A1: Classical Larmor precession

We use the classical expression for the evolution of a magnetic moment in a static magnetic field to understand an electron with a magnetic moment $\vec{\mu}$ in the presence of a magnetic field $\vec{B}$. In spherical coordinates $\vec{\mu}$ and $\vec{B}$ are given as

$$\begin{aligned}\vec{\mu} &= (\mu, \theta, \varphi)\\ \vec{B} &= (B, 0, 0)\end{aligned}$$

(a) What is the classical expression for the torque exerted by $\vec{B}$ on $\vec{\mu}$?

(b) Calculate the time evolution of the angular momentum $\vec{L} = -2m\vec{\mu}/g_{\mathrm{L}}e$, with m the electron mass, $-e$ it's charge, and g_{L} is the Landé-factor.

(c) Show that the trajectory of $\vec{L}$ is a circle with constant θ and L and find the rotation frequency. Draw schematically the trajectory in a unit sphere.

A2: Quantum Larmor precession

Imagine a particle of spin 1/2 in a uniform magnetic field pointing in the z-direction. The Hamiltonian has the following form:

$$\hat{H} = -\gamma \vec{B}\vec{S}.$$

The spin operators are defined the following way: $\hat{S}_i = \frac{\hbar}{2}\sigma_i$, where

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) The spin wavefunction is given at $t = 0$ by $\chi(t = 0) = \begin{pmatrix} a \\ b \end{pmatrix}$. Calculate the wavefunction $\chi(t)$ at time t.

(b) Calculate the time dependent expectation value $\hat{S}_x$.