Homework problem sheet 3

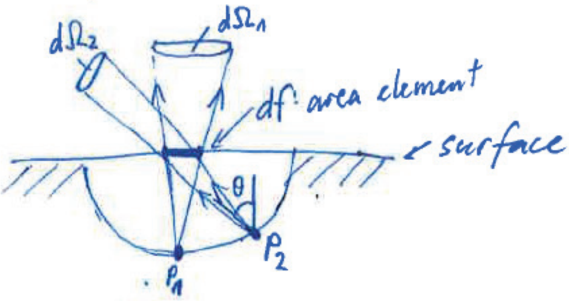
A1

energy density : u , speed of light (photons) : c

solid angle (Raumwinkel) of a sphere : 4π

$\Rightarrow$ power spectral density of emitted radiation: $S^*(\nu,T) = \frac{c}{4\pi} u(\nu,T)$

a)



Intuitive: the radiation emitted throug a small hole in the surface is proportional to the area of the hole in the direction of the emitted particles.

$$\tilde{df} = df \cdot \cos(\theta)$$

$\Rightarrow$ emitted power per solid angle is $\underline{\underline{S^* \cdot \cos(\theta)}}$

b)

$$S = \underbrace{\int_0^\infty d\nu}_{\text{integrate full spectrum}} \underbrace{\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta}_{\text{half a sphere}} \underbrace{\cos(\theta)}_{\text{from a)}} \cdot \frac{c}{4\pi} u(\nu,T) \cdot \underbrace{\sin(\theta)}_{\text{solid angle element: } d\Omega = \sin(\theta)\cdot d\theta \cdot d\varphi}$$

only half space

$$= \frac{c}{4\pi} \cdot \frac{8\pi h}{c^3} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \cdot \underbrace{\int_0^{\pi/2} \sin(\theta)\cos(\theta) d\theta}_{\frac{1}{2}\sin^2\theta \Big|_0^{\pi/2} = \frac{1}{2}} \cdot \int_0^\infty \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} d\nu = \frac{2\pi h}{c^2} \int_0^\infty \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} d\nu$$

$$\underset{\substack{x = \frac{h\nu}{kT} \\ \nu = \frac{kT}{h}\cdot x \\ d\nu = \frac{kT}{h}\cdot dx}}{=} \frac{2\pi h}{c^2} \cdot \int_0^\infty \frac{(kT/h)^3 \cdot x^3}{e^x - 1} \cdot \underbrace{kT/h \cdot dx}_{d\nu} = \frac{2\pi h}{c^2} \left(\frac{kT}{h}\right)^4 \cdot \underbrace{\int_0^\infty \frac{x^3}{e^x - 1} dx}_{\text{look up: } \pi^4/15} = \underline{\underline{\underbrace{\frac{2\pi^5 k_B^4}{15 \cdot c^2 h^3}} \cdot T^4 \quad \propto T^4}}$$

Stefan - Boltzmann constant:

$\underline{\underline{\sigma = 5.67 \cdot 10^{-8} \frac{W}{K^4 m^2}}}$

c) - Emitted power from the sun:

$$P_{sun} = \underbrace{S}_{\text{emitted power per area}} \cdot \underbrace{4\pi R_{sun}^2}_{\text{surface area}} = 4\pi R_{sun}^2 \cdot \sigma T_{sun}^4$$

- power arriving from the sun on the area A at a distance D (earth):

$$P_{earth} = A \cdot \frac{P_{sun}}{4\pi D^2} = \sigma T_{sun}^4 \cdot \left(\frac{R_{sun}}{D}\right)^2 \cdot A$$

- losses in the conversion to electricity:

$$\underline{P_{el}} = P_{earth} \cdot \underbrace{\eta_{atm} \cdot \eta_{conv} \cdot \eta_{BW}}_{\text{"efficiency" of solar cell} \approx 20\%} = \underline{\underline{1.39\ MW}}$$

d) Power emitted from the sun: $P_\odot = \underbrace{S}_{\text{emitted power/area}} \cdot \underbrace{A_\odot}_{\text{surface of the sun}} = \underbrace{\sigma}_{\text{Stefan-Boltzmann}} \cdot T_\odot^4 \cdot 4\pi R_\odot^2$

Fraction of light enering the eye: $\tilde{P}_{eye} = P_\odot \cdot \frac{A_{eye}}{4\pi D^2} = P_\odot \frac{(d/2)^2 \pi}{4\pi D^2}$

Factor to account for atmoshere, spectral efficiency of the eye...: $\eta = 0.4$

$\Rightarrow$ power entering the eye: $P_{eye} = \eta \cdot \sigma T^4 \cdot \cancel{4\pi} R_\odot^2 \cdot \frac{d^2/4\, \pi}{\cancel{4\pi} D^2}$

$\Rightarrow$ number of photons: $\underline{\underline{r}} = P_{eye}/h\nu_{max} = P_{eye} \frac{\lambda_{max}}{hc} = \frac{\lambda_{max}}{4hc} \eta \sigma T^4 \frac{R_\odot^2 d^2 \pi}{D^2} \approx \underline{\underline{4.38 \cdot 10^{15}\ \text{photons/s}}}$

e)

$$P_{max} = 1 \frac{mW}{cm^2} = 10 \frac{W}{m^2}$$

$$\Rightarrow r_{max} = \underbrace{\frac{P_{max}}{h\nu}}_{\text{photon rate per area}} \cdot \underbrace{(d/2)^2 \pi}_{\text{area}} \approx 7.90 \cdot 10^{13}\ \text{photons/s} \ll r_{(d)}$$

$\Rightarrow$ it is dangerous to stare into the sun (ask Galileo!)

A2

a) $E_\gamma = \frac{hc}{\lambda} = 17.45\ keV$

b) complete backscattering: $\vartheta = \pi$ $(\cos = -1) \Longrightarrow \underline{\underline{\lambda_f}} = \lambda_i + \frac{2h}{mc} = \underline{\underline{0.07595\ nm}}$

c) $E_f = \frac{hc}{\lambda_f} = 16.34\ keV$

d) $p = \hbar k = \hbar \cdot \frac{2\pi}{\lambda} = \frac{h}{\lambda}$

$p_i = \frac{h}{\lambda_i} = 9.31 \cdot 10^{-24}\ \frac{kg \cdot m}{s}$

$p_f = \frac{h}{\lambda_f} = 8.72 \cdot 10^{-24}\ \frac{kg \cdot m}{s}$ (in opposite direction)