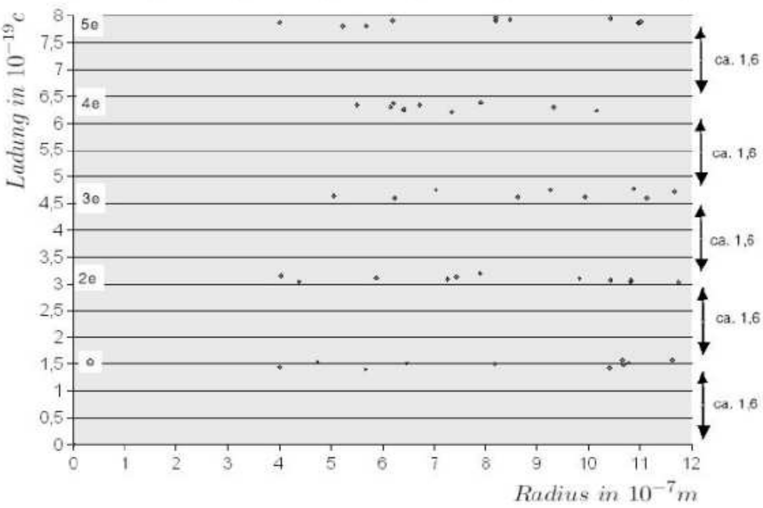
A1: Electric charge comes quantized

The goal of Millikan's experiment is to determine the absolute size of the "quantum of charge", which is the unit charge carried by electrons and protons. For this, tiny droplets of oil are charged with a small but random amount of charge, and their trajectories as they fall through air are observed in a microscope: first (a) without external electric field, then (b) with a vertically oriented electric field.



(a) In the absence of an electric field, an oil droplet of radius r and density ρ_{oil} quickly reaches its terminal velocity v_{t} when falling through air of viscosity η_{air} and density ρ_{air}. This terminal velocity can be calculated considering the buoyant force (Auftrieb) in air and Stokes' formula for the viscous drag of a sphere, $F = 6\pi\eta_{\text{air}} r v_{\text{t}}$. Calculate the droplet radius from the observed terminal velocity.

(b) Once a specific droplet's terminal velocity has been measured, a vertical electric field E is switched on and adjusted exactly such that the droplet's fall is stopped and it remains floating in the microscope's field of view. Calculate the absolute charge Q in the droplet as a function of the observed quantities (E, v_{t}) and the known macroscopic properties of oil and air.

(c) In the above graph the results of a typical Millikan experiment are shown. Interpret these results.

A2: Classical Electromagnetic Waves

The wave equation for electromagnetic waves can be derived from Maxwell's equations in the absence of free charges and currents;

$$\nabla \cdot \mathbf{D} = 0, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t},$$

where $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$ in a perfectly linear and isotropic medium. The permittivities ϵ and μ are related to the index of refraction and the speed of light in vacuum by $\epsilon\mu = (n/c)^2$.

(a) Derive the electromagnetic wave equation

$$\nabla^2 \mathbf{E} - \left(\frac{n}{c}\right)^2 \frac{\partial^2}{\partial t^2} \mathbf{E} = 0$$

by considering $\nabla \times (\nabla \times \mathbf{E})$ and using the vector identity $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$. Assume the medium is perfectly linear and isotropic.

(b) Show that a wave of the form $\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_o f(\mathbf{k} \cdot \mathbf{x} - \omega t)$ (where $\mathbf{k}$ is the wave-vector and ω is the angular frequency) is a solution of the electromagnetic wave equation if $\omega = \frac{c}{n} k$ $(k = |\mathbf{k}|)$. A relation between frequency (or energy) of a wave and its wave-vector is commonly called a *dispersion relation* and is frequently encountered in wave phenomena.