**A1: Infinite square well**

We again study the infinite square well potential:

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq L \\ \infty & \text{for } x < 0, x > L \end{cases}$$

with eigenfunctions

$$\phi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & \text{for } 0 \leq x \leq L \\ 0 & \text{for } x < 0, x > L \end{cases}$$

and eigen-energies $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$ for $n = 1, 2, 3, \ldots$.

(a) Show that these eigenfunctions are orthonormal, *i.e.*, $\int_{-\infty}^{\infty} \phi_n^*(x)\phi_{n'}(x)\mathrm{d}x = \delta_{nn'}$

(b) For a given wavefunction $\chi(x)$ find formally the decomposition into eigenfunctions $\phi(x)$, $\chi(x) = \sum_{n=1}^{\infty} c_n \phi_n(x)$, i.e., solve for $c_n$ by evaluating $\int_{-\infty}^{\infty} \phi_n^*(x)\chi(x)\mathrm{d}x$ and using the result of problem 2a.

(c) Assume that at time $t = 0$ the particle is confined to the right half of the well:

$$\psi(x, t=0) = \begin{cases} 0 & \text{for } x \leq L/2 \\ \frac{2}{\sqrt{L}} \sin\left(2\pi \frac{x - L/2}{L}\right) & \text{for } L/2 < x < L \\ 0 & \text{for } x \geq L \end{cases}$$

Express this initial state in terms of the eigenfunctions of the full well, *i.e.*, find the coefficients $a_n$ such that $\psi(x, t=0) = \sum_{n=1}^{\infty} a_n \phi_n(x)$. Pay special attention to the case $n = 2$.

(d) Find the time-dependent wavefunction $\psi(x, t)$ which satisfies the time-dependent Schrödinger equation (do not evaluate any occurring sums explicitly).

(e) What is the probability that a measurement of the energy would yield the value $E_1$?

(f) What is the expectation value of the energy? (Hint: $\sum_{n\,\text{odd}} \frac{n^2}{(n^2-4)^2} = \frac{\pi^2}{16}$).

**A2: Ehrenfest theorem**

A particle of mass $m$ moves along the $x$ axis in a potential $V(x)$. Newton's (classical) equation of motion is ("$ma = F$")

$$\frac{\mathrm{d}}{\mathrm{d}t} p = -\frac{\partial V(x)}{\partial x}.$$

Using Schrödinger's equation, demonstrate that it's quantum-mechanical equivalent called Ehrenfest theorem

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle \hat{p} \rangle = \langle -\frac{\partial V(x)}{\partial x} \rangle$$

holds for all quantum-mechanical states $\psi(x, t)$. *Hint:* use

$$\langle \hat{p} \rangle = \int_{-\infty}^{\infty} \psi^*(x, t) \left(-\mathrm{i}\hbar \frac{\partial}{\partial x}\right) \psi(x, t)\mathrm{d}x$$

and note that only the wave function $\psi(x, t)$ depends on time, but not the operator $\hat{p} = -\mathrm{i}\hbar\frac{\partial}{\partial x}$.