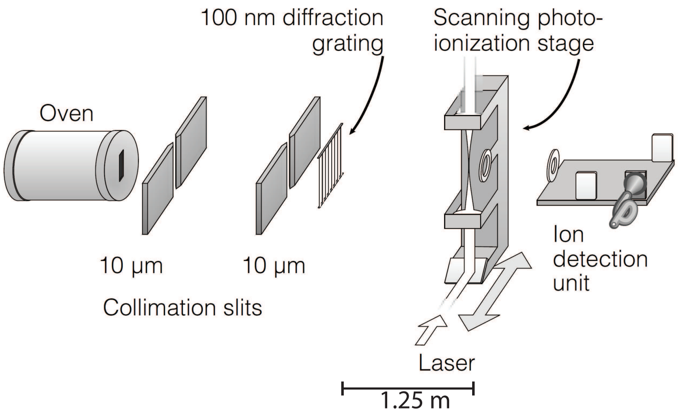
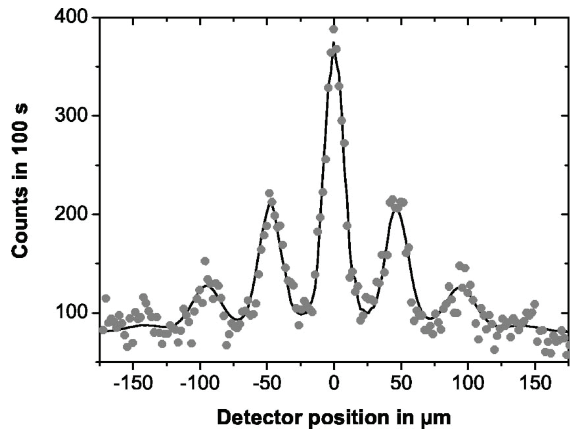
A1: de Broglie Wavelength

With modern experimental techniques, the wave nature of particles can be explored with large and complex objects. In this way, Zeilinger and coworkers were able to observe interference between C_{60} molecules ("Buckyballs"), see American Journal of Physics **71**, 319 (2003) and Nature **401**, 680 (1999). In their experiment, a beam of C_{60} molecules was directed from an oven to a diffraction grating with 100 nm slit spacing as shown in the figure below. At a distance of 1.25 m from the grating, the distribution of molecules perpendicular to the propagation direction was measured.





(a) From the observed interference pattern, calculate the de Broglie wavelength of the buckyballs. Compare this to the diameter of a buckyball.

(b) Using the observed de Broglie wavelength, calculate the average exit velocity of the buckyballs from the oven.

(c) By reducing the average velocity of the particles, the de Broglie wavelength can be increased. While extremely low temperatures cannot be reached at present with buckyballs, gases of alkali atoms at extremely low temperatures can be generated using laser cooling. Determine the average de Broglie wavelength of ^{87}Rb atoms with a temperature of 100 nK. Compare the de Broglie wavelength to the diameter of a ^{87}Rb atom.

A2: Radiation Pressure

Because photons carry momentum, they can exert a mechanical force when absorbed in a material. The force per area is called radiation pressure and plays an important role in today's research.

(a) Derive an expression for the radiation pressure by considering a beam of light with intensity I and wavelength λ fully absorbed by an object. Hint: force is change in momentum per time. Think of the beam as being composed of individual photons (arriving at a certain rate) that transfer momentum to the mirror.

(b) Even though you used the concept of photons to derive the radiation pressure force, comment on whether or not photons are necessary to describe radiation pressure.

(c) Calculate the radiation pressure exerted by a 1 W beam of light that is fully absorbed with wavelength 633 nm and diameter of 2 mm.

(d) Suppose this beam of light is focused to a diameter of 20 μm and absorbed by an AFM cantilever in vacuum. Neglect any heating and strain effects. If the spring constant of this cantilever is $k = 0.15\,\mathrm{N/m}$, how much is it displaced from its equilibrium position by the laser beam?