A1: Collisional Cross Section

(a) A beam of fast atoms moves through a cold gas, and is scattered by it. Show that the number $N(x)$ of fast atoms in the beam after a distance x follows the differential equation $N'(x) = -nN(x)\sigma$, where n is the density of the background gas and σ the collisional cross section.

(b) Solve this differential equation in order to find the number of fast atoms which remain in the beam after a distance L.

(c) A beam of fast helium atoms is scattered in a cold helium gas at 0.1 mbar and 300 K. After 30 cm only about 30% of the beam arrives at a detector. Estimate the He–He scattering cross section; estimate the radius of a helium atom.

A2: New definition of the kg

Currently, the kilogram is defined as the mass of the International Prototype Kilogram (IPK), a Pt-Ir cylinder created in 1889 and held in a secure basement of a laboratory near Paris. At the time of its creation, several copies were made and distributed to other countries. Periodic comparisons of the mass of the IPK and its copies have shown variations on the order of 10 μg (realtive variation of 10^{-5}). As most other SI units are defined by fundamental physical constants which can be measured everywhere and anytime, a new definition of the kg is required. One proposal is to define the kg as a specific number of ^{12}C atoms (the definition of the atomic mass unit is such that the mass of one mole of ^{12}C) is exactly 12 g, which ties the kg definition to a number, i.e. Avogadro's constant, N_A with no uncertainty. Thus, N_A needs to be measured as accurately as possible before it can be fixed exactly. So far, the best measurement of N_A has been made using a 1 kg single-crystal sphere of ultrahigh purity ^{28}Si. Measurement of N_A using such a sphere is reduced to knowledge of the molar mass of ^{28}Si (measured to be $M_W = 27.976\,926\,5325$ g/mol with a relative uncertainty of 6.8×10^{-11}), the number of Si atoms in a unit cell ($n = 8$ with no uncertainty; a unit cell is a box which builds up the complete crystal when shifted periodically), the lattice constant (measured by x-ray interferometry to be $a = 543.102\,0504 \times 10^{-12}$ m with a relative uncertainty of 1.6×10^{-8}), and a measurement of the macroscopic density of the sphere, ρ. The macroscopic density can be determined from measurements of the sphere mass and diameter. N_A can then be written in terms of these quantities as

$$N_A = \frac{M_W n}{\rho a^3}. \quad (1)$$

Please round all solutions to three significant digits.

(a) How many ^{28}Si atoms are needed to create a 1 kg sphere, if $N_A = 6.022\,141\,79 \times 10^{23}$?

(b) What is the volume and diameter of a 1 kg sphere of ^{28}Si?

(c) Provided that ρ can be measured arbitrarily well (with zero uncertainty), which of the remaining parameters (M_W or a) will dominate the relative uncertainty of N_A? What is the relative uncertainty of N_A with respect to this parameter?

For the next questions, assume that there is no uncertainty in the values of M_W and a.

(d) The new kg definition, and therefore N_A, are required to have a relative uncertainty smaller than 10^{-8} in order to be useful. What fraction of contamination from ^{29}Si would cause a mass increase that such that the specification cannot be reached? (Each ^{29}Si atom takes the place of a ^{28}Si atom)

(e) Aside from ^{29}Si substitutional defects, there will be other defects such as C, O, B, ^{30}Si, and vacancies (missing atoms) that take the place of ^{28}Si atoms. Estimate the maximum concentration of vacancies which still allows a relative uncertainty as specified.

(f) Unterminated Si surfaces always result in a thin layer of SiO_2 around the surface when exposed to oxygen. Estimate the maximum thickness this oxide layer can have so that the uncertainty specification is not nulled. Neglect the contribution due to the increase in mass and consider only the increase in sphere diameter.

(g) Slight variations in temperature during measurements will cause variations in the measured diameter because of thermal expansion. What is the maximum allowable temperature change to meet the relative uncertainty specification? (The thermal expansion coefficient for Si near 300 K is $\alpha = \Delta l/(l\Delta T) = 2.6\,\mu$m/(m K))