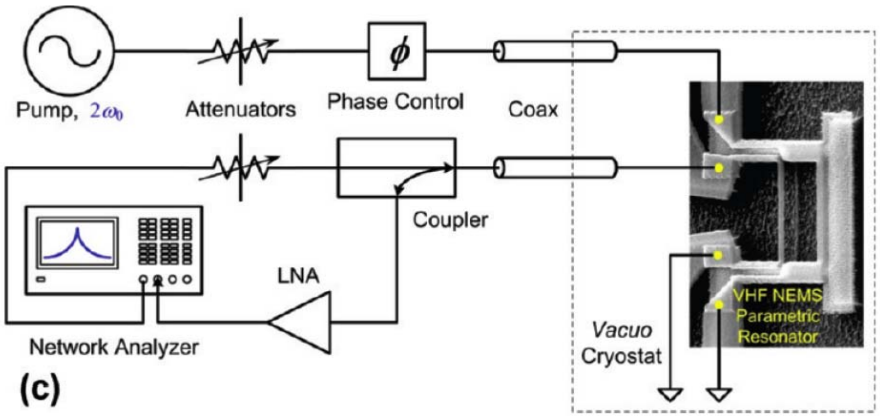
Nanoelectromechanical Systems: applications

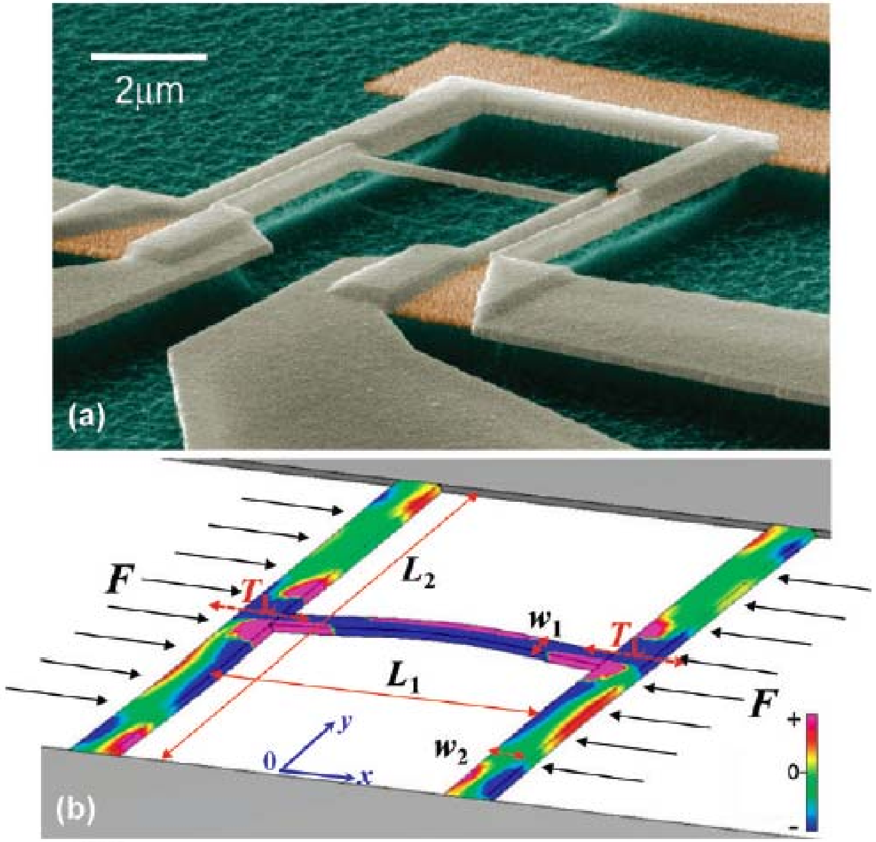
Parametric radio-frequency mechanical Amplifier

Large out-of plane field

Excitation: $F = LBI(t)$

Distortion modulates strain (spring constant)
-> parametric amplification





Karabalin *et al.*, Nano Lett. **9**, 3116 (2009)

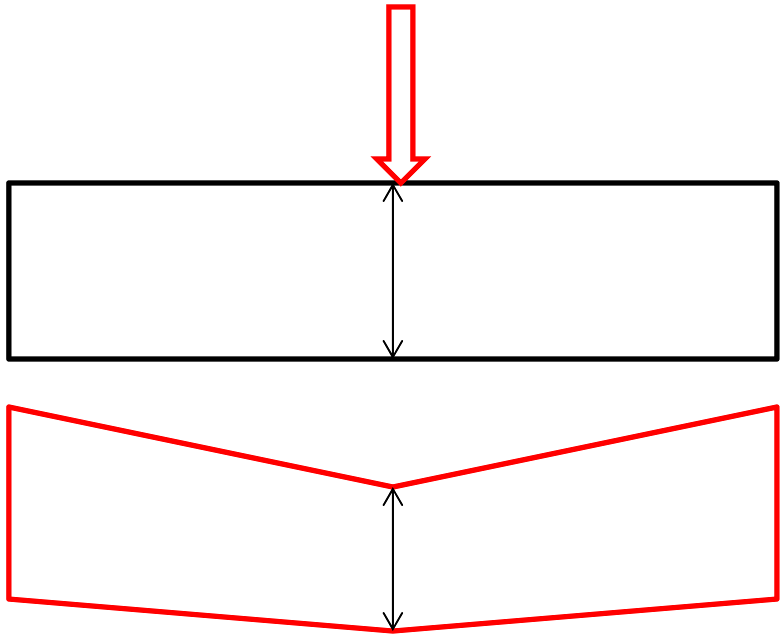
Continuums mechanics Poot et al., Physics Reports 511, 273 (2012)

OK for > 10 monolayers (perpendicular to vibration)

Infinitesimal volume element:
displacement **u**(x,y,z) -> deformation / elongation of volume element (not only displacement):
strain tensor γ

e.g. no strain for **u**=const.

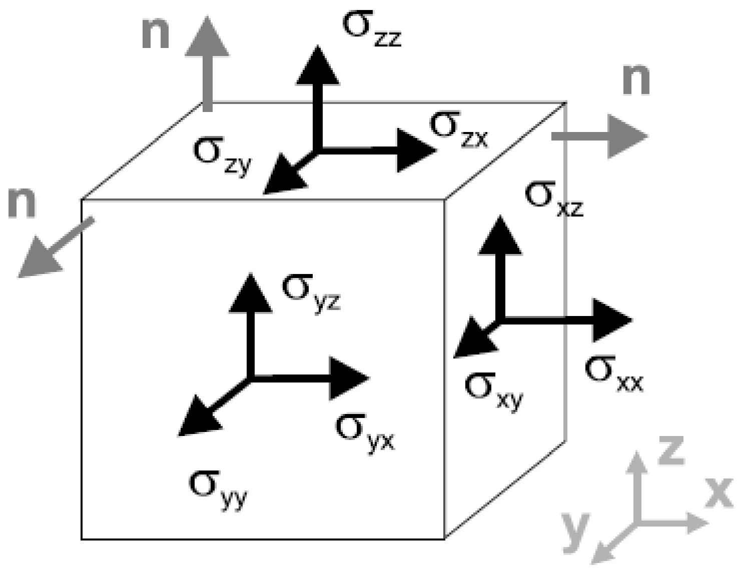
$$\gamma_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \cancel{\frac{\partial u_m}{\partial x_i}\frac{\partial u_m}{\partial x_j}}\right)$$



Continuums mechanics Poot et al., Physics Reports 511, 273 (2012)

$$\delta F_i = \sigma_{ij} n_j \delta A$$

strain tensor



Consider momentum of element of the material with mass Δm and volume ΔV speed $\mathbf{v} = \dot{\mathbf{u}}$,

$$\Delta\mathbf{p} \equiv \int_{\Delta m} \mathbf{v}\, \mathrm{d}m = \int_{\Delta V} \rho \mathbf{v}\, \mathrm{d}V$$

$$\frac{\mathrm{d}\Delta\mathbf{p}}{\mathrm{d}t} = \int_{\Delta V} \mathbf{F}_\mathrm{b}\, \mathrm{d}V + \int_{\delta(\Delta V)} \sigma\, \mathrm{d}A$$

Gauss integral:

$$\int_{\delta(\Delta V)} \sigma\, \mathrm{d}A = \int_{\Delta V} \partial\sigma_{ij}/\partial x_i \cdot \hat{\mathbf{x}}_i\, \mathrm{d}V$$

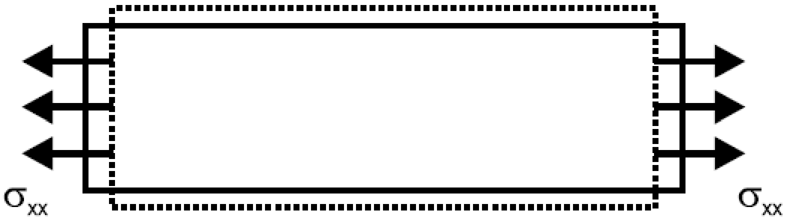
material forces + body forces

Cauchy's first law of motion $$\rho \ddot{u}_j = \frac{\partial \sigma_{ij}}{\partial x_i} + F_{b,j}$$

similar for angular momentum: $\sigma_{ij}=\sigma_{ji}$

Continuums mechanics

Elasticity: $$\sigma_{ij} = E_{ijkl}\gamma_{kl}$$



From symmetries (81 elements -> 21 indep. Elements)

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xz}\\ \sigma_{yz}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix} E_{xxxx} & E_{xxyy} & E_{xxzz} & E_{xxxz} & E_{xxyz} & E_{xxxy}\\ E_{xxyy} & E_{yyyy} & E_{yyzz} & E_{yyxz} & E_{yyyz} & E_{yyyx}\\ E_{xxzz} & E_{yyzz} & E_{zzzz} & E_{zzzx} & E_{zzzy} & E_{zzxy}\\ E_{xxxz} & E_{yyxz} & E_{zzzx} & E_{xzzx} & E_{xzzy} & E_{yxxz}\\ E_{xxyz} & E_{yyyz} & E_{zzzy} & E_{xzzy} & E_{yxxz} & E_{xyyz}\\ E_{xxxy} & E_{yyyx} & E_{zzxy} & E_{yxxz} & E_{xyyz} & E_{xyyx}\end{bmatrix} \begin{bmatrix}\gamma_{xx}\\ \gamma_{yy}\\ \gamma_{zz}\\ 2\gamma_{xz}\\ 2\gamma_{yz}\\ 2\gamma_{xy}\end{bmatrix}$$

or $$[\sigma] = [E][\gamma]$$

compliance tensor **C** $\gamma_{ij} = C_{ijkl}\sigma_{kl}$

isotropic material
→ only two indep. elements!

$$[C] = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0\\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0\\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0\\ 0 & 0 & 0 & 1/G & 0 & 0\\ 0 & 0 & 0 & 0 & 1/G & 0\\ 0 & 0 & 0 & 0 & 0 & 1/G\end{bmatrix} \qquad G = \frac{E}{2+2\nu}$$

Young's modulus E and Poisson's ratio ν

G is the shear modulus

Continuums mechanics

energy: $$U = \int_V U' \, \mathrm{d}V \quad \text{with } U' = \frac{1}{2} E_{ijkl} \gamma_{ij} \gamma_{kl}$$

$$U = U_B + U_T + U_F$$

depending on geometry: bending (perpendicular to long dimensions)
+ tension (along long dimensions)
+ external

minimize U (variational methods) -> differential equation for **u**

$$u \rightarrow u + \delta u$$

e.g. Euler Bernoulli equation for a beam:

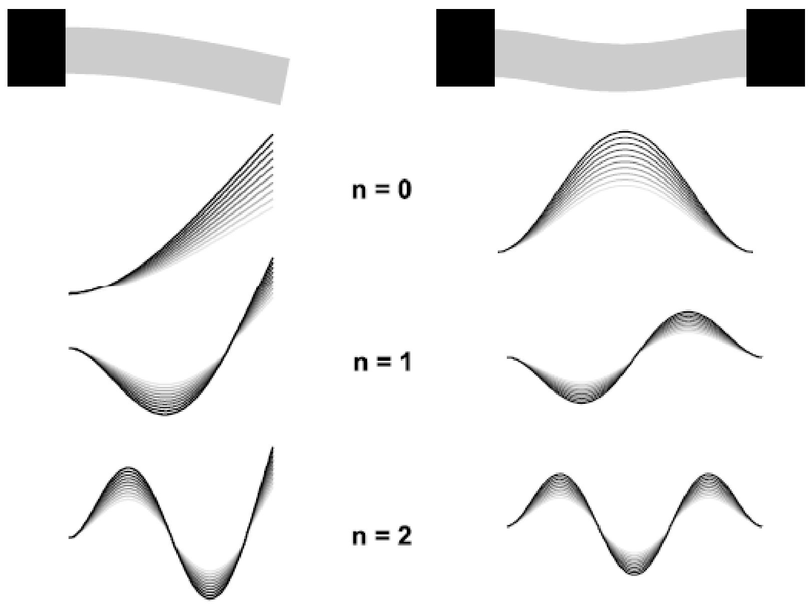
$$\rho A \frac{\partial^2 u}{\partial t^2} + D \frac{\partial^4 u}{\partial x^4} - T \frac{\partial^2 u}{\partial x^2} = F$$

Continuums mechanics

(doubly) clamped beam

neglect tension -> small amplitudes

$$\rho A \frac{\partial^2 u}{\partial t^2} + D \frac{\partial^4 u}{\partial x^4} - T \cancel{\frac{\partial^2 u}{\partial x^2}} = F$$



$$u_n(x) = c_4 \left(\sin\left(\beta_n \frac{x}{\ell}\right) - \sinh\left(\beta_n \frac{x}{\ell}\right) - \frac{\sin(\beta_n) + \sinh(\beta_n)}{\cos(\beta_n) + \cosh(\beta_n)} \left[\cos\left(\beta_n \frac{x}{\ell}\right) - \cosh\left(\beta_n \frac{x}{\ell}\right) \right] \right)$$

and

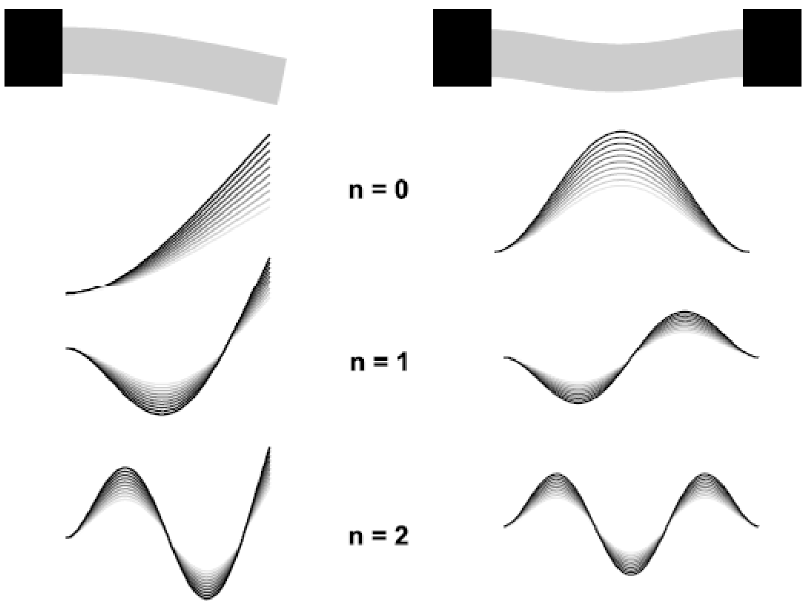
$$\cos(\beta_n)\cosh(\beta_n) - 1 = 0, \qquad \omega_n = 2\pi f_n = \beta_n^2 \ell^{-2} \sqrt{D/\rho A}.$$

Continuums mechanics

(doubly) clamped beam

neglect tension -> small amplitudes

$$\rho A \frac{\partial^2 u}{\partial t^2} + D \frac{\partial^4 u}{\partial x^4} - \cancel{T \frac{\partial^2 u}{\partial x^2}} = F$$



Hermitean

$$m_{\mathrm{eff}} \ddot{u}(\mathbf{r}, t) = -\gamma \dot{u}(\mathbf{r}, t) + \mathcal{L}[u(\mathbf{r}, t)]$$

$$u(x,t) = \sum_n u^{(n)}(t) \xi_n(x)$$

-> eigenfunctions form complete orthogonal basis -> harmonic oscillators!

Mass: beam-mass only for ortho-normal ξ: $\ell^{-1} \int_0^\ell \xi_n^2 \, \mathrm{d}x = 1$

Continuums mechanics

Euler Bernoulli equation

Tension-dominated (guitar string)

$$\rho A \frac{\partial^2 u}{\partial t^2} + \cancel{D \frac{\partial^4 u}{\partial x^4}} - T \frac{\partial^2 u}{\partial x^2} = F$$

standard harmonic oscillator, usual solutions, i.e.

$$f_n = \sqrt{T/\rho A} \times (n+1)/2\ell$$

$$\xi_n(x) = \sqrt{2}\sin(\pi n x/\ell)$$

Harmonic oscillator, classical

differential equation: $\ddot{x}(t) + \frac{\omega_0}{Q}\dot{x}(t) + \omega_0^2 x(t) = \frac{F}{m}\cdot\cos(\omega t)$

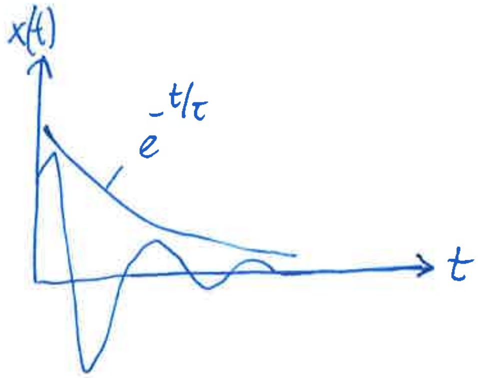
quality factor — eigenfrequency of un-damped oscillator — driving force

under-damped oscillator: $Q > 1/2$

no drive, $F=0$: $x(t) = e^{-t/\tau}\cdot[A\cdot\cos(\omega t) + B\cdot\sin(\omega t)]$

from initial conditions

ring-down time $\tau = \frac{2Q}{\omega_0}$ $\hat{=}$ bandwidth of the system $\approx \tau^{-1} = \frac{\omega_0}{2Q}$

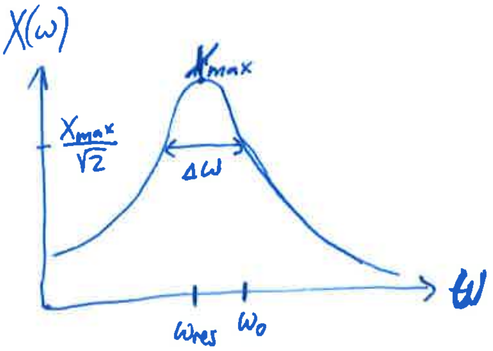


[over-damped: $Q < 1/2$ → double-exponential damping, no oscillations]

driven oscillator; $F \neq 0$: stationary solution $x(t) = X(\omega)\cdot\cos(\omega t - \alpha[\omega])$

amplitude

$$X(\omega) = \frac{F/m}{(\omega^2-\omega_0^2)^2 + \frac{1}{Q^2}\omega^2\omega_0^2}, \quad \tan(\alpha) = \frac{1}{Q}\cdot\frac{\omega\omega_0}{\omega_0^2-\omega^2}$$



$\omega_{res} = \omega_0\cdot\sqrt{1-\frac{1}{2Q^2}}$

$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$ → bandwidth [for $Q \gg 1$]

Energy loss per period: $\frac{\Delta E}{E} \approx \frac{2\pi}{Q}$ [for $Q \gg 1$]

⇒ prospective device: energy loss / pumping $> k_B T$ (otherwise thermally driven)

⇒ power consumption $\approx \frac{k_B T}{\tau} = \frac{k_B T\omega_0}{2Q} \approx 10^{-17}\,W \ll$ electronic devices

quantum harmonic oscillator

no damping.

Hamiltonian $\hat{H} = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2} m\omega_0^2 \hat{x}$

creation and annihilation operators $\hat{a}^+ = \frac{m\omega_0 \hat{x} - i\hat{p}}{\sqrt{2m\hbar\omega_0}}$, $\hat{p} = -i\hbar\frac{\partial}{\partial x}$

$$\hat{a} = \frac{m\omega_0 \hat{x} + i\hat{p}}{\sqrt{2m\hbar\omega_0}}$$

properties: $\left.\begin{aligned} \hat{a}^+|n\rangle &= \sqrt{n+1}\cdot|n+1\rangle \\ \hat{a}|n\rangle &= \sqrt{n}\cdot|n-1\rangle \\ [\hat{a},\hat{a}^+] &= 1 \end{aligned}\right\}$ $\hat{H} = \hbar\omega_0(\hat{n}+1/2)$ with $\hat{n} = a^+a$ (number operator)

$E_n = \hbar\omega_0(n+1/2)$, $n \in \mathbb{N}_0$

- zero-point motion: ground state $n=0$, $E_0 = \frac{1}{2}\hbar\omega_0$

$$\Rightarrow \underline{\Delta x} := \sqrt{\langle x^2\rangle} = \sqrt{\langle 0|\hat{x}^2|0\rangle} = \sqrt{\frac{\hbar}{2m\omega_0}}\cdot\left(\langle 0|(a^+ + a)^2|0\rangle\right)^{1/2} = \sqrt{\frac{\hbar}{2m\omega_0}}\left(\langle 0|(a^+)^2|0\rangle + \langle 0|a^+a|0\rangle + \langle 0|aa^+|0\rangle + \langle 0|a^2|0\rangle\right)^{1/2}$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega_0}}(a^+ + a)$$

$$= \sqrt{\frac{\hbar}{2m\omega_0}}\Big(\underbrace{\langle 0|2\rangle}_{=0} + \underbrace{\langle 0|\cancel{a}a^+ + 1|0\rangle}_{\text{commutator}} + \langle 0|\cancel{aa^+}|0\rangle + \langle 0|\underbrace{(a|0\rangle)}_{0}\Big)^{1/2} \overset{\langle 0|0\rangle = 1}{=} \underline{\underline{\sqrt{\frac{\hbar}{2m\omega_0}}}}$$

omitted

- Coupling between electronic (two-level) system and phonons (bosonic field)

Example: coherent oscillations between electronic qubit and a mechanical oscillator, O'Connell et al., Nature 464, 697 (2010)

Formally: $\hat{H} = H_{el} + H_{field} + H_{interaction}$ with $H_{el} = \frac{1}{2}\hbar\omega_{el}\cdot\hat{\sigma}_z$ $\longleftrightarrow$ $|1\rangle$, $\updownarrow \frac{1}{2}\hbar\omega_{el}$, $\updownarrow \frac{1}{2}\hbar\omega_{el}$, $|0\rangle$

$H_{field} = \hbar\omega_0\cdot\hat{n} = \hbar\omega_0 a^+a$ (const. $\frac{1}{2}\hbar\omega_0$ omitted)

$$H_{int} = \underbrace{\frac{\hbar\Omega}{2}}_{\text{interaction strength}}\underbrace{(a^+ + a)}_{\sim\hat{x}}\cdot\underbrace{(\sigma^+ + \sigma)}_{\text{ladder operators} \,\hat{=}\, \text{polarization}}$$

compare to electric field: $\Delta E = \vec{E}\cdot\vec{p}$ (dipole coupling)

rotating frame: $H_0 = H_{el} + H_{field}$ $\Rightarrow a^+ \to e^{iH_0t/\hbar}a^+e^{-iH_0t/\hbar}$ $\to$ with commutation relations

$\Rightarrow \hat{a}^+(t) = a^+\cdot e^{i\omega_0 t}$; $\hat{a}(t) = a\cdot e^{-i\omega_0 t}$; $\sigma^+(t) = \sigma^+\cdot e^{i\omega_{el}t}$; $\sigma(t) = \sigma\cdot e^{-i\omega_{el}t}$ and $[a^+,\sigma^+] = [a^+,\sigma] = \ldots = 0$

$$\Rightarrow \underline{H_{int}(t)} = \frac{\hbar\Omega}{2}\left[a^+\sigma^+e^{i(\omega_0+\omega_{el})t} + a\sigma^+e^{i(\omega_{el}-\omega_0)t} + a^+\sigma e^{-i(\omega_{el}-\omega_0)t} + a\sigma e^{-i(\omega_{el}+\omega_0)t}\right] \underset{\uparrow}{=} \underline{\underline{\frac{\hbar\Omega}{2}\left[a\sigma^+ + a^+\sigma\right]}}$$

- neglect fast $\sim 2\omega$ terms

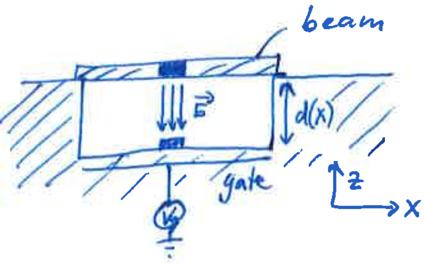
| Jaynes Cummings Hamiltonian, intuitive (swap)! |

on blackboard:

Coupling mechanisms: Actuation / detection by electrical means

→ capacitive (see below)
→ magneto-motive (see below)
→ piezoelectric / resistive (e.g. scanning probe experiments)
→ non-linear elements (SETs, SQUIDs, ...)
→ rf-resonators
→ ...

Capacitively:



1 area/volume element: $E_{charge} = \frac{1}{2}\frac{Q^2}{C} - Q \cdot V_g$

$$= \frac{1}{2}\frac{C^2 V_g^2}{C} - C V_g^2 = -\frac{1}{2} C V_g^2$$

$Q = C \cdot V_g$

force on this element: $f(x) = -\frac{\partial E_c}{\partial z} = \frac{1}{2} V_g^2 \cdot \frac{\partial C}{\partial z}$

simplest version: plate capacitors (transl. invariant along x and wide → negligible stray fields)

$$\Rightarrow C(z) = C(d) = \frac{\varepsilon \cdot \overset{\text{area}}{A}}{d(x)} \quad ; \quad d(x) = d_0 - \underset{\text{deformation}}{u(z)}$$

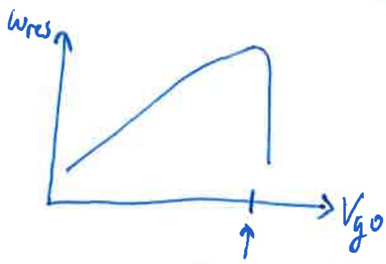
Drive: $V_g(t) = V_{g0} + \delta V_g \cdot e^{-i\omega_d t}$

$$\Rightarrow f(x) = \frac{1}{2} V_{g0}^2 \frac{\partial}{\partial d}\left(\frac{\varepsilon A}{d}\right) + 2 \cdot \frac{1}{2} V_{g0} \cdot \delta V_g \cdot e^{-i\omega_d t} \cdot \frac{\partial}{\partial d}\left(\frac{\varepsilon A}{d}\right) + \mathcal{O}(\delta V_g)$$

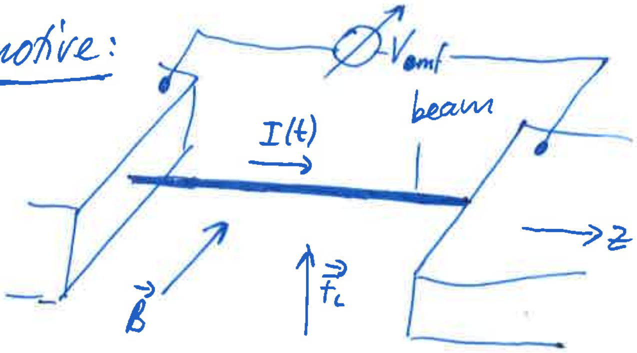
$$\simeq \underbrace{-\frac{\varepsilon A}{2 d(t)^2} \cdot V_{g0}^2}_{\text{tune } f} + \frac{\varepsilon A}{d(t)^2} V_{g0} \cdot \delta V_g \cdot \underbrace{e^{-i\omega_d t}}_{\text{oscillates in time}}$$

- small forces: $d(t) \approx d_0$ → homogeneous $f(x,t) = f(t)$
- larger forces: $d(t) = d(x,t)$ → $f = f(x,t)$ → requires exact $u(x,t)$, ...

↳ intuitive / qualitative: f → tension → larger restoring force → larger ω_{res}



"snap-in" → material gets deformed to another average position magnetomotive:



excitation
- current $\vec I(t)$ through beam
- large (!) magnetic field $\vec B \perp \vec I$

$\Rightarrow$ Lorentz force $\vec F_L \propto \vec I \wedge \vec B$

↳ oscillation

Detection: electromagnetic force → measurable voltage $V_{emf}(t)$

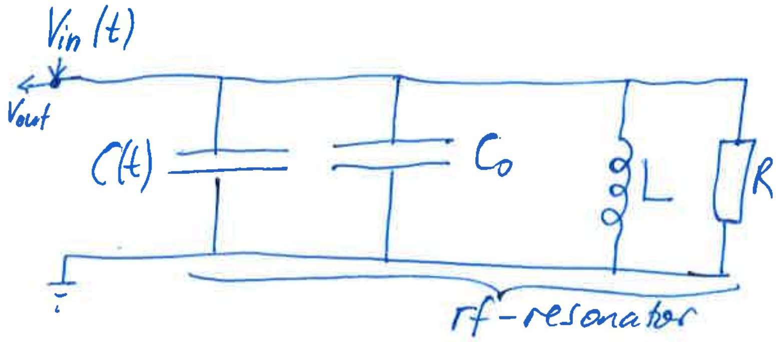
↳ $V_{emf}(t) = \underbrace{\frac{d\Phi_m}{dt}}_{\text{change in magnetic flux}} = \underbrace{B}_{\text{const.}} \cdot \int \frac{\partial \overset{\text{position}}{u}(z,t)}{\partial t}$

↳ resonance → maximum u → maximum V_{emf}

SQUIDS: (see intro) current depends strongly on magnetic flux through loop → oscillating beam changes loop area: $A = A(t)$

SET: oscillating beam capacitively coupled (see above) to SET, SET is used as very sensitive electrometer.

rf-resonator:



$C(t)$: oscillating capacitance of a beam, e.g. to backgate

↳ coupled to an LCR-resonator given by C_0, L, R:

↳ changes in resonance frequency and phase

↳ read-out by reflection of $V_{in}(t)$ into $V_{out}(t)$

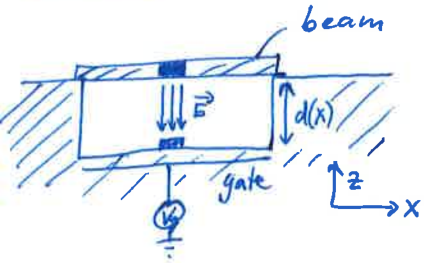
↳ very sensitive to changes in $C_{tot} = C_0 + C(t) \Rightarrow f_{res}(t), \varphi(t)$

on blackboard:

Coupling mechanisms: Actuation / detection by electrical means

→ capacitive (see below)
→ magneto-motive (see below)
→ piezoelectric / resistive (e.g. scanning probe experiments)
→ non-linear elements (SETs, SQUIDs, ...)
→ rf-resonators
→ ...

Capacitively:



1 area/volume element: $E_{charge} = \frac{1}{2}\frac{Q^2}{C} - Q\cdot V_g$

$$= \frac{1}{2}\frac{C^2V_g^2}{C} - CV_g^2 = -\frac{1}{2}CV_g^2$$

$Q = C\cdot V_g$

force on this element: $f(x) = -\frac{\partial E_c}{\partial z} = \frac{1}{2}V_g^2\cdot\frac{\partial C}{\partial z}$

simplest version: plate capacitors (transl. invariant along x and wide → negligible stray fields)

$$\Rightarrow C(z) = C(d) = \frac{\varepsilon\cdot \overset{\text{area}}{A}}{d(x)} \; ; \; d(x) = d_0 - \underset{\text{deformation}}{u(z)}$$

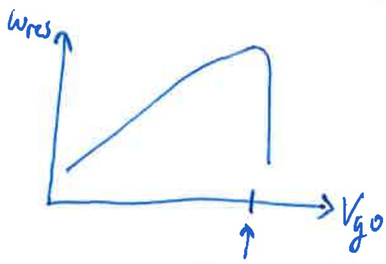
Drive: $V_g(t) = V_{g0} + \delta V_g \cdot e^{-i\omega_d t}$

$$\Rightarrow f(x) = \frac{1}{2}V_{g0}^2\frac{\partial}{\partial d}\left(\frac{\varepsilon A}{d}\right) + 2\cdot\frac{1}{2}V_{g0}\cdot\delta V_g\cdot e^{-i\omega_d t}\cdot\frac{\partial}{\partial d}\left(\frac{\varepsilon A}{d}\right) + O(\delta V_g)$$

$$\simeq \underbrace{-\frac{\varepsilon A}{2d(t)^2}\cdot V_{g0}^2}_{\text{tune } f} + \frac{\varepsilon A}{d(t)^2}V_{g0}\cdot\delta V_g\cdot \underbrace{e^{-i\omega_d t}}_{\text{oscillates in time}}$$

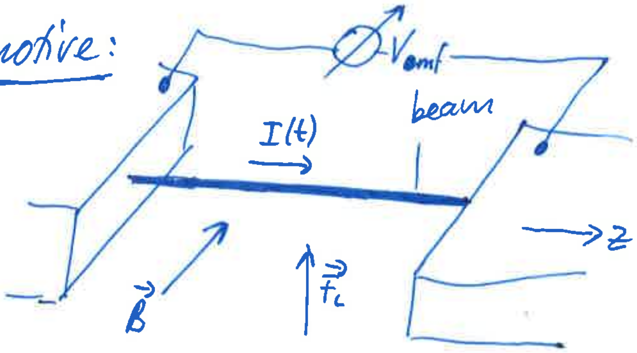
- small forces: $d(t) \approx d_0$ → homogeneous $f(x,t) = f(t)$
- larger forces: $d(t) = d(x,t)$ → $f = f(x,t)$ → requires exact $u(x,t)$, ...

↳ intuitive / qualitative: f → tension → larger restoring force → larger ω_{res}



"snap-in" → material gets deformed to another average position

magnetomotive:



excitation:
- current $\vec{I}(t)$ through beam
- large (!) magnetic field $\vec{B} \perp \vec{I}$

$\Rightarrow$ Lorentz force $\vec{F}_L \propto \vec{I} \wedge \vec{B}$

↪ oscillation

Detection: electromagnetic force → measurable voltage $V_{emf}(t)$

↪ $V_{emf}(t) = \underset{\substack{\uparrow \\ \text{change in} \\ \text{magnetic flux}}}{\frac{d\phi_m}{dt}} = \underset{\substack{\uparrow \\ \text{const.}}}{B} \cdot \int \frac{\partial u(\overset{\text{position}}{z},t)}{\partial t}$

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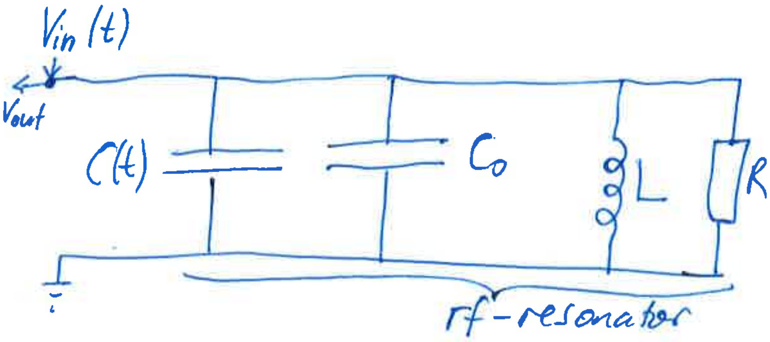

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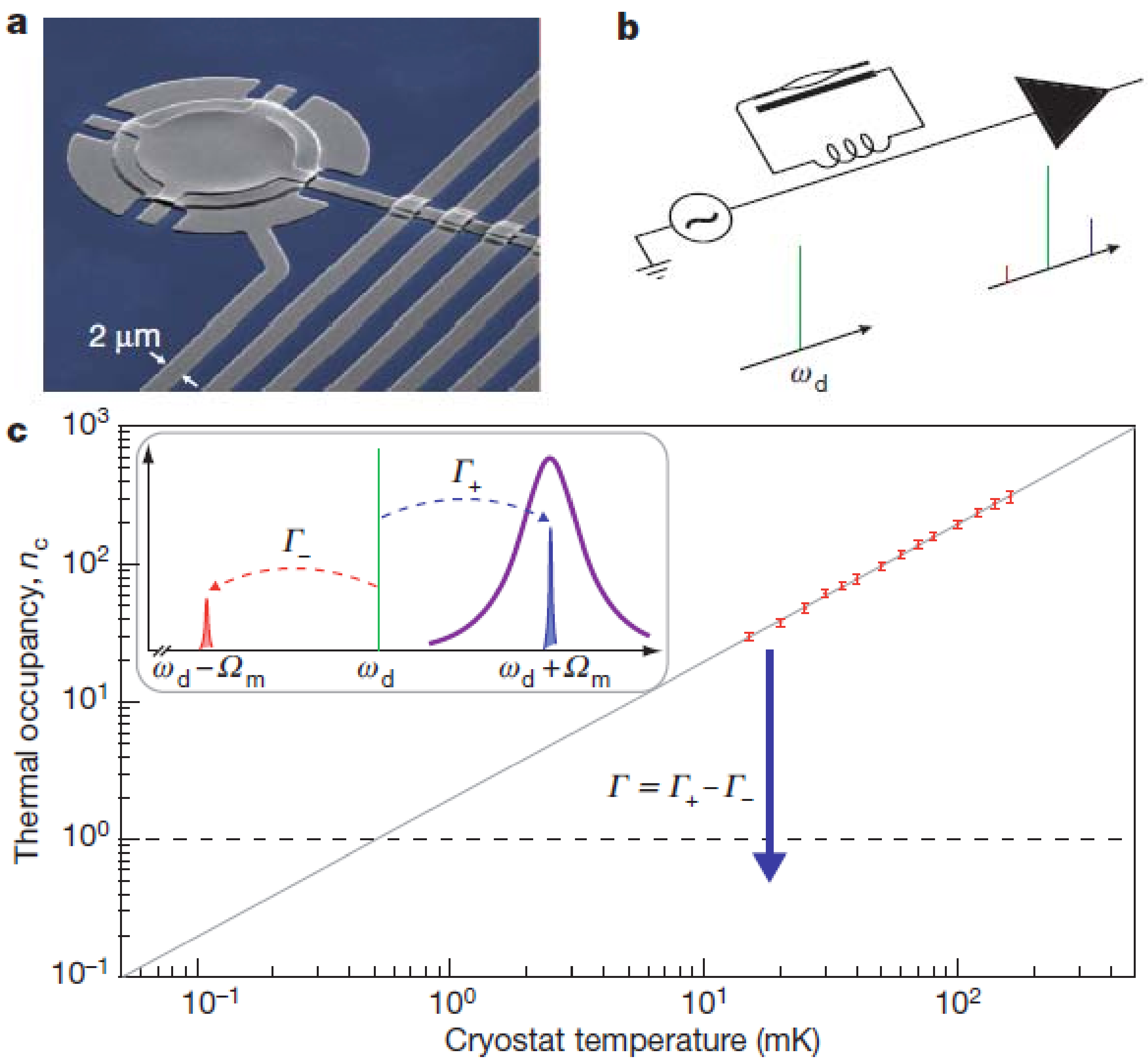
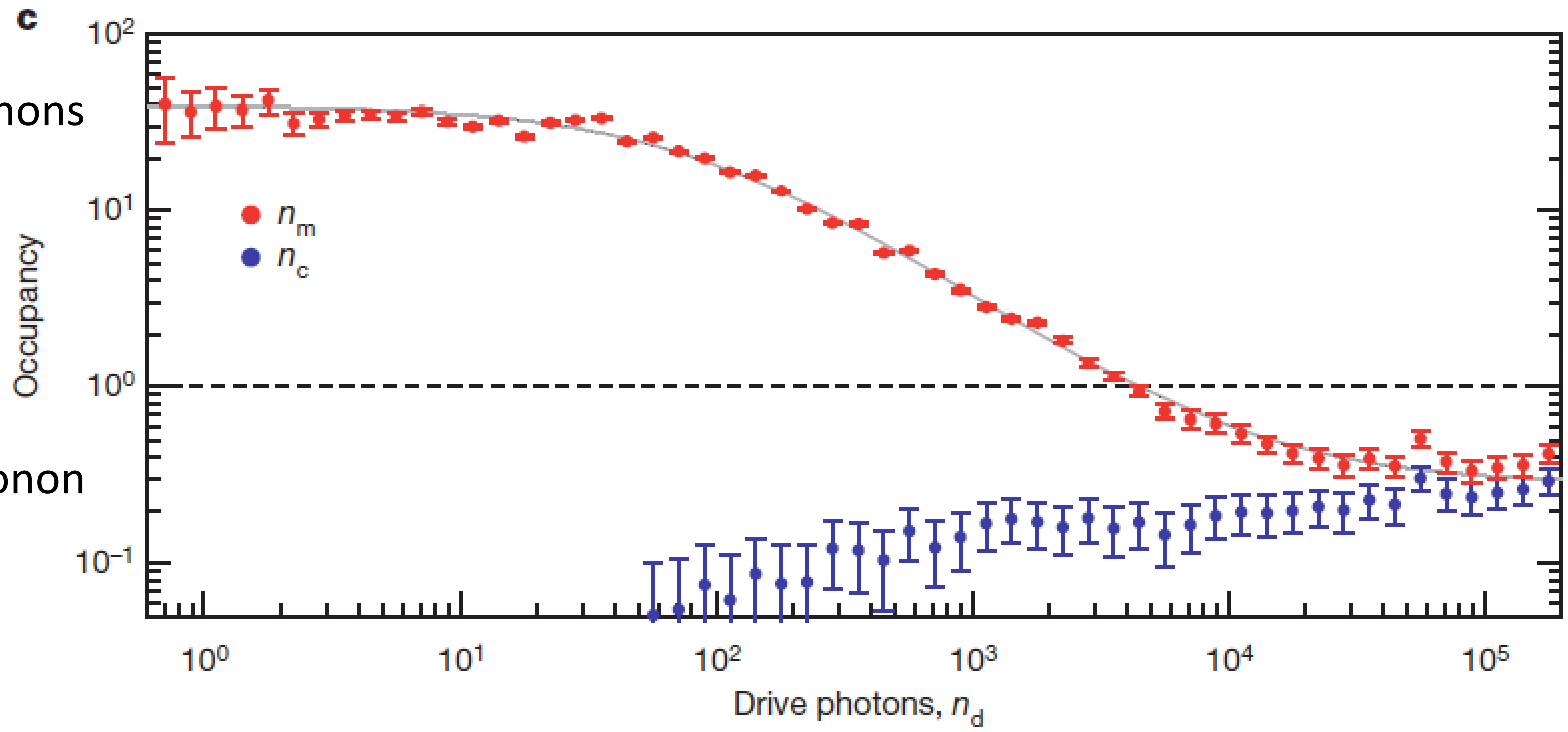
Electrical side-band cooling of a resonator



Teufel *et al.*, Nature 475, 359 (2011)

Electrical side-band cooling of a resonator

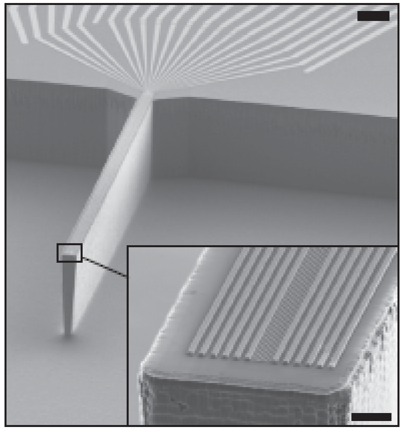
40 phonons

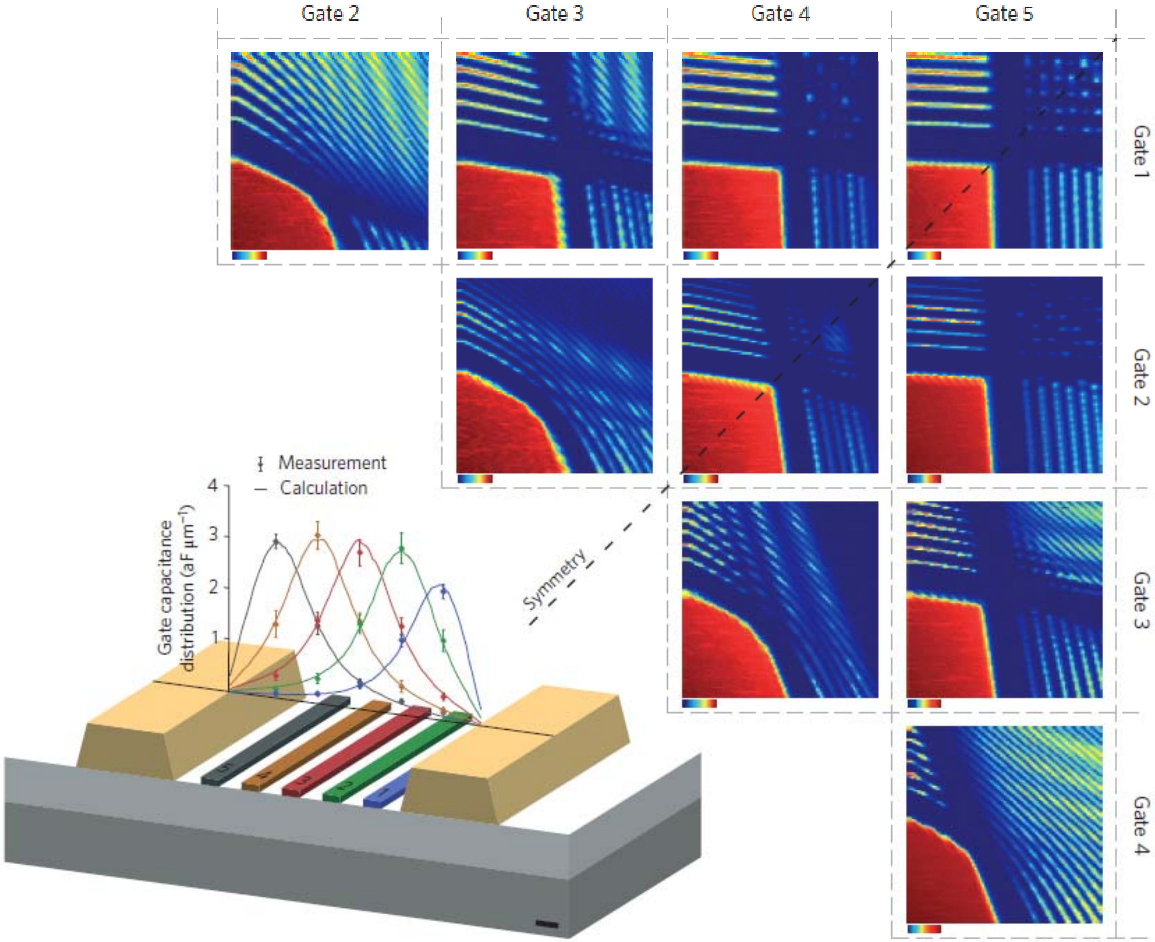


<1 phonon

Teufel *et al.*, Nature 475, 359 (2011)

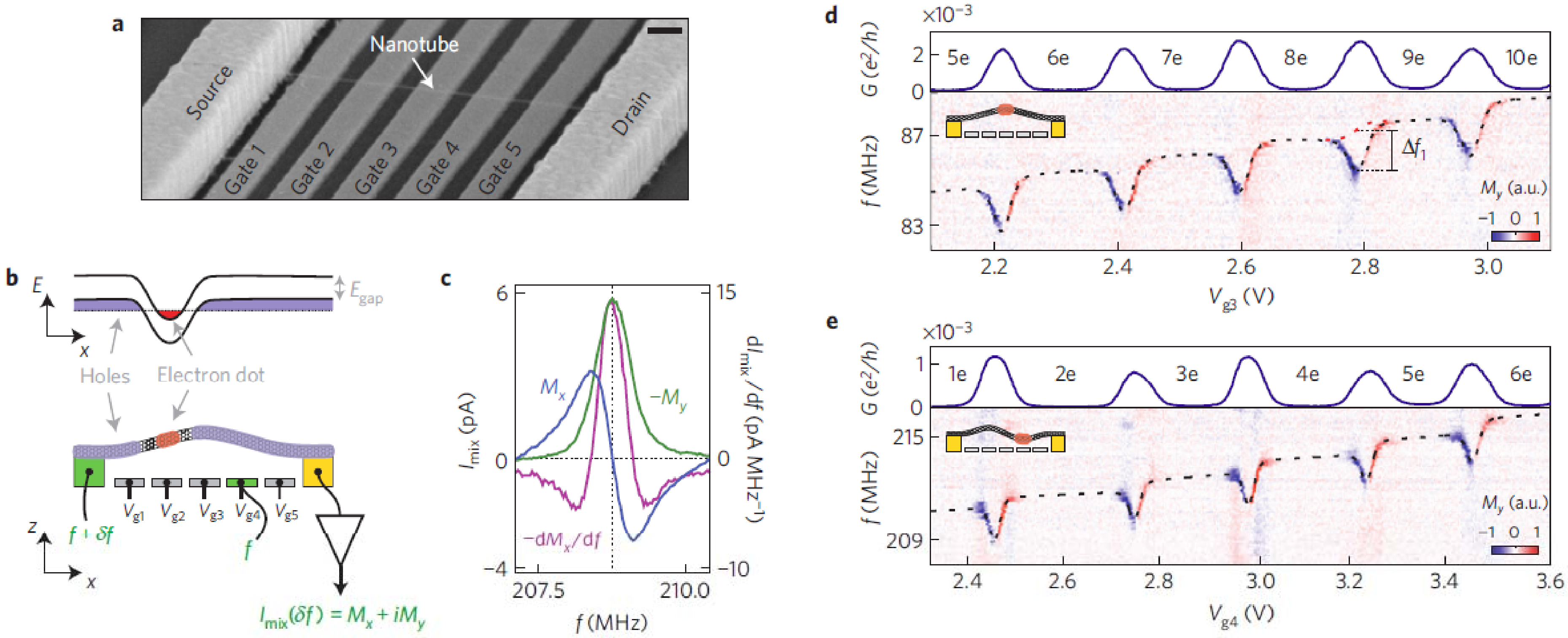
Tailoring the electron-phonon interaction





Waissman *et al.*, Nature Nano 8, 569 (2013)

Tailoring the electron-phonon interaction



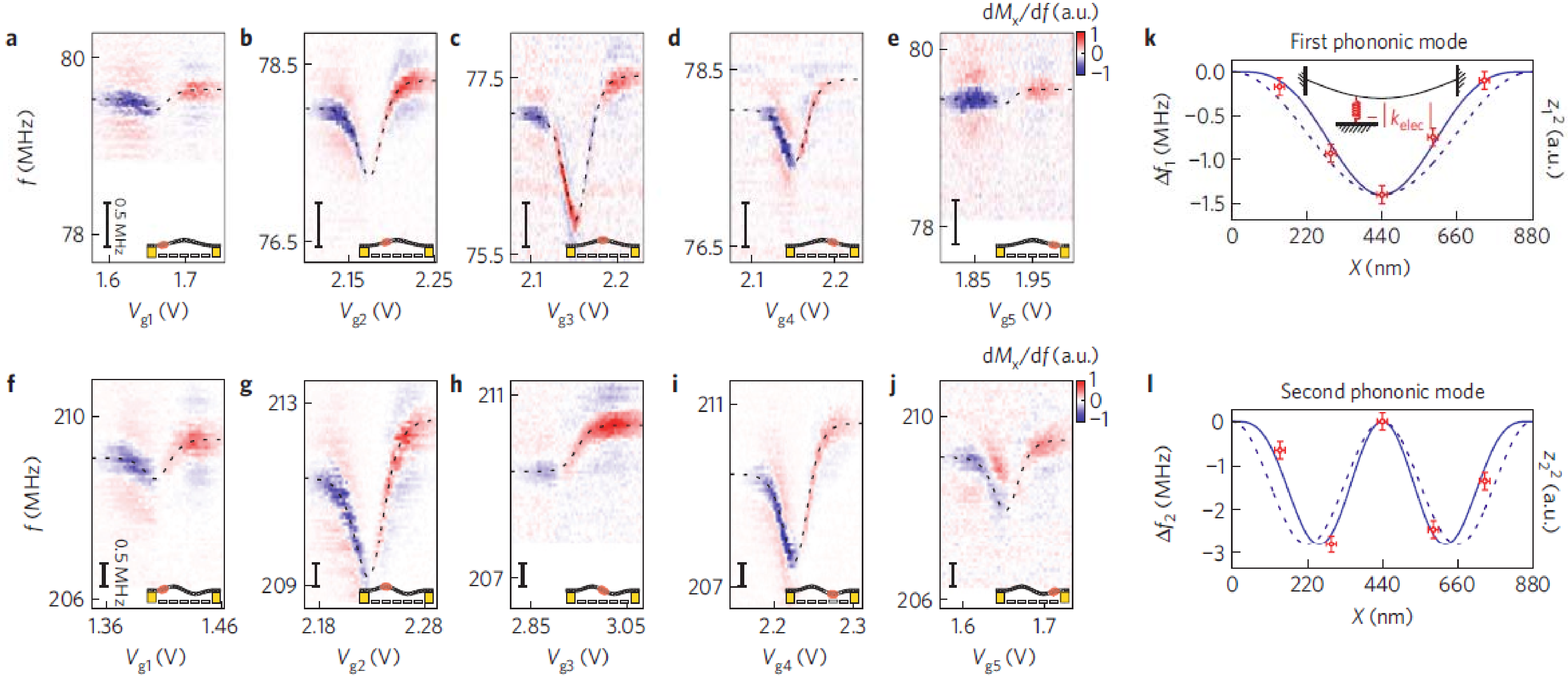
$$\partial^2 I / \partial V_{SD} \partial V_g$$

Detection: frequency mixing

Dynamic softening by tunneling to and from leads -> like a (negative) spring constant at the QD position

Benyamini *et al.*, Nature Phys. 10, 151 (2014)

Tailoring the electron-phonon interaction



Then: double QD!

Benyamini *et al.*, Nature Phys. 10, 151 (2014)