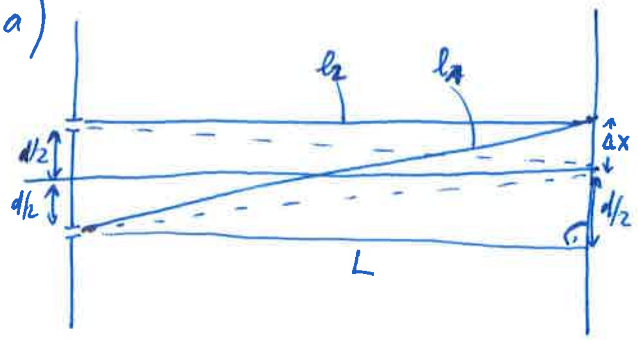
Physik III **Problem Sheet 4** ①

1a)



- first interference maximum in center ($\Delta x = 0$)
- second interference maximum: $\Delta l = l_2 - l_1 \overset{!}{=} n \cdot \lambda_{dB}$, $n = 1$

de Broglie wave length

$$l_1 \underset{\text{Pythagoras}}{=} \sqrt{L^2 + (d/2 + \Delta x)^2} = L \sqrt{1 + \left(\frac{d/2 + \Delta x}{L}\right)^2}$$

$$\underset{L \gg d/2 + \Delta x}{\approx} L \cdot \left(1 + \frac{1}{2}\left(\frac{d/2 + \Delta x}{L}\right)^2\right)$$

$$= L + \frac{1}{2L}\left(d^2/4 + d\Delta x + \Delta x^2\right)$$

$$l_2 = \sqrt{L^2 + (d/2 - \Delta x)^2} \underset{\text{similar to above}}{=} L + \frac{1}{2L}\left(d^2/4 - d\Delta x + \Delta x^2\right)$$

From plot: $\Delta x \approx 50 \mu m$

$$\Rightarrow \underline{\underline{\Delta l}} = l_2 - l_1 = \frac{1}{2L} \cdot 2d\Delta x = \underline{\underline{\frac{d}{L} \cdot \Delta x}}$$

$$\Rightarrow \underline{\underline{\lambda_{dB}}} = \Delta l = \underline{4 pm} \ll 0.7 nm = 700 pm$$ (diameter of buckyball)

b) momentum $p = \underset{\text{mass}}{m}\bar{v} = \hbar \underset{\text{wave vector}}{k} = \hbar \frac{2\pi}{\lambda_{dB}} = \frac{h}{\lambda_{dB}}$ $\Rightarrow \underline{\underline{\bar{v}}} = \frac{h}{m \lambda_{dB}} \underset{m_{C60} = 60 \cdot \frac{12 g}{N_A}}{\approx} \underline{\underline{138.5 \, m/s}}$

c) The average velocity in a classical gas (Maxwell-Boltzmann distribution) is

$$\bar{v} = \sqrt{\frac{8 k_B T}{m \pi}} \Rightarrow \underline{\underline{\lambda_{dB}}} \underset{\text{from b)}}{=} \frac{h}{m\bar{v}} = \frac{h}{m \cdot \sqrt{\frac{8kT}{m\pi}}} = h \cdot \sqrt{\frac{\pi}{8mkT}} \underset{m_{87} = \frac{87 g}{N_A};\ T = 10^{-7} K}{=} 9.3 \cdot 10^{-7} m = \underline{\underline{0.93 \mu m}} \gg \text{Rb atom}$$

2a) initial photon momentum : $p_i = \hbar k \; \left(= \hbar \frac{2\pi}{\lambda} = \frac{h}{\lambda}\right)$

final photon momentum : $p_f = 0$ (i.e. fully absorbed)

momentum conservation : momentum transfer to object per photon

$\Delta p_{ob} = p_i - p_f = \hbar k = \frac{h}{\lambda}$

number of photons per time (rate)

Radiation force $F_{rad} = \Delta p_{ob} \cdot r$

$= \frac{h}{\lambda} \cdot \frac{P}{h\nu}$ ← power

$= \frac{P}{\nu \cdot \lambda} = \frac{P}{c}$

→ radiation pressure (Force per area):

$$\underline{\underline{p_{rad}}} = \frac{F_{rad}}{A} = \frac{P}{c \cdot A} = \frac{1}{c} \cdot \underbrace{\frac{P}{A}}_{\substack{\text{power/area} \\ = \text{intensity}}} = \underline{\underline{\frac{I}{c}}}$$

↑ area

2b) a) can also be derived from the classical wave equations :

$I = \langle S \rangle = \frac{1}{2} |\vec{E}|^2 \cdot \varepsilon_0 \; ; \; |\langle p \rangle| = \frac{1}{2c} |\vec{E}|^2 \cdot \varepsilon_0 = \frac{\langle S \rangle}{c}$

↑ pointing vector

$\Rightarrow \underline{\underline{\Delta p_{ob}}} = \langle p \rangle = \frac{1}{2c} |\vec{E}|^2 \varepsilon_0 = \underline{\underline{\frac{I}{c}}}$

A good indication is that there is no $\hbar$ in the expressions.

2c) $p_{rad} = \frac{I}{c} = \frac{1}{c} \frac{P}{A} = \frac{1}{c} \frac{P}{(d/2)^2 \pi} = \underline{\underline{1.06 \cdot 10^{-3} \frac{N}{m^2} \; (Pa)}}$

$F_{rad} = p_{rad} \cdot A = \frac{P}{c}$ (only powers matters!)

↑ from c)

2d) $F_{spring} = k_s \cdot \Delta x \overset{!}{=} F_{rad} \Rightarrow \Delta x = \frac{F_{rad}}{k_s} = \frac{P}{k_s \cdot c} = 2.22 \cdot 10^{-8}\, m = \underline{\underline{22.2\, nm}}$

↑ spring constant