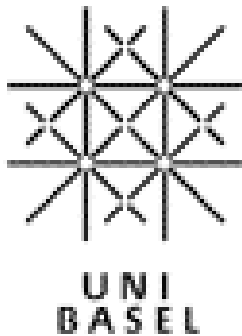


# *Surface Physics 2010*

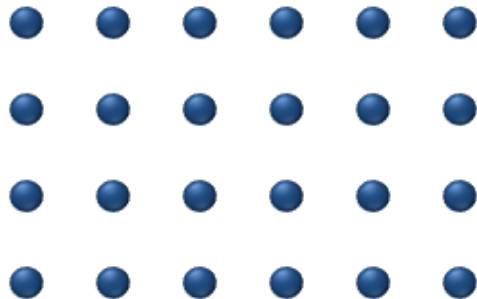
## Surface Diffusion



Lecturer: Dr. Enrico Gnecco  
NCCR Nanoscale Science

# Random-Walk Motion

- Thermal motion of an adatom on an ideal crystal surface:



- Thermal excitation  $\rightarrow$  the adatom can hop from one adsorption site to the next

- Mean square displacement at time  $t$ :

$$\langle \Delta r^2 \rangle = \nu a^2 t$$

$a$  = jump distance;  $\nu$  = hopping frequency

(Note that  $\nu t$  = number of hops!)

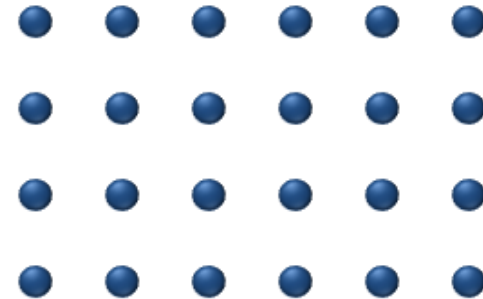
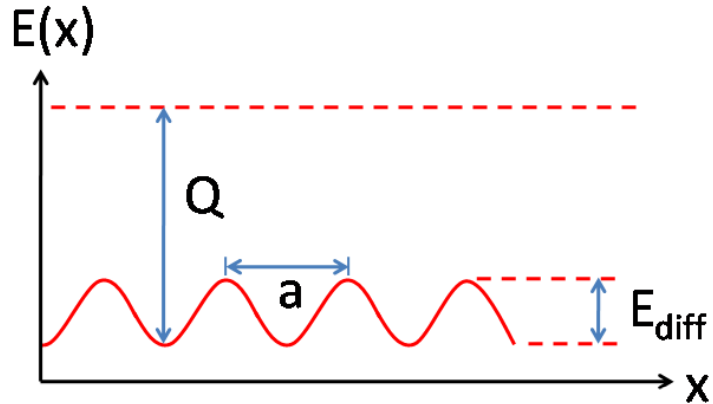
- **Diffusion coefficient:**

$$D = \frac{\langle \Delta r^2 \rangle}{z t} = \frac{\nu a^2}{z}$$

$z$  = number of first neighbors =  $\begin{cases} 2 \text{ in 1D diffusion} \\ 4 \text{ on a square lattice} \\ 6 \text{ on a hexagonal lattice} \end{cases}$

# Random-Walk Motion

- Hopping → surmounting a potential barrier



- Arrhenius law:

$$v = v_0 \exp\left(-\frac{E_{diff}}{k_B T}\right)$$

$v_0$  = oscillation frequency of the atom in the well;

$E_{diff}$  = barrier height

Typically  $E_{diff} \sim 5\text{-}20\%$  of  $Q$  (heat of desorption)

- For chemisorbed species:  $E_{diff} \gg k_B T$
- If  $E_{diff} < k_B T$ : 2D gas (only a few physisorbed species)

# Fick's Laws

- **Fick's First Law** (for 1D diffusion):

$$J = -D \frac{\partial c}{\partial x}$$

diffusion flux

concentration gradient

(flux  $\rightarrow$  region of lower concentration)

- **Fick's Second Law** (for 1D diffusion):

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

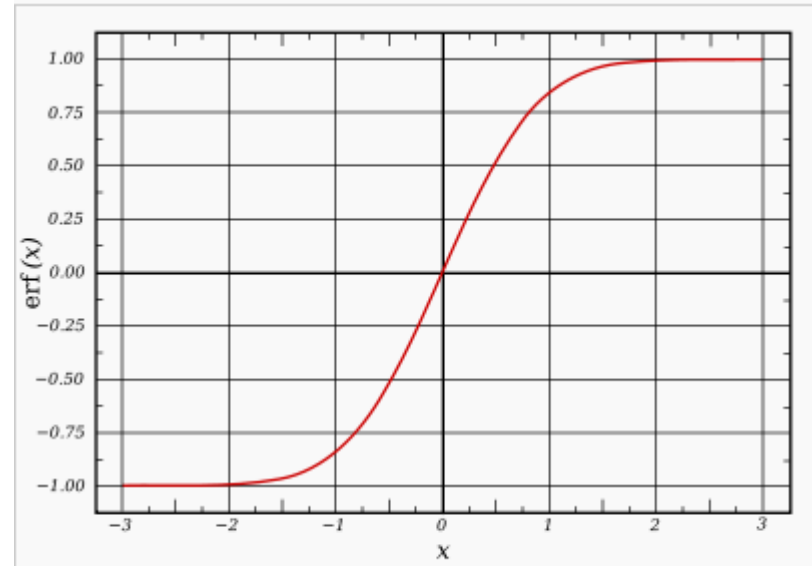
$\leftarrow$  from equation of continuity

- Analytical solutions can be found for specific initial and boundary conditions!

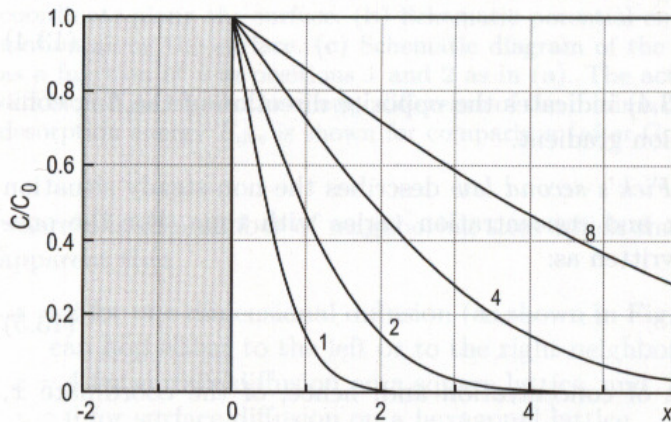
# Analytical Solutions of Fick's Laws

- We introduce the **error function**

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$$



- Source of constant concentration:



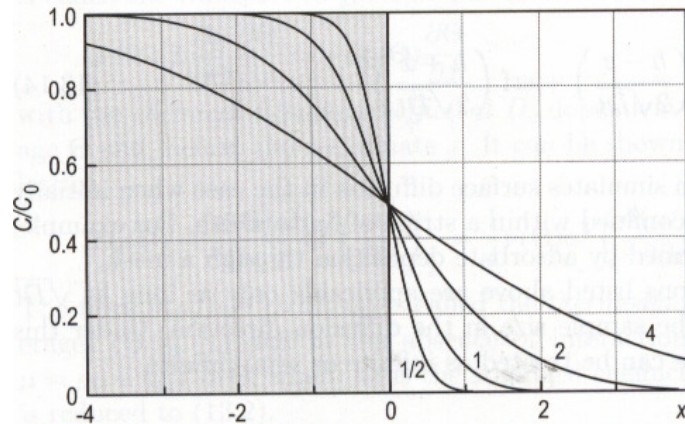
$$c(x, t) = c_0 \left[ 1 - \text{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \right]$$

$2\sqrt{Dt}$  : **diffusion length**

- Example: Submonolayer film with 3D islands supplying mobile adatoms

# Analytical Solutions of Fick's Laws

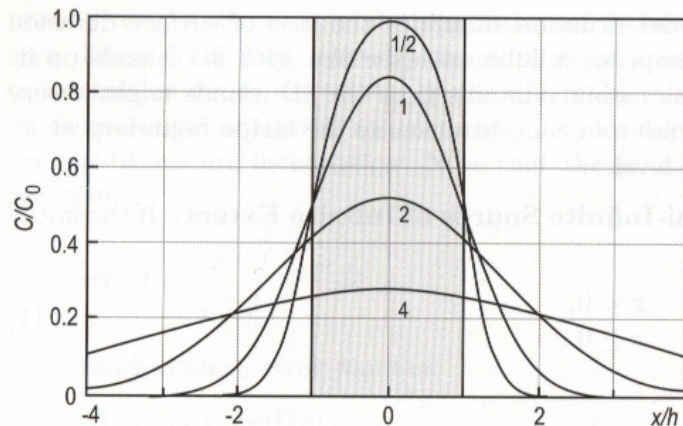
- Source of infinite extent:



$$c(x,t) = \frac{c_0}{2} \left[ 1 - \operatorname{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \right]$$

- Example: Submonolayer film

- Source of limited extent:



$$c(x,t) = \frac{c_0}{2} \left[ \operatorname{erf} \left( \frac{h-x}{2\sqrt{Dt}} \right) + \operatorname{erf} \left( \frac{h+x}{2\sqrt{Dt}} \right) \right]$$

- Example: Submonolayer film confined in a stripe of finite width

# Diffusion Mechanisms

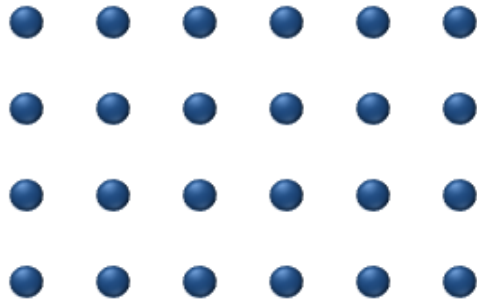
- Depending on the coverage  $\Theta$ :
  - Tracer diffusion (low  $\Theta$ )
  - Chemical diffusion (intermediate to high  $\Theta$ )



# Tracer Diffusion

- **Tracer Diffusion:**

- Low coverage (<0.01 ML)
- Individual adparticles



- Fick's first law is valid:

$$D = \frac{\langle \Delta r^2 \rangle}{z t}$$

- For an ensemble of many particles:

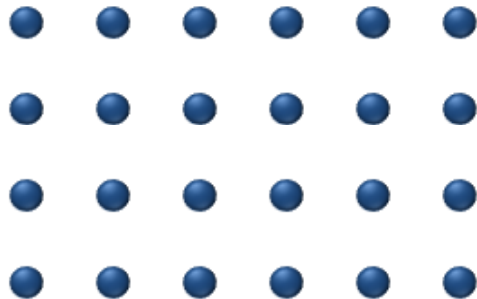
$$D = \frac{1}{z N t} \sum_i \langle \Delta r_i^2 \rangle$$

(no relation to radioactive tracers!)



# Chemical Diffusion

- **Chemical Diffusion:**
  - Higher coverage
  - Attraction or repulsion between adatoms



- Fick's first law can be generalized:

$$J = -D_c(\Theta) \frac{\partial \Theta}{\partial x}$$

chem. diff. coefficient

coverage

- Strong dependence on adsorbate coverage is expected, especially when ordered phases are formed (Naumovets-Vedula, 1986)

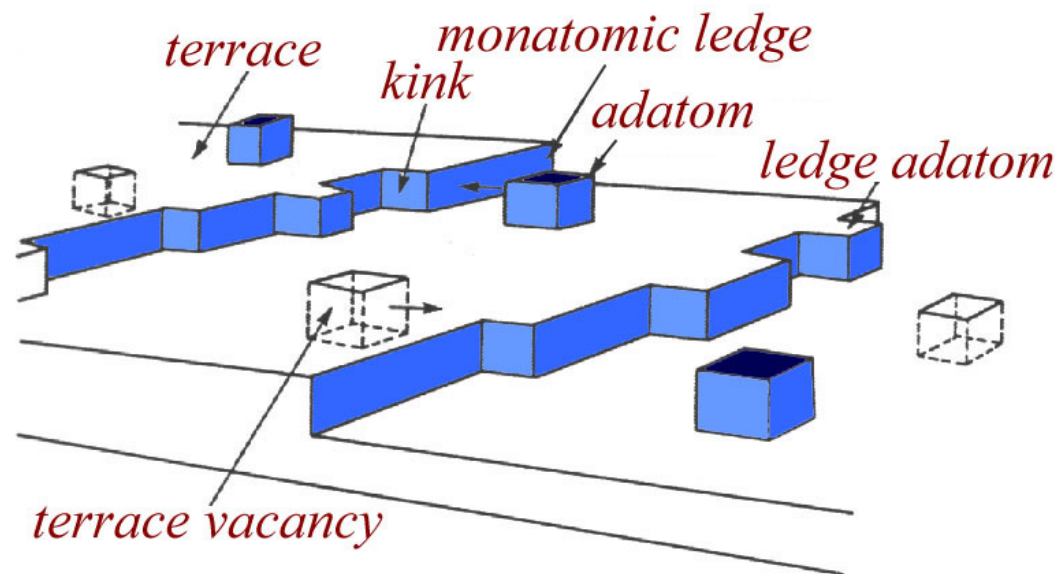
# Diffusion Mechanisms

- Depending on the landscape:
  - Intrinsic diffusion (no sources and traps)
  - Mass transfer diffusion (generation and/or trapping)



# Intrinsic Diffusion

- Adparticle motion is monitored within a single terrace → Spatial limit  $\sim 100$  nm
- In practice: no strong distinction from tracer diffusion



# Mass Transfer Diffusion

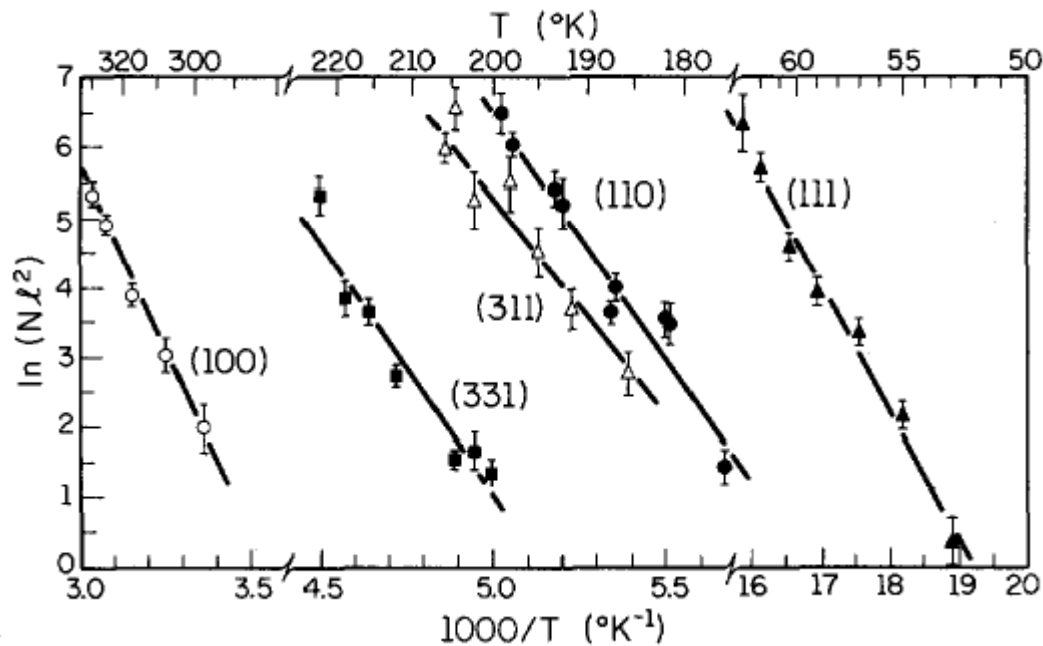
- Real surfaces contain defects (steps, kinks, adatoms or vacancy clusters, etc.)
- If average separation between defects < diffusion length  $\rightarrow$  number of mobile particles (and diffusion) become strongly temperature dependent
- If adatoms and substrate are the same chemical species:

$$D = \frac{v_0 a^2}{z} \exp\left(-\frac{\Delta G + E_{diff}}{k_B T}\right) \quad \Delta G = \text{energy of adatom formation}$$

( $\rightarrow$  two types of energy barriers!)

# Anisotropy of Surface Diffusion

- **Orientalional Anisotropy:** the diffusion coefficient depends on the orientation of the surface
- Example: Rh surfaces at different T (Ayrault & Ehrlich, JCP 1974)



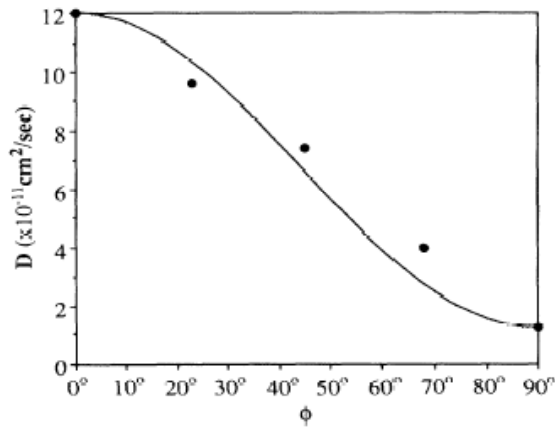
N: number of jumps in 3 min

ℓ: jump distance

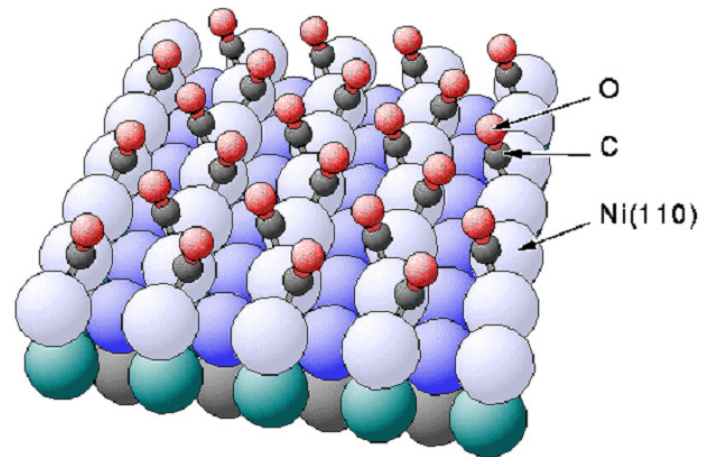
- At given T the differences in the diff. coeff. can be several orders of magnitude!

# Anisotropy of Surface Diffusion

- **Directional Anisotropy:** the diffusion coefficient depends on the direction at the surface
- Rectangular lattice → directional anisotropy (Xiao et al., PRL 1991)

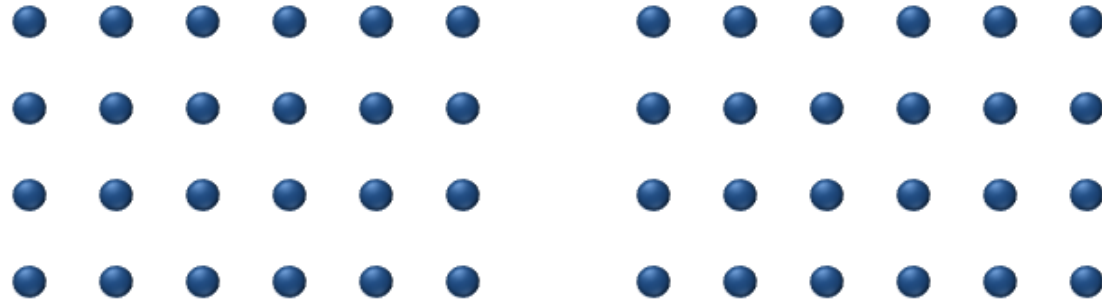


$$D(\phi) = D_x \cos^2 \phi + D_y \sin^2 \phi$$

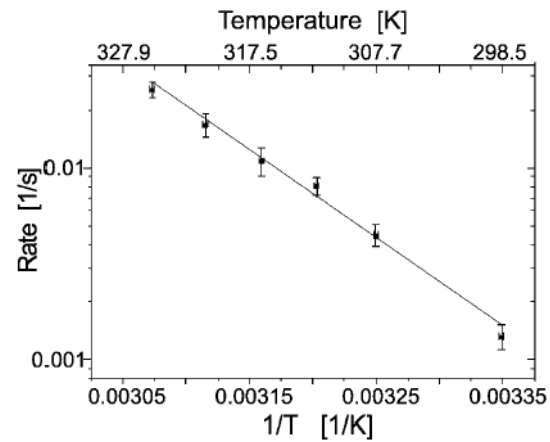
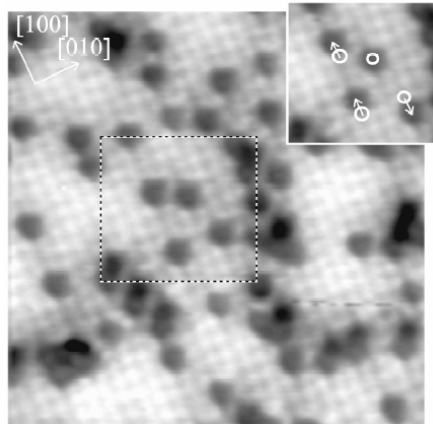


# Atomistic Mechanisms

## 1) Hopping mechanism:



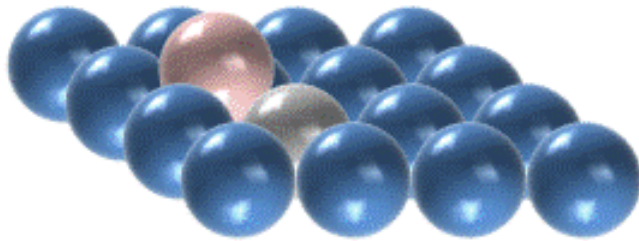
- Example: N adatoms on Fe(100) (Pedersen et al., PRL 2000)



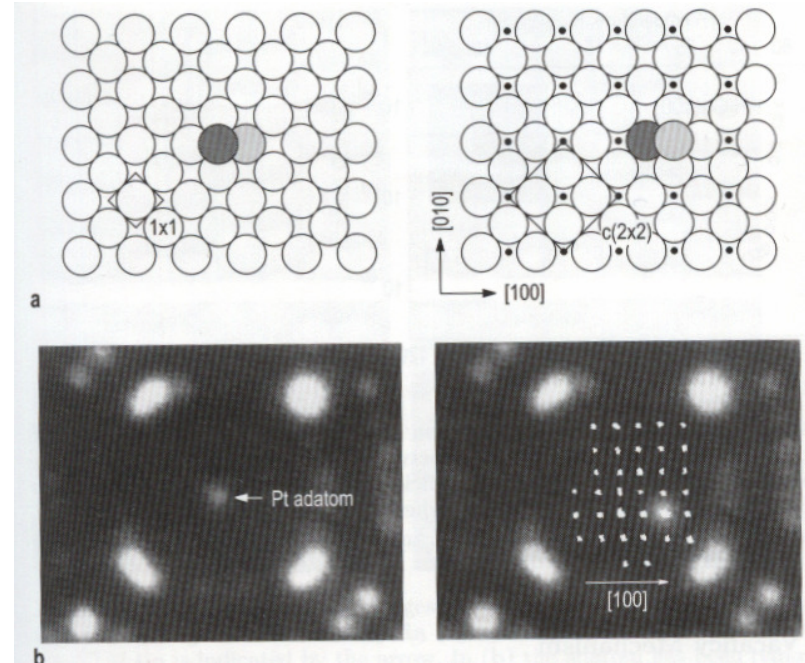
→ Arrhenius law with  $\nu \sim 10^{12} \text{ s}^{-1}$ ,  $E_{diff} = 0.92 \text{ eV}$

# Atomistic Mechanisms

## 2) Atomic exchange mechanism:



- Example: Pt adatom on Pt(100)  
(Kellogg, SSR 1994)



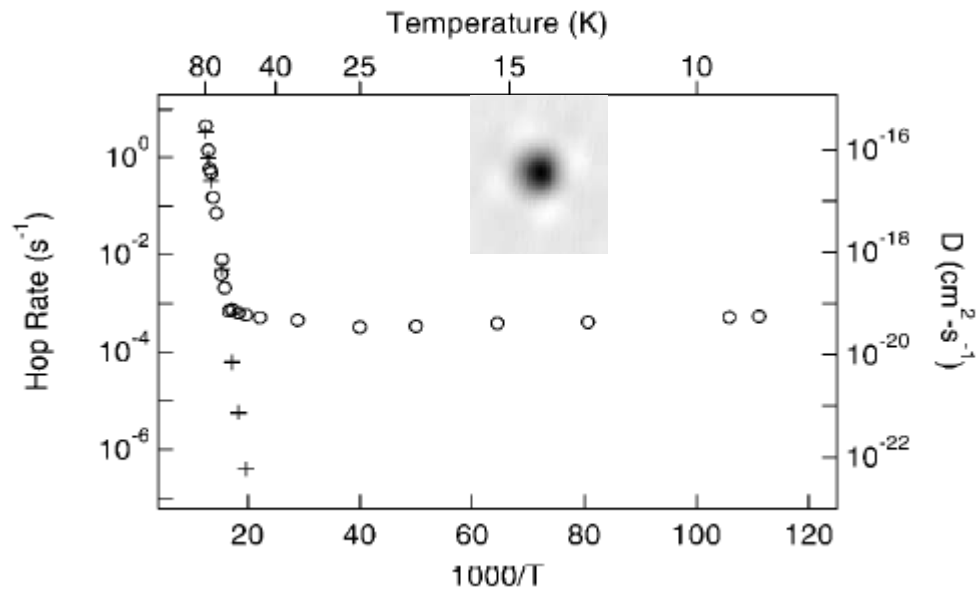
- Observed also on heterosystems [Pt on Ni(110), Ir on Pt(100), Re on Ir(100)]



# Atomistic Mechanisms

## 3) Tunneling mechanism:

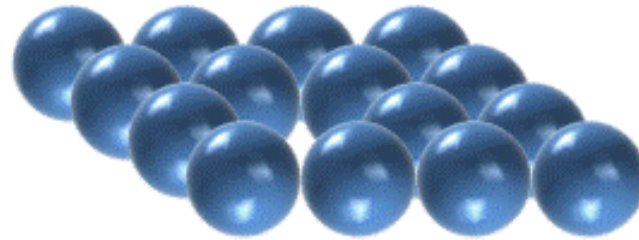
- Diffusing particle with small mass
- Low potential barrier against diffusion
- Example: Hydrogen on Cu(100) (Lauhon & Ho, PRL 2000)



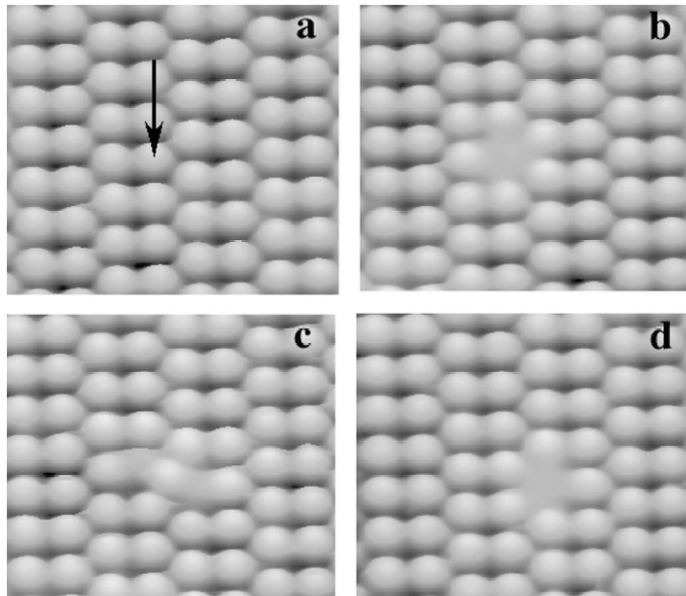
- Above 60 K: Arrhenius law with  $\nu \sim 10^{13} \text{ s}^{-1}$ ,  $E_{diff} = 0.20 \text{ eV}$
- Below 60 K: quantum tunnelling, T independent

# Atomistic Mechanisms

## 4) Vacancy mechanism:



- Example: Ge(111)c(2x8) (Mayne et al., SS 2001)



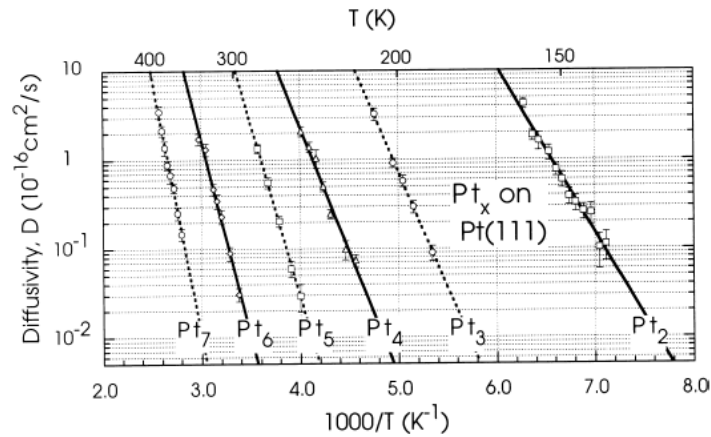
- Vacancy created with the STM tip

- T-activated hopping of neighboring atoms

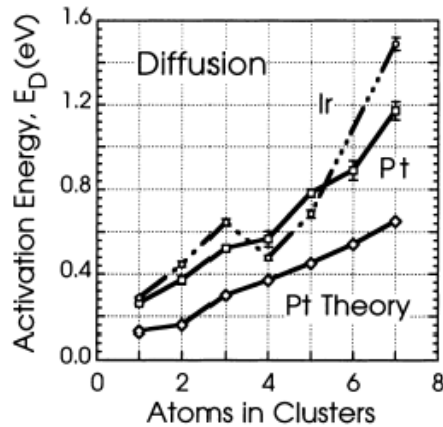
- Heterodiffusion by vacancy-exchange also reported

# Cluster Diffusion

- The larger the cluster, the lower its mobility:

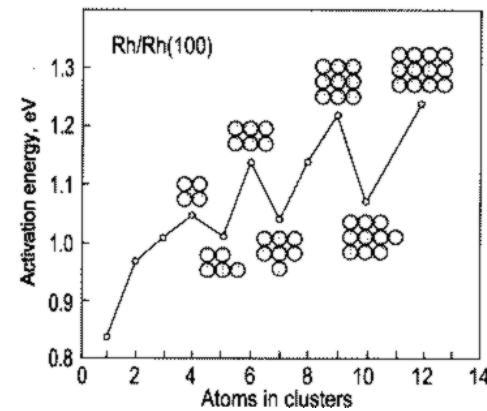


- Activation energy increases with cluster size:



(Kyuno & Ehrlich, SS 1999)

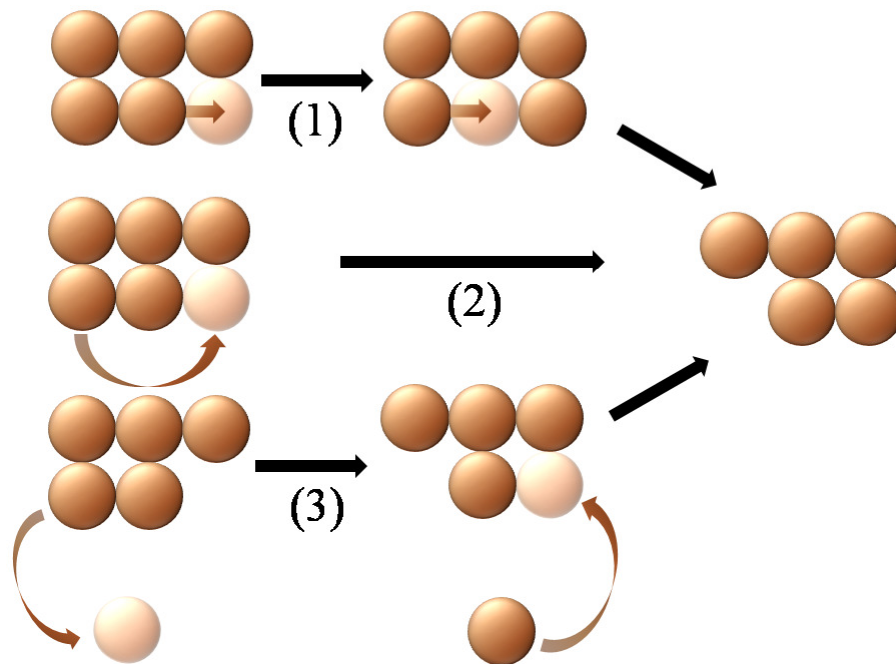
- Compact shapes are less mobile...



(Kellogg, PSS 1996)

# Cluster Diffusion

Individual mechanisms:



(1) Sequential displacement

(2) Edge diffusion

(3) Evaporation-condensation

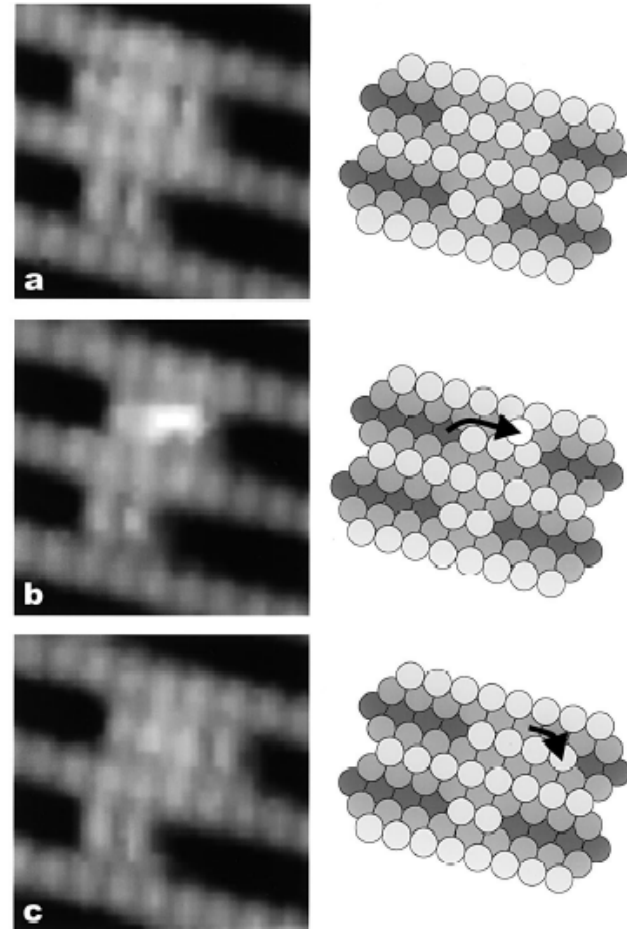
# Cluster Diffusion

## Individual mechanisms:

(4) “Leapfrog” mechanism:



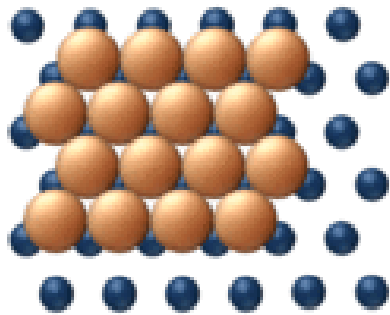
- Example: Pt(110)2x1  
(Linderoth et al., PRL 1999)



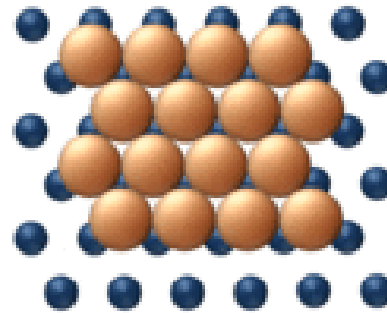
# Cluster Diffusion

## Concerted mechanisms:

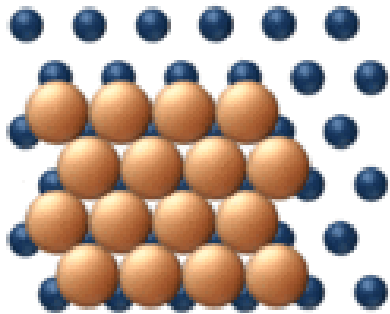
(1) Glide:



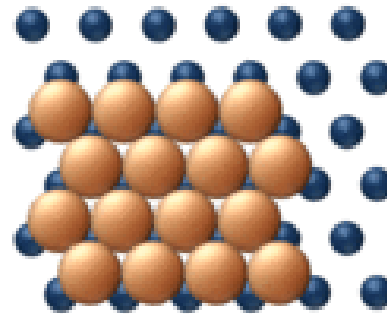
(2) Shear:



(3) Reptation:

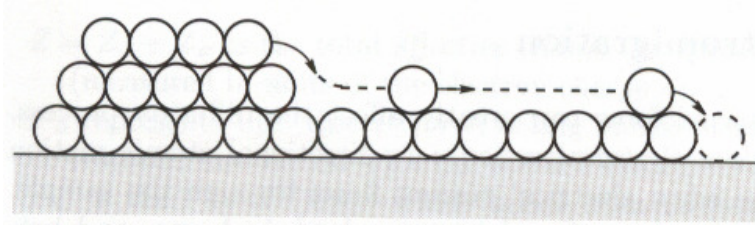


(4) Dislocation:



# Phase Formation

- Coverage  $\Theta \sim 0.1-1$  ML  $\rightarrow$  formation of surface phases
- First layer atoms are usually immobile  $\rightarrow$  “unrolling carpet” mechanism



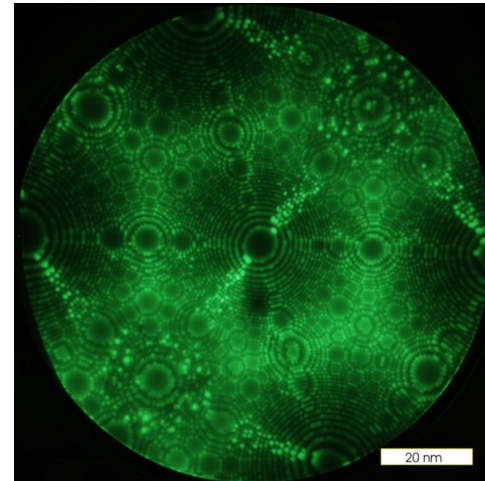
# Surface Electromigration

- Electric current through the sample  $\rightarrow$  Directional atomic motion on the surface
- Self-electromigration  $\rightarrow$  Changes in the step structure
- Hetero-electromigration  $\rightarrow$  Mass transfer towards cathode or anode

# Experimental Techniques

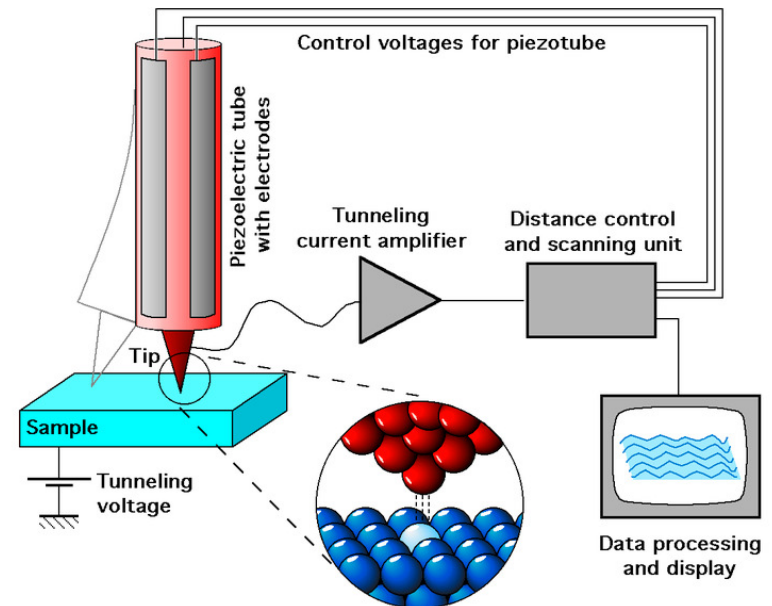
## 1) Direct observation:

- Field ion microscopy (FIM)  
→ “image-anneal-image” technique
  - Limited to refractory or noble metal surfaces



- Scanning tunneling microscopy (STM)  
→ “image-while-hot” technique

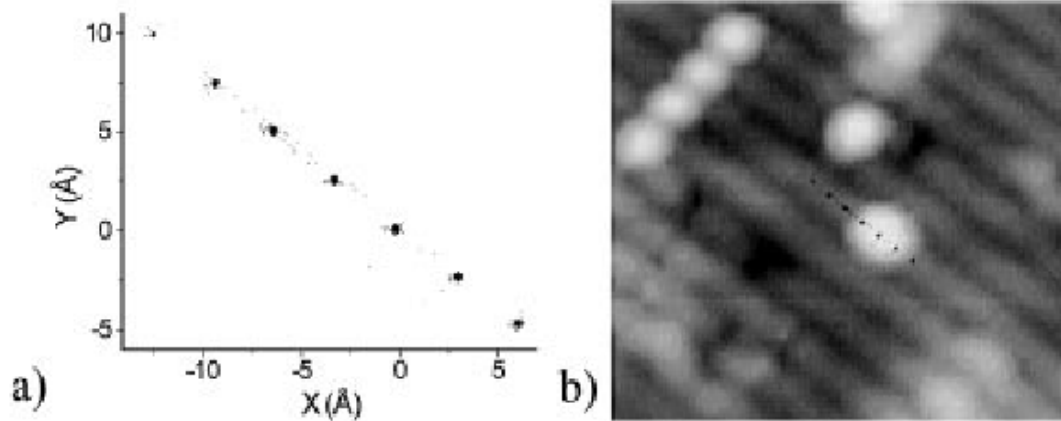
- STM “movies” can be recorded (at 0.01-1 frames per second)





# Experimental Techniques

- Scanning tunneling microscopy (STM) → “atom-tracking” technique
  - STM tip locked onto an adparticle by 2D lateral feedback
  - Example: Si on Si(100) (Swartzentruber, PRL 1996)

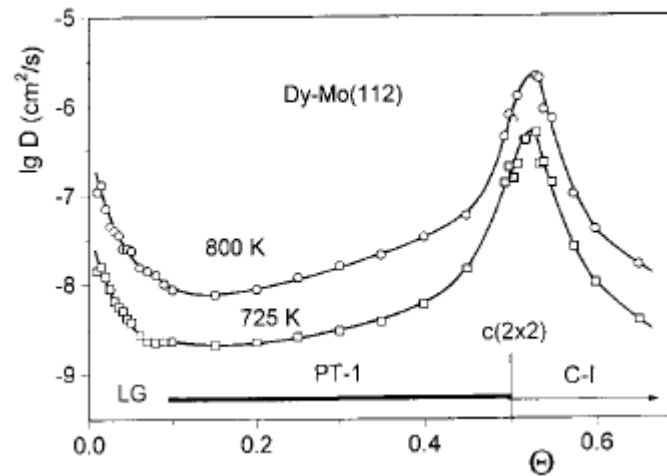
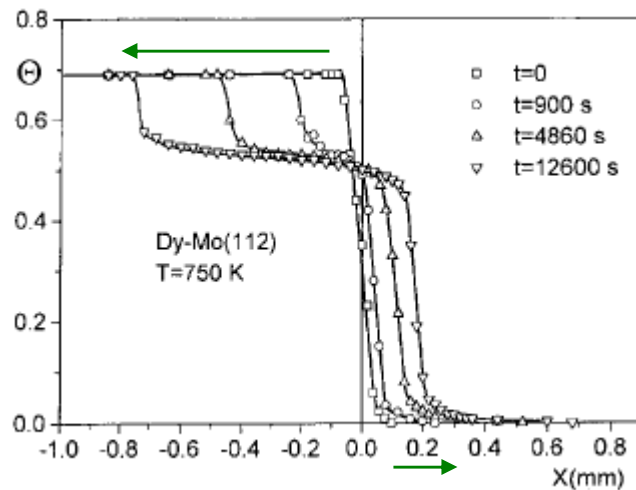


- Electric fields from the STM tip influence surface diffusion!

# Experimental Techniques

## 2) Profile evolution method:

- Smearing of a sharp initial concentration profile is monitored
  - Initial profile deposited using a mask
  - AES, SIMS, SEM or local work-function...
  - $D(\Theta)$  can be evaluated
- Example: Dy on Mo(112) (Loburets et al., SS 1998)

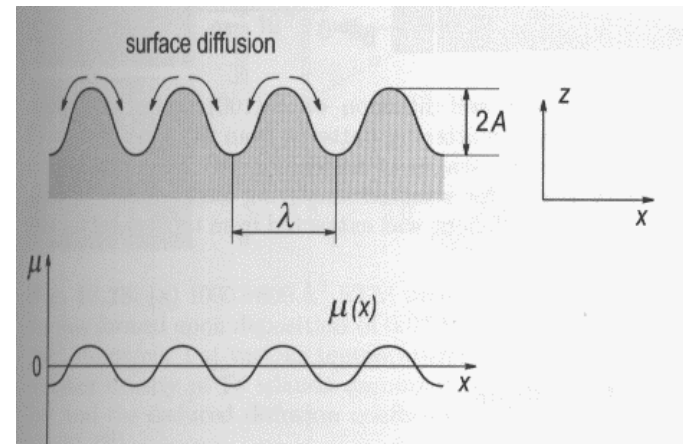


# Experimental Techniques

## 3) “Capillarity” techniques:

- A surface is perturbed from its lowest energy configuration...  
... and allowed to relax via diffusion
- Relaxation rate  $\rightarrow$  Coefficient of diffusion
- For a sinusoidal profile (Mullins, JAP 1999):

$$A(t) = A_0 \exp\left[-\frac{\gamma D n_0 V^2}{k_B T} \left(\frac{2\pi}{\lambda}\right)^4 t\right]$$

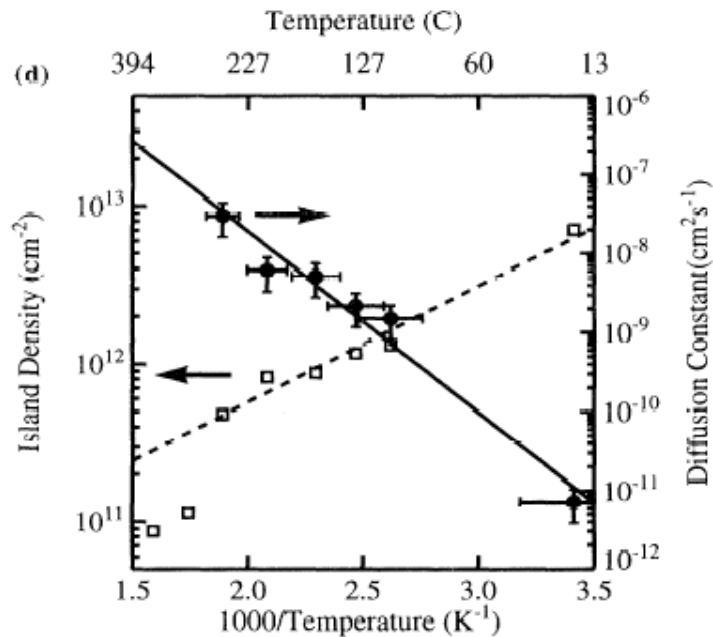
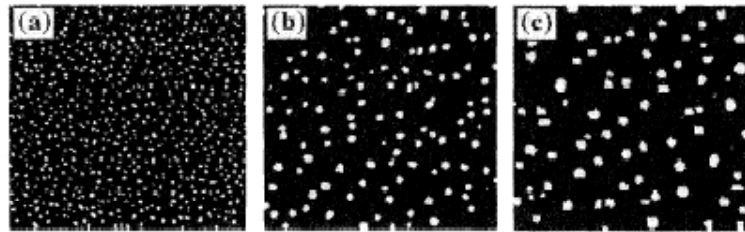


$\gamma$  = surface tension;  $V$  = atomic volume;  $n_0$  = surface density

# Experimental Techniques

## 4) Island growth techniques:

- Number density of islands after submonolayer deposition is monitored
- Example: Fe on Fe(100) (Stroscio et al., PRL 1993)



$$N \propto \left( \frac{R\Theta}{\nu} \right)^{1/3}$$

deposition rate

hopping rate

## Further Reading

- K. Oura et al., Surface Science, Springer 2003, chapter 13
- A.G. Naumovets & Yu.S. Vedula, Surf. Sci. Rep. 4 (1985) 365
- R. Gomer, Rep. Prog. Phys. 53 (1990) 917
- G.L. Kellogg, Surf. Sci. Rep. 21 (1994) 1