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Exercises and Complements for the Introduction to Physics I  
for Students  
of Biology, Pharmacy and Geoscience

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Sheet 9 / November 12, 2019

**Solutions**

**Exercise 41.**

(a) The frequency is calculated by:

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = 0.13 \text{ Hz}$$

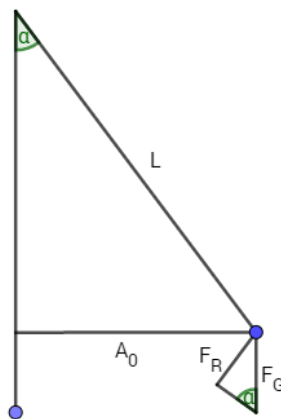
(b) The oscillation period is the reciprocal of the frequency:

$$T = f^{-1} = 7.8 \text{ s}$$

(c) The total distance  $s_{ges}$  of a period in x-direction can be divided into four sections. According to the sketch, each section  $A_0$  can be calculated using the sine:

$$A_0 = \sin(5^\circ) \cdot 15 \text{ m}$$

$$s_{ges} = 4 \cdot A_0 = 5.23 \text{ m}$$



(d) The  $x(t)$  equation can be formed as follows from the amplitude  $A_0$  and the angular frequency  $\omega$ :

$$x(t) = A_0 \cdot \cos(\omega \cdot t)$$

With  $A_0 = s_{\frac{1}{4}} = 1.31 \text{ m}$  and  $\omega = 0.81 \text{ s}^{-1}$  it follows:

$$x(t) = 1.31 \text{ m} \cdot \cos(0.81 \text{ s}^{-1} \cdot t)$$

(e) The velocity can be formed by the time derivative of the  $x(t)$  equation:

$$v(t) = \dot{x}(t) = -A_0 \cdot \omega \cdot \sin(\omega \cdot t)$$

With  $A_0 = 1.31 \text{ m}$  and  $\omega = 0.81 \text{ s}^{-1}$  it follows:

$$|v(5 \text{ s})| = |-1.31 \text{ m} \cdot 0.81 \text{ s}^{-1} \cdot \sin(0.81 \text{ s}^{-1} \cdot t)| = 0.84 \text{ m/s}$$

(f) The restoring force  $F_R$  can be calculated as follows according to the sketch:

$$F_R = F_G \cdot \sin(\alpha) = 8 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \sin(5^\circ) = 6.84 \text{ N}$$

### Exercise 42.

After the clock has run 12 hours, it shows 11.5 h. So, it made just  $11.5/12 = 95,8\%$  of the required pendulum motions, respectively the time of oscillation  $T_0$  is too big. To be on time, it must be:

$$T = T_0 \cdot 0.958$$

The equation for a mathematical pendulum (pendulum clock) describing the time of oscillation is:

$$T_0 = 2\pi \sqrt{\frac{l_0}{g}}$$

and from this it follows:

$$\frac{T}{T_0} = \sqrt{\frac{l'}{l_0}} = 0.958 \quad \Rightarrow \quad l' = l_0 \cdot 0.958^2 = 0.459 \text{ m}$$

### Exercise 43.

(a) The eigenfrequency of a mathematical pendulum is:

$$\omega_P = 2\pi/T_P = \sqrt{g/l}$$

The eigenfrequency of the combined system of spring and pendulum is:

$$\omega_F = 2\pi/T_F = \sqrt{D/m + g/l}$$

for the time of contact  $t$  it must be  $t = T_F/2$  (since the spring is considered as massless) and for  $T_F$ :

$$T_F = \frac{2\pi}{\sqrt{D/m + g/l}}$$

and consequently  $t$  is:

$$t = T_F/2 = \frac{1}{2} \cdot \frac{2\pi}{\sqrt{D/m + g/l}} = 0.32 \text{ s}$$

(b) Since the time of the oscillation of the pendulum (for small deflections) is independent of the angle and  $T_P = T_F \cdot \sqrt{2}$ , is the time of contact independent of  $\alpha$ .

#### Exercise 44.

(a) The following force is needed to bring the cuboid out of equilibrium by pushing it by the distance  $\Delta h$  into the water:

$$F = m_{water}g = V_{water}\rho_{water}g = A\Delta h\rho_{water}g$$

whereby the force is proportional to the deflection  $\Delta h$  (compare with the behavior of a spring) and the systems is oscillating harmonically.

(b) the constant of proportionality (spring constant):

$$c = \frac{F}{\Delta h} = A\rho_{water}g$$

The following equation describes the harmonic oscillation:

$$T = 2\pi\sqrt{\frac{m_{cuboid}}{c}} = 2\pi\sqrt{\frac{Ah\rho_{cuboid}}{A\rho_{water}g}} = 2\pi\sqrt{\frac{h\rho_{cuboid}}{\rho_{water}g}}$$

(c) In the case of a wooden sphere (instead of the cuboid) the cross-sectional area is not constant in height. As a result, the buoyant force is not proportional to the depth of immersion (deflection) and as a consequence the sphere is not oscillating harmonically. Therefore the result obtained in (b) is not valid for a sphere.

#### Exercise 45.

It is given that (script 108-6):

$$x(t) = c_0 e^{-\delta t} \sin(\omega t - \phi_0)$$

accordingly is:

$$\begin{aligned} x(t_0) &= c_0 e^{-\delta t_0} \sin(\omega t_0) \\ x(t_0 + 5T) &= c_0 e^{-\delta(t_0 + 5T)} \sin(\omega(t_0 + 5T)) = c_0 e^{-\delta(t_0 + 5T)} \sin(\omega t_0) \end{aligned}$$

The ratio is:

$$\frac{x(t_0 + 5T)}{x(t_0)} = \frac{1}{2} = e^{-5\delta T}$$

and therefore

$$\delta = \frac{\ln 2}{5T} = 0.0462 \text{ s}^{-1}$$