Solutions of the Test Exam

Fall Semester 2019

1 Oscillation (10 points)

(a) The period (= duration of an oscillation) can be read from the graph:

$$T = 2.5\,\text{s}\ \textbf{(1 point)}$$

(b) For the angular frequency, the following applies:

$$\begin{aligned}\omega &= \frac{2\pi}{T}\ \textbf{(1 point)}\\ &= \frac{2\pi}{2.5\ \text{s}} = \frac{4\pi}{5}\,\text{s}^{-1} = 2.5\,\text{s}^{-1}\ \textbf{(1 point)}\end{aligned}$$

(c) The amplitude (= deflection of an oscillation) can be read from the graph:

$$A = 5\,\text{cm}\ \textbf{(1 point)}$$

(d) For the position as a function of time applies:

$$\begin{aligned}y(t) &= A \cdot \sin(\omega \cdot t + \varphi_0)\ \textbf{(1 point)}\\ &= 5\,\text{cm} \cdot \sin(\frac{4\pi}{5}\,\text{s}^{-1} \cdot t + \pi)\\ &= -5\,\text{cm} \cdot \sin(\frac{4\pi}{5}\,\text{s}^{-1} \cdot t)\ \textbf{(1 point)}\end{aligned}$$

(e) The velocity $v(t)$ is the temporal derivative of the position:

$$\begin{aligned}v(t) &= \frac{dy(t)}{dt}\ \textbf{(0.5 points)}\\ &= -4\pi\,\frac{\text{cm}}{\text{s}} \cdot \cos(\frac{4\pi}{5}\,\text{s}^{-1} \cdot t)\ \textbf{(1.5 points)}\end{aligned}$$

(f) The maximum value of the velocity is the prefactor of $v(t)$, since

$$|\cos_{max}| = 1\ \textbf{(1 point)}$$

Therefore:

$$|v_{max}| = 4\pi\,\frac{\text{cm}}{\text{s}} = 12.6\,\frac{\text{cm}}{\text{s}}\textbf{(1 point)}$$

2 Velocity (6 points)

(a) The total path of the car is composed of two sections:

1. the path which the car drives during the reaction time t_r
2. the path during breaking

Path 1 is a uniform motion:

$$\begin{aligned} v &= \frac{s}{t} \\ \Rightarrow s &= vt_r \quad \textbf{(1 point)} \\ &= 30\ \text{m/s} \cdot 0.8\,\text{s} \\ &= 24\,\text{m} \end{aligned}$$

The car is 90 m - 24 m = 66 m away from the truck. **(1 point)**

(total 2 points)

(b) Path 2 is a uniform accelerated motion:

$$s = \frac{a}{2}t^2 \quad \textbf{(1 point)}$$

The time till the car is standing still is:

$$t = \frac{v}{a}$$

put in the formula for s:

$$\begin{aligned} s &= \frac{v^2}{2a} \quad \textbf{(1 point)} \\ &= 72.6\,\text{m} \quad \textbf{(1 point)} \end{aligned}$$

From this it follows that the total stopping distance is 96.6 m. Since the distance to the truck was just 90 m, the car does not manage to stop before reaching the truck. **(1 point)**

(total 4 Points)

3 Gymnastics hoop (8 points)

(a) The kinetic energy is:

$$\begin{aligned} E_{\text{kin}} &= \frac{1}{2}mv^2 \quad \textbf{(1 point)} \\ &= \frac{1}{2} \cdot 0.5 \text{ kg} \cdot (6 \text{ m/s})^2 \\ &= 9 \text{ J} \quad \textbf{(1 point)} \end{aligned}$$

(total 2 points)

(b) The rotational energy of the hoop is:

$$\begin{aligned} E_{\text{rot}} &= \frac{1}{2}J\omega^2 \quad \textbf{(1 point)} \\ &= \frac{1}{2} \cdot mr^2 \cdot (\frac{v}{r})^2 \\ &= \frac{1}{2}mv^2 \quad \textbf{(1 point)} \\ &= \frac{1}{2} \cdot 0.5 \text{ kg} \cdot (6 \text{ m/s})^2 \\ &= 9 \text{ J} \quad \textbf{(1 point)} \end{aligned}$$

(total 3 points)

(c) The total energy must be equated with the potential energy to get the height:

$$\begin{aligned} E_{\text{tot}} &= mgh \\ h &= \frac{E_{\text{kin}} + E_{\text{rot}}}{mg} \quad \textbf{(1 point)} \\ h &= 3.67\,\text{m} \quad \textbf{(1 point)} \end{aligned}$$

Then, the sine can be used to calculate the distance of the hoop:

$$s = \frac{h}{\sin 10°} = \frac{3.67 \text{ m}}{\sin 10°} = 21.13 \text{ m} \quad \textbf{(1 point)}$$

(total 3 points)

4 Mixed (8 points)

(a) **(i)** Since the density of fresh water is smaller, it must be $d_{fresh} > d_{sea}$ **(1 point)**

(ii) The ship with the cross-section area A enters d_{sea} in the seawater. The mass of the ship is:

$$m = \rho_{sea} \cdot A \cdot d_{sea} \quad \textbf{(1 point)}$$

with ρ_{sea} the density of seawater. After unloading, the ship enters as deep in fresh water as before(with the load) in seawater. It has now the following mass:

$$m - \Delta m = \rho_{fresh} \cdot A \cdot d_{sea} \quad \textbf{(1 point)}$$

From the first equation it follows

$$A \cdot d_{sea} = m/\rho_{sea}$$

inserted into the second equation, it follows:

$$m - \Delta m = \frac{\rho_{SW} \cdot m}{\rho_{MW}} \quad \textbf{(1 point)}$$

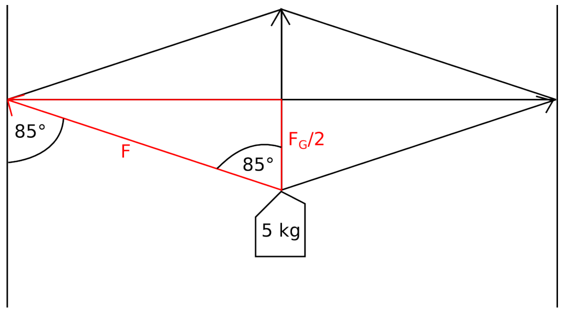
Therefore:

$$m = \frac{\Delta m}{1 - \frac{\rho_{SW}}{\rho_{MW}}} = 2.06 \cdot 10^7\,\text{kg} \quad \textbf{(1 point)}$$

(b) The gravitational force is:

$$F_G = mg = 49.05\,\text{N} \quad \textbf{(1 point)}$$

The adjacent side of the red triangle in the figure corresponds to half of the gravitational force. From this it follows:



$$\cos 85° = \frac{\frac{F_G}{2}}{F}$$

$$\Rightarrow F = \frac{\frac{F_G}{2}}{\cos 85°} \quad \textbf{(1 point)}$$

$$= 281.4\,\text{N} \quad \textbf{(1 point)}$$

5 Warming up water (4 points)

(a) The needed energy is:

$$\begin{aligned}\Delta E &= (m_{cup}c_{quartz} + m_{water}c_{water}) \cdot \Delta T \quad \textbf{(1 point)} \\ &= (0.2\,\text{kg} \cdot 710\,\frac{\text{J}}{\text{kg} \cdot \text{K}} + 0.2\,\text{kg} \cdot 4182\,\frac{\text{J}}{\text{kg} \cdot \text{K}}) \cdot 70\,\text{K} \\ &= 68.49\,\text{kJ} \quad \textbf{(1 point)}\end{aligned}$$

(b) The usable power of the microwave oven ($P = 600$ W) can be specified depending on the heat energy and time:

$$P = \frac{\Delta E}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

Thus, the duration Δt is:

$$\begin{aligned}\Delta t &= \frac{\Delta E}{P} \quad \textbf{(1 point)} \\ &= \frac{(0.2\,\text{kg} \cdot 710\,\frac{\text{J}}{\text{kg}\cdot\text{K}} + 0.2\,\text{kg} \cdot 4182\,\frac{\text{J}}{\text{kg}\cdot\text{K}}) \cdot 70\,\text{K}}{600\,\text{W}} \\ &= 114.1\,\text{s} \quad \textbf{(1 point)}\end{aligned}$$

(total 4 points)

total score: 36 points