A1: Early quantum mechanics: Planck's law
Show schematically the power spectral density of the radiation emitted by a black body for two different temperatures as a function of the wavelength. (0.5 points)

A2: Free particles
What is the solution to the Schrödinger equation for a given energy E in free space in one dimension? (0.5 points) What is the degeneracy? (0.5 points) What is the momentum and the velocity of a particle in each state? (0.5 points)

A3: Confined particles
An electron is confined to a square in 2D with zero potential inside and infinite potential energy outside (infinite walls). Calculate the eigenfunctions. (1 point) What is the degeneracy of the 3 lowest energy levels? (1 point)

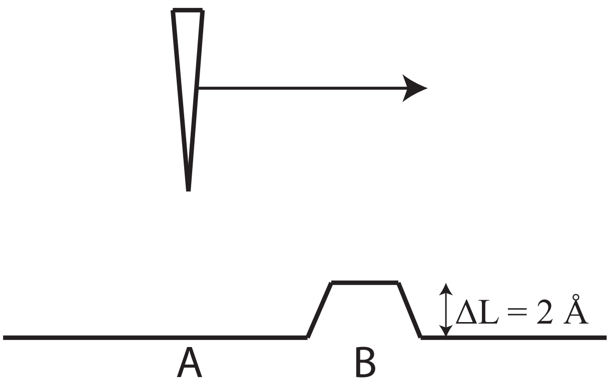
A4: Scattering problems: the principle of the Scanning Tunneling Microscope
A scanning tunneling microscope (STM) measures the tunneling current (which is proportional to the tunneling transmission T) between a sharp tip and the surface of another material. Consider a sharp tip made of Tungsten operating in the low-bias regime so that the energy for an electron to leave the structure is given by the work function of Tungsten $\phi_{\mathrm{w}} = 4.5\,\mathrm{eV}$, i.e., $V_0 - E = \phi_{\mathrm{w}}$. We neglect here the work function of the target material.

(a) In the lecture, the tunneling transmission through a barrier in the weak tunneling ($L\sqrt{2m(V_0-E)}/\hbar \gg 1$) limit was derived as

$$T = 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)\mathrm{e}^{-2L\sqrt{2m(V_0-E)}/\hbar}.$$

Rewrite the exponential part of this equation in the form e^{-L/L_0} and determine the decay constant L_0 for the Tungsten tip. (0.5 points)

(b) Let us assume that the tip follows the trajectory indicated in the diagram above a sample surface across a surface with an object of height $\Delta L = 2\,\mathring{A}$, made of the same material as the rest of the surface, at the position B. What is the ratio of the tunneling currents at position B and position A? (1 point)



A5: Harmonic oscillator
A harmonic oscillator at time $t = 0$ is at state

$$\psi(t=0) = \frac{1}{\sqrt{3}}\left(\Phi_0 + \Phi_1 + \Phi_2\right),$$

where Φ_n is the n-th eigenstate of the harmonic oscillator. Calculate i) $\psi(t)$, (1 point) ii) $\langle E(t)\rangle$ (1 point) and iii) $\langle x(t)\rangle$. (1 point) Use the ladder operators for the calculation!

A6: Spherical harmonics and hydrogen atom

(a) We consider a spin-less particle which only moves on the surface of a sphere (no radial degree of freedom) with the Hamiltonian $H = \frac{\hat{L}}{2mr^2}$. What are the energy eigenvalues and the degeneracy? (1 point)

(b) What is the wavelength of light that is absorbed in the optical transition between the state (3,1,0) to state (2,0,0) in a hydrogen atom? The indices are the usual quantum numbers (n,l,m). Do not account for any relativistic or other corrections. (1 point)

A7: Spin 1/2 The z-component of a spin one half prepared in state was measured and was found to be $\hbar/2$.

(a) Afterwards the the x-component of the wave-function was measured. What were the possible measurement outcomes and with what probability? (0.5 points)

(b) Now the z-component is measured again. What were the possible measurement outcomes and with what probability? (0.5 points) The spin operators are defined the following way: $S_i = \frac{\hbar}{2}\sigma_i$, where

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(c) What would have been the outcome without the first measurement? (0.5 points)