A1: Harmonic Oscillator

Consider a 1-dimensional harmonic oscillator potential as discussed in the lecture.

(a) Calculate the expectation values $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, and $\langle p^2\rangle$ for the *ground state*. In addition, show that the ground state is a minimum uncertainty state by calculating $\Delta x \Delta p$ where $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ and $\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$.

(b) A coherent state of a harmonic oscillator is defined such that for a (random) number α

$$\hat{a}_-\Psi_\alpha = \alpha\Psi_\alpha,$$

i.e. the state is an eigenstate of the lowering operator $\hat{a}_-$. These are interesting states because they are something like the 'most classical' states of a harmonic oscillator for which the expectation values for the momentum and position oscillate while maintaining minimal uncertainty in position and momentum. Please note that these states are not more (or less) coherent than any superposition state of the harmonic oscillator and that the name is more historically motivated. Using the ladder operators $\hat{a}_-$ and $\hat{a}_+ = \hat{a}_-^\dagger$ (with $[\hat{a}_-, \hat{a}_+] = 1$) calculate the expectation value $< H >_{\Psi_\alpha}$ for a state Ψ_α of the Hamiltonian

$$\hat{H} = \hbar\omega(\hat{a}_+\hat{a}_- + \frac{1}{2}).$$

Similarly, calculate $< H^2 >_{\Psi_\alpha}$ and with that the variance of the energy, $\Delta H = \sqrt{< H^2 >_\alpha - < H >_\alpha^2}$. Discuss briefly your findings.